



W:H:HUTCHINSON.

AN
ACCOUNT
OF
Sir ISAAC NEWTON'S
Philosophical Discoveries,
In FOUR BOOKS.

At a Meeting of the ROYAL SOCIETY,

June 9, 1748.

IMPRIMATUR.

M. FOLKES, Pr. R. S.

The author's design, with regard to this work
was to have been to explain only those parts
of Isaac's philosophy, which have been and which
for some time, controverted, which was
probably the reason that his discoveries concern
and colours were scarcely touched on."

Dr Rees. art. optac: vol. 2

A N

A C C O U N T

Math. & Phys.
Lib. Coll. Dublin
O F

Sir ISAAC NEWTON's

Philosophical Discoveries,

In F O U R B O O K S.

B Y

COLIN MACLAURIN, *A. M.*

Late Fellow of the Royal Society, Professor of Mathematics
in the University of EDINBURGH, and Secretary
to the Philosophical Society there.

Published from the Author's Manuscript Papers,

By PATRICK MURDOCH, *M. A.* and *F. R. S.*

T. Hutchinson.

The THIRD EDITION.

L O N D O N :

Printed for J. NOURSE, W. STRAHAN, J. and F. RIVINGTON,
W. JOHNSTON, D. WILSON, T. LOWNDES,
T. CADELL, T. BECKET, W. RICHARDSON, T. LONGMAN,
and W. OTRIDGE.



IN MEMORY
OF THAT JUST VENERATION
WHICH
THE AUTHOR MY DECEASED HUSBAND
ALWAYS EXPRESSED
FOR HIS ROYAL HIGHNESS
THE DUKE,
THIS ACCOUNT OF SIR ISAAC NEWTON'S
PHILOSOPHICAL DISCOVERIES
IS HUMBLY INSCRIBED
TO HIS ROYAL HIGHNESS
BY HIS
MOST OBEDIENT SERVANT,
ANNE MACLAURIN.



Digitized by the Internet Archive
in 2019 with funding from
Wellcome Library

<https://archive.org/details/b30526474>

THE
CONTENTS.

An Account of the Life and Writings of the
Author. i

BOOK I.

Of the method of proceeding in natural philosophy, and the various systems of philosophers.

CHAP. I. *A general view of Sir Isaac Newton's method, and of his account of the system of the world* 3

CHAP. II. *Of the systems of the ancient philosophers* 25

CHAP. III. *Of the modern philosophers before Des Cartes* 43

CHAP. IV. *Of the philosophical principles of Des Cartes, the emendations of his followers, and the present controversies in natural philosophy* 66

The CONTENTS.

CHAP. V. <i>Conclusions from the foregoing observations</i>	94
---	----

B O O K II.

Of the theory of motion, or rational mechanics.

CHAP. I. <i>Of space, time, matter, and motion</i>	102
--	-----

CHAP. II. <i>Of the laws of motion, and their general corollaries</i>	118
---	-----

CHAP. III. <i>Of the mechanical powers</i>	154
--	-----

CHAP. IV. <i>Of the collision of bodies</i>	192
---	-----

CHAP. V. <i>Of the motion of projectiles in vacuo; of the cycloid, and the motion of a pendulum in it</i>	207
---	-----

B O O K III.

Gravity demonstrated by analysis.

CHAP. I. <i>Of the theory of gravity as far as it appears to have been known before Sir Isaac Newton</i>	232
--	-----

CHAP. II. <i>The moon is a heavy body, and gravitates towards the earth in the same manner as terrestrial bodies</i>	254
--	-----

CHAP.

The C O N T E N T S.

CHAP. III. *Of the solar system: and the parallaxes of the planets and fixed stars* 268

CHAP. IV. *Of the general gravitation of matter* 288

CHAP. V. *Of the quantity of matter, and density, of the sun and planets* 303

B O O K IV.

The effects of the general power of gravity deduced synthetically.

CHAP. I. *Of the centre of the solar system* 310

CHAP. II. *Shewing how gravity produces some small irregularities in the motions of the planets* 315

CHAP. III. *Of the approach and recess of the planets to and from the sun, in every revolution* 320

CHAP. IV. *Of the motion of the moon* 338

CHAP. V. *Of the path of a secondary planet upon an immoveable plane; with an illustration of Sir Isaac Newton's account of the motions of the satellites, from the theory of gravity* 352

CHAP. VI. *Of the figure of the earth, and the precession of the equinoxes* 362

The C O N T E N T S.

CHAP. VII. *Of the ebbing and flowing of the sea* 368

CHAP. VIII. *Of the comets* 387

CHAP. IX. *Of the Supreme Author and Governor of the Universe, the True and Living God* 396



A N
A C C O U N T
O F T H E
L I F E and W R I T I N G S
O F T H E
A U T H O R.

COLIN MACLAURIN was descended of an ancient family, which had been long in possession of the island of *Tirrie*, upon the coast of *Argyleshire*. His grandfather, *Daniel*, removing to *Inverara*, greatly contributed to restore that town, after it had been almost entirely ruined in the time of the civil wars; and, by some memoirs which he wrote of his own times, appears to have been a person of worth and superior abilities. *John*, the son of *Daniel*, and father of our author, was minister of *Glenderule*; where he not only distinguished himself by all the virtues of a faithful and diligent pastor, but has left, in the register of his provincial synod, lasting monuments of his talents for business, and of his public spirit. He was likewise employed by that synod in completing the version of the Psalms into *Irish*, which is still used in those parts of the country where divine service is performed in that language. He married a gentleman of the family of *Cameron*, by whom he had three sons; *John*, who is still living, a learned and pious divine, one of the ministers of the city of *Glasgow*; *Daniel*, who died young, after having given proofs of a most extraordinary genius; and *Colin*, born at *Kilmoddan* in the month of *February* 1698.

His father died six weeks after ; but that loss was in a good measure supplied to the orphan family, by the affectionate care of their uncle Mr. *Daniel Maclaurin*, minister of *Kilfinnan*, and by the virtue and prudent œconomy of Mrs. *Maclaurin*. After some stay in *Argyleshire*, where her sisters and she had a small patrimonial estate, she removed to *Dumbarton*, for the more convenient education of her children : but dying in 1707, the care of them devolved entirely to their uncle.

In 1709 *Colin* was sent to the university of *Glasgow*, and placed under the care of one of the best men, and most eminent professors of this age, the learned Mr. *Gershom Carmichael*. Here he continued five years, applying himself to his studies with that success which might be expected from parts like his, cultivated with the most indefatigable care and diligence. We find, amongst his oldest manuscripts, fragments of a diary in which he kept an account of every day, and of almost every hour of the day ; of the beginning and success of every particular study, enquiry or investigation : of his conversations with learned men, the subjects of them, and the arguments on either side. Here we read the names of Professor *Carmichael*, the celebrated Mr. *Robert Simson*, Dr. *Johnston*, and several other gentlemen of learning and worth ; who all vied who should most encourage our young philosopher, by opening to him their libraries, and admitting him into their most intimate society and friendship. He could not, afterwards, find time to keep so formal a register of his life, but we are assured the habit never left him ; and that every hour of it was continually filled up with something which he could review with pleasure.

His genius for mathematical learning discovered itself so early as at twelve years of age, when, having accidentally met with a copy of *Euclid* in a friend's chamber, in a few days he became master of the first six books without any assistance : and thence, following his natural bent, made such a surprizing progress, that very soon after we find him engaged in the most curious and difficult problems. Thus much is certain, that in his sixteenth year, he had already
invented

invented many of the propositions afterwards published under the title of *Geometria Organica*.

In the fifteenth year of his age he took his degree of Master of Arts, with great applause; on which occasion he composed and publicly defended a *Thesis* on the power of gravity: and after having spent a year in the study of divinity, he quitted the university, and lived, for the most part, in an agreeable country retirement at his uncle's house, till near the end of 1717. In this retirement, he pursued his studies with the same assiduity as he had done at the university; continuing his favourite researches in mathematics and philosophy, and at other times reading the best classic authors; for which he naturally had an exceeding good taste.

In the intervals of his studies, the lofty mountains amidst which he lived would often invite him abroad, to consider the numberless natural curiosities they contain, and the infinite variety of plants that grow on them; or to climb to their tops, and enjoy the most extensive and most diversified prospects. And here his fancy being warmed by the grand scenes which presented themselves, he would sometimes break out into a hymn or poetic rhapsody on the beauties of nature, and the perfections of its Author. Of these some fragments still remain; which, though so unfinished that it can be only thro' forgetfulness they have not been destroyed, yet shew a genius capable of much greater things in that way. His friends, however, are obliged to the accidents that have preserved them, together with some others of his juvenile performances; for however unfit they may be for the public view, they shew the progress he had made in the several parts of learning, at the time they were written: and what can be more delightful, than to observe the gradual openings and improvements of a mind like that of Mr. *Maclaurin*?

In the autumn of 1717, he presented himself a candidate for the professorship of mathematics in the marishal college of *Aberdeen*, which he obtained after a comparative trial of ten days with a very able competitor: and being fixed in his chair, he soon revived the taste of mathematical

cal learning, and raised it higher than it had ever been in that university.

During the vacations of 1719 and 1721, he went to *London*, with a view of improving himself, and of being introduced to the illustrious men there. In his first journey, besides Dr. *Hoadly* then bishop of *Bangor*, Dr. *Samuel Clarke*, and several other eminent men, he became acquainted with Sir *Isaac Newton*; whose friendship he ever after reckoned the greatest honour and happiness of his life. He was admitted a member of the *Royal Society*; two papers of his were inserted in their transactions, and his book intitled *Geometria Organica* was published with the approbation of their president.

In his second journey to *London* in 1721, he became acquainted with *Martin Folkes*, Esq; now president of the *Royal Society*; with whom he thenceforth cultivated a most entire and unreserved friendship, frequently interchanging letters with him, and communicating all his views and improvements in the sciences.

In 1722, Lord *Polwarth*, Plenipotentiary of the King of *Great Britain* at the congress of *Cambray*, engaged Mr. *Maclaurin* to go as tutor and companion to his eldest son, who was then to set out on his travels.

After a short stay at *Paris*, and visiting some other towns in *France*, they fixed in *Lorrain*; where, besides the advantage of a good academy, they had that of the conversation of one of the most polite courts in *Europe*. Here Mr. *Maclaurin* gained the esteem of the most distinguished persons of both sexes, and at the same time quickly improved that easy genteel behaviour which was natural to him, both from the temper of his mind, and from the advantages of a graceful person.

It was here likewise that he wrote his piece on the percussion of bodies, which gained the prize of the *Royal Academy of Sciences* for 1724; the substance of this tract is inserted in his *Treatise of Fluxions*, and also in *Book II. Chap. 2.* of the following work.

Mr.

Mr. *Maclaurin* and his pupil having quitted *Lorrain*, were got as far on their tour as the southern provinces of *France*, when Mr. *Hume* was seized with a fever, and died at *Montpelier*. An event so shocking must have affected a heart less sensible and tender than Mr. *Maclaurin*'s: in some letters written on this occasion, he appears quite inconsolable. His own grief for his pupil, his companion, and friend; and his sympathy with a family to which he owed great obligations, and which had suffered an irreparable loss in the death of this hopeful young nobleman, rendered him altogether unhappy. Travelling and every thing else was become distasteful, so he set out immediately on his return to his profession at *Aberdeen*.

But being now universally distinguished as one of the first genius's of the age, some of the curators of the university of *Edinburgh*, were desirous of engaging him to supply the place of Mr. *James Gregory*, whose age and infirmities had rendered him incapable of teaching. Several difficulties retarded this design for some time; particularly the competition of a gentleman eminent for mathematical abilities, who had good interest with the patrons of the university; and the want of an additional fund for the new professor. But both these difficulties were got over, upon the receipt of two letters from Sir *Isaac Newton*. In one, addressed to Mr. *Maclaurin*, with allowance to shew it to the patrons of the university, Sir *Isaac* expresses himself thus; "I am very glad to hear that you have a
" prospect of being joined to Mr. *James Gregory* in the
" professorship of the mathematics at *Edinburgh*, not
" only because you are my friend, but principally because
" of your abilities, you being acquainted as well with the
" new improvements of mathematics, as with the former
" state of those sciences; I heartily wish you good suc-
" cess, and shall be very glad of hearing of your being
" elected. I am, with all sincerity, your faithful friend
" and most humble servant."

In a second letter to the then Lord Provost of *Edinburgh*, which Mr. *Maclaurin* knew nothing of till some years after Sir *Isaac*'s death, he thus writes, "I am glad
to

“ to understand that Mr. *Maclaurin* is in good repute
 “ amongst you for his skill in mathematics, for I think
 “ he deserves it very well; and to satisfy you that I do
 “ not flatter him, and also to encourage him to accept the
 “ place of assisting Mr. *Gregory*, in order to succeed him,
 “ I am ready (if you please to give me leave) to contri-
 “ bute twenty pounds *per annum* towards a provision for
 “ him, till Mr. *Gregory*’s place become void, if I live so
 “ long, and I will pay it to his order in *London*.”

In *November* 1725, he was introduced into the university: as was at the same time his learned colleague and intimate friend, Dr. *Alexander Monro*, professor of anatomy. After this the mathematical classes soon became very numerous, there being generally upwards of a hundred young gentlemen attending his lectures every year: who being of different standings and proficiency, he was obliged to divide them into four or five classes, in each of which he employed a full hour every day, from the first of *November* to the first of *June*.

In the first or lowest class (sometimes divided into two) he taught the first six books of *Euclid*’s Elements, plain trigonometry, practical geometry, the elements of fortification, and an introduction to algebra. The second class studied algebra, the 11th and 12th books of *Euclid*, spherical trigonometry, conic sections, and the general principles of astronomy. The third class went on in astronomy and perspective, read a part of Sir *Isaac Newton*’s *Principia*, and had a course of experiments for illustrating them, performed and explained to them. He afterwards read and demonstrated the elements of fluxions. Those in the 4th class read a system of fluxions, the doctrine of chances, and the rest of *Newton*’s *Principia*.

All Mr. *Maclaurin*’s lectures on these different subjects were given with such perspicuity of method and language, that his demonstrations seldom stood in need of repetition: such, however, was his anxiety for the improvement of his scholars, that if at any time they seemed not fully to comprehend his meaning, or if, upon examining them, he found they could not readily demonstrate the propo-
 tions

tions which he had proved, he was apt rather to suspect his own expressions to have been obscure, than their want of genius or attention; and therefore would resume the demonstration in some other method, to try if, by exposing it in a different light, he could give them a better view of it.

Besides the labours of his public profession, he had frequently many other employments and avocations. If an uncommon experiment was said to have been made anywhere, the curious were desirous of having it repeated by Mr. *Maclaurin*: if an eclipse or comet was to be observed, his telescopes were always in readiness. The ladies too would sometimes be entertained with his experiments and observations; and were surprized to find how easily and familiarly he could resolve the questions they put to him. His advice and assistance, especially to the young gentlemen who had been his pupils, was never wanting; nor was admittance refused to any, except in his teaching hours, which were kept sacred. His acquaintance and friendship was likewise courted by the ingenious of all ranks; who, by their fondness for his company, took up a great deal of his time, and left him not master of it, even in his country retirements. Notwithstanding the necessary labour and the many interruptions and avocations which he had, he continued to pursue his own studies with the utmost assiduity, reading whatever was published, from which he could expect any information or improvement. But to have time for so much study and writing, he was obliged to take from the ordinary hours of sleep, what he bestowed on his scholars and friends; and, by this, no doubt, greatly impaired his health.

Sir *Isaac Newton* dying in the beginning of the year 1728, his nephew Mr. *Conduitt* proposed to publish an account of his life, and desired Mr. *Maclaurin*'s assistance; who, out of gratitude to his great benefactor, cheerfully undertook and soon finished the history of the progress which philosophy had made before Sir *Isaac*'s time. This was the first draught of the following work; which was immediately sent up to *London*, and had the approbation of some of the best judges. Dr. *Rundle*, in particular, af-

terwards

terwards bishop of *Derry*, was so pleased with the design, that he mentioned it to her late Majesty ; who did it the honour of a reading, and expressed a desire to see it published. But Mr. *Conduitt*'s death having prevented the execution of his part of the proposed work, Mr. *Maclaurin*'s manuscript was returned to him. To this he afterwards added the more recent proofs and examples, given by himself or others, on the subjects treated of by Sir *Isaac*, and left it in the state in which it now appears.

Mr. *Maclaurin* had lived a bachelor to the year 1733 : but being formed for society as well as for contemplation, and desirous of mixing more delicate and interesting delights with those of philosophy, he married *Anne*, daughter of Mr. *Walter Stewart*, solicitor-general to his late Majesty for *Scotland* ; by whom he had seven children, of which, two sons *John* and *Colin*, and three daughters, have survived him.

Dr. *Berkley* bishop of *Cloyne*, having taken occasion from some disputes that had arisen concerning the grounds of the fluxionary method, in a treatise entitled the *Analyst*, published in 1734, to explode the method itself, and, at the same time, to charge mathematicians in general with infidelity in religion ; Mr. *Maclaurin* found it necessary to vindicate his favourite study, and repel an accusation in which he was most unjustly included. He began an answer to the bishop's book ; but as he proceeded, so many discoveries, so many new theories and problems occurred to him, that instead of a vindictory pamphlet, his work came out a complete system of fluxions, with their application to the most considerable problems in geometry and natural philosophy.

This work was published at *Edinburgh* in 1742, in two volumes in quarto ; in which we are at a loss what most to admire, his solid and unexceptionable demonstrations of the grounds of the method itself, or its application to such a variety of curious and useful problems.

His demonstrations had been, several years before, communicated to Dr. *Berkley*, and Mr. *Maclaurin* had treated him with the greatest personal respect and civility: notwithstanding which, in his pamphlet on tar-water, he renews the charge, as if nothing had been done; for this excellent reason, that different persons had conceived and expressed the same thing in different ways.

A society having subsisted some years at *Edinburgh* for improving medical knowledge, Mr. *Maclaurin* proposed to have their plan made more extensive, so as to take in all the parts of physics, together with the antiquities of the country. This was readily agreed to; and Mr. *Maclaurin*'s influence engaged several noblemen and gentlemen of the first rank and character, to join themselves, for that purpose, to the members of the former society. The Earl of *Morton* did them the honour to accept of the office of president; Dr. *Plummer*, professor of chymistry, and Mr. *Maclaurin* were appointed secretaries; and several gentlemen of distinction, *English* and foreigners, desired to be admitted members.

At the monthly meetings of the society, Mr. *Maclaurin* generally read some performance or observation of his own, or communicated the contents of his letters from foreign parts; by which means the society was informed of every new discovery or improvement in the sciences.

Several of the papers read before this society, are printed in the 5th and 6th volumes of the *Medical Essays*. Some of them are likewise published in the *Philosophical Transactions*, and Mr. *Maclaurin* had occasion to insert a great many more in his Treatise of Fluxions, and in his account of Sir *Isaac Newton*'s philosophy. By which means the publication of any volume of the works of the society has been retarded: but we may hope their labours will still be continued with success, notwithstanding the loss they have sustained by Mr. *Maclaurin*'s death.

He

He likewise proposed the building an astronomical *observatory*, and a convenient school for experiments in the university; of which he drew an elegant and well-contrived plan: and as this work was to be carried on by private contributions, employed all his influence to raise money for that purpose; with so much success, that had not the unhappy disorders of that country intervened, the fabric might by this time have been far advanced. The Earls of *Morton* and *Hoptoun* shewed their liberality as well as their love of the sciences, upon this occasion; as did the honourable Baron *Clerk*, vice-president of the *philosophical* society: and several noblemen and gentlemen offered to contribute what instruments of value they were possessed of, as soon as the observatory should be ready to receive them.

The Earl of *Morton* being to set out for *Orkney* and *Shetland* in 1739, to visit his estates there, wanted at the same time to settle the geography of these countries, which is very erroneous in all our maps; to examine their natural history, to survey the coasts, and to take the measure of a degree of the meridian: and, for this purpose, desired Mr. *Maclaurin's* assistance. But his family affairs not permitting him to take such a journey, he could do no more than draw a memorial of what he thought necessary to be observed, furnish the proper instruments, and recommend Mr. *Short*, the famous optician, as a fit operator for managing them.

The account which he received of this voyage, made him still more sensible of the erroneous geography we have of those parts, by which many shipwrecks have been occasioned; and therefore he employed several of his scholars, who were then settled in the northern counties, to survey the coasts.

The reverend Mr. *Bryce* composed from observations a map of the coast of *Caithness* and *Strathnaver*, with remarks on the natural history and rarities of the country, together with directions for sea-faring people. This map was presented to the *Philosophical Society* at *Edinburgh*,
and

and published by their order. The reverend Mr. *Bonnar* drew likewise a map of the three most northerly islands of *Shetland*, which is among Mr. *Maclaurin*'s papers; and we expect soon the geography of the *Orkneys* corrected by Mr. *Mackenzie*. It was from observations like these, made by skilful persons, and with the best instruments, that Mr. *Maclaurin* expected to see a good map of *Scotland*; not from the slavish copying of map-sellers, nor from a painful collecting, and patching together of old draughts and surveys of little authority; which he thought must contribute more to perpetuate than to rectify errors.

Mr. *Maclaurin* had still another scheme for the improvement of geography and navigation, of a more extensive nature. After reading all the accounts he could procure of voyages, both in the south and north seas, he imagined the sea was open all the way from *Greenland* to the south sea, by the north pole. Of this he was so much persuaded, that he has been heard to say, if his situation could admit of such adventures, he would undertake the voyage even at his own charges. But when schemes, for finding out such a passage, were laid before the parliament in 1744, and he was consulted concerning them by several persons of high rank and influence; before he could finish the memorials which he proposed to have sent, the præmium was limited to the discovery of a north-west passage, and Mr. *Maclaurin* used to regret that the word *west* was inserted, because he thought that passage, if at all to be found, must lie not far from the pole.

Such was the zeal of this worthy person for the public good, in every instance; the last, and most remarkable, is that which we are now going to relate.

When it was certainly known, in 1745, that the rebels, after having got between *Edinburgh* and the King's troops, were continuing their march southwards, Mr. *Maclaurin* was among the first to rouse the friends of our happy constitution, from the unlucky security they had hitherto continued in: and tho' he was sensible that the city of *Edinburgh*, far from being able to stand the attack of a regular army, could not even hold out any consider-

b

able

able time against the undisciplined and ill-armed force that was coming against it; yet, as he foresaw of how much advantage it would be to the rebels, to get possession of that capital; and, the King's forces under the command of Sir *John Cope* being daily expected; he made plans of the walls, proposed the several trenches, barricades, batteries, and such other defences as he thought could be got ready before the arrival of the rebels, and by which, he hoped, the town might be kept till the King's forces should come to its relief. The whole burden, not only of contriving, but also of overseeing the execution, of these hasty fortifications fell to Mr. *Maclaurin's* share; he was employed night and day, in making plans, and running from place to place; and the anxiety, fatigue, and cold to which he was thus exposed, affecting a constitution naturally of weak nerves, laid the foundation of the disease of which he died.

How this plan came to be neglected, and in what manner the rebels got possession of the town, is not a proper enquiry for this place. They got possession of it! and, their spirits being raised by this unaccountable success, and by the supply of arms and provisions which it gave them, they soon after defeated the King's troops at *Preston*. The moderation which they had affected before that unhappy battle was now laid aside, and obedience was to be given to whatever proclamations or orders they thought fit to issue, under pain of military execution. Among other despotic orders, one was, commanding all who had been volunteers in defence of the town, before a stated time, to wait on their secretary of state, to subscribe a recantation of what they had done, and a promise of submission to their pretended government, under the pain of being deemed and treated as rebels. Mr. *Maclaurin* had been too active and distinguished a volunteer, to think he could escape the severest treatment, if he fell into their hands, after neglecting to make the submission required; he therefore withdrew privately into *England*, before the last day of receiving the submissions; but, previous to his escape, found means to convey a good telescope into the castle, and concerted a method of supplying the garrison with provisions.

As soon as his Grace, Dr. *Thomas Herring*, then Lord Archbishop of *York*, was informed that Mr. *Maclaurin* had fled to the north of *England*, he invited him in a most friendly and polite manner, to reside with him during his stay in that country. Mr. *Maclaurin* gladly accepted of the invitation, and soon after expresses himself thus in a letter to a friend ; “ Here (says he) I live as happily as a
“ man can do, who is ignorant of the state of his family,
“ who sees the ruin of his country.” His Grace, of whose merit and goodness, Mr. *Maclaurin* ever retained the highest sentiments, afterwards kept a regular correspondence with him ; and when it was suspected that the rebels might once more take possession of *Edinburgh*, after their retreat from *England*, invited his former guest again to take refuge with him.

At *York* he had been observed to be more meagre than ordinary, and with a sickly look ; though not being apprehensive of any danger at that time, he did not call in the assistance of a physician : but having had a fall from his horse on his journey southward, and, when the rebel army marched into *England*, having on his return home been exposed to most tempestuous cold weather, upon his arrival he complained of being much out of order. In a little time his disease was discovered to be a dropsey of the belly, to remove which, variety of medicines, prescribed by the most eminent physicians at *London*, as well as those of *Edinburgh*, and three tappings, were used without making a cure.

His behaviour, during this tedious and painful distemper, was such as became a philosopher and a christian ; calm, chearful, and resigned ; his senses and judgment remaining in their full vigour, till within a few hours of his death. Then, for the first time, his amanuensis to whom he was dictating the last chapter of the following work (in which he proves the wisdom, the power, goodness, and other attributes of the Deity) observed some hesitation or repetition : no pulse could then be felt in any part of his body, and his hands and feet were already cold. Notwithstanding this extremely weak condition, he sat in his
b 2 chair,

chair, and spoke to his friend Dr. *Monro* with his usual serenity and strength of reason, desiring the Doctor to account for a phænomenon which he then observed in himself: flashes of fire seeming to dart from his eyes, while in the mean time his sight was failing, so that he scarce could distinguish one object from another. In a little time after this conversation, he desired to be laid upon his bed; where, on *Saturday* the 14th of *June*, 1746, aged 48 years and 4 months, he had an easy passage from this world to that state of bliss, which he had the most elevated ideas of, and which he most ardently longed to possess.

The grief for the loss of this excellent person was as general as the esteem which he had acquired, with all ranks of men: but those of greatest worth, and who had most intimately known him, were the most deeply affected. Dr. *Monro*, in an oration spoken at the first meeting of the university after Mr. *Maclaurin*'s death (from which the substance of the foregoing account is taken) gives, particularly, a very moving picture of the grief of the late Lord President *Forbes*, on this occasion. A likeness of character, and a perfect harmony of sentiments and views, had closely united them in their lives; in their deaths, they were alas! too little divided: the president likewise, worn out in the service of his country, was soon to be the subject of a general mourning.

In the same discourse the Doctor shews, in a variety of instances, that acute parts and extensive learning were, in Mr. *Maclaurin*, but inferior qualities; that he was still more nobly distinguished from the bulk of mankind, by the qualities of the heart; his sincere love to God and Men, his universal benevolence and unaffected piety; together with a warmth and constancy in his friendships, that was in a manner peculiar to himself. He professes likewise, that after an intimacy with him for so many years, he had but half known his worth; which then only disclosed itself in its full lustre, when it came to suffer the severe test of that distressful situation, in which every man must at last find himself; and which only minds prepared like his, armed with virtue and christian hope, can bear with dignity.

But

But the bounds we are confined to, do not permit us to follow the professor in this delightful track; nor would the modesty of Mr. *Maclaurin*'s surviving friends bear with our being so particular. We must content ourselves to consider him in the character in which he was universally known; by giving a short account of his works, and of the taste and manner in which he cultivated the mathematical sciences; pursuing with such indefatigable pains, studies that seem, to many, rather curious than useful.

His first work, composed in his early youth, was the *Geometria Organica*, in which he treats of the description of curve lines by continued motion. The first and simplest of curves is described by the motion of a right line on a plane, round one of its extremities. Sir *Isaac Newton* had shewn, that the *Conic Sections* might all be described by assuming two centres or poles in a plane, and moving round them two given angles, so as the intersection of two legs be always found in a straight line, given in position in the same plane; for thus the intersection of the other two will trace some conic section. In a similar way, he describes such lines of the third order, as have a double point, that is to say, which returning upon themselves, pass twice through the same point; but the description of the far greater number of those lines, which have no such point, Sir *Isaac* declares to be a problem of much more difficulty. This was reserved for Mr. *Maclaurin*; who not only happily resolved it, but carried the same method of description much higher. By assuming more poles, or by moving the angular points along more lines given in position, or, lastly, by carrying the intersections along curve lines, instead of straight, he has extended, or given hints of extending, the method as far as it can go. And because, by the motion of rulers actually combined, as the case requires, such descriptions may be effected, he calls them by the general name of *Organical*. When he wrote this treatise, the subjects being new and entertaining, his invention in its prime, and the ardor of his curiosity continually urging him on to farther discoveries, he did not take time to finish every demonstration in so elegant a manner as he might have done. His page, we

must own, is incumbered with algebraical calculations, and these have offended the delicate eyes of some critics; but, in answer to this, we may say that what offends them, may be very acceptable to younger students: nor indeed should we at all have mentioned this blemish in so great a work, if himself had not somewhere hinted at it, and, in a letter to one of his friends, expressed an intention of resuming, with his first leisure, that whole theory, and adding to it a *supplement*; the greatest part of which had been printed several years ago, but whereof we have only an abstract in the *Philosophical Transactions*, N^o. 439. In the same volume, he gives a new theory of the curves which may be derived from any given curve, by conceiving perpendiculars to its tangents to be drawn continually through a given point, whose intersections with the tangents will form a new curve; from which last a third may be formed in the same manner, and so on *in infinitum*. This furnishes many curious theorems: there are likewise some propositions concerning centripetal forces and other subjects, which, with the quotations he uses, shew the great progress he had already made in every part of mathematical learning, and how well acquainted he was with the writings of the best authors.

We shall not here repeat what has been said concerning his piece which gained the prize of the *Royal Academy of Sciences* in 1724. In the year 1740, the *Academy* adjudged him a prize which did him still more honour, for accounting for the motion of the *Tides*, from the theory of gravity; a question which had been given out the former year, without receiving any solution. He happened to have only ten days time to draw up this paper, and could not find leisure to transcribe a fair copy, so that the *Paris* edition of it is incorrect; but he afterwards revised the whole, and inserted it in his *Treatise of Fluxions*.

Nor need we mention the occasions on which several pieces which he sent to the *Royal Society* were written: the following list will shew their dates, and the subjects treated of in them.

1. *Of the construction and measure of curves*, Phil. Trans. N^o. 356.
2. *A new method of describing all kinds of curves*, N^o. 359.
3. *A Letter to Martin Folkes, Esq; on equations with impossible roots*, May, 1726. N^o. 394.
4. ——— *Continuation of the same*, March, 1729. N^o. 408.
5. Decem. 21st, 1732. *On the description of curves; with an account of farther improvements, and a paper dated at Nancy, 27th Nov. 1722.* N^o. 439.
6. *An account of the annular eclipse of the sun, at Edinburgh, Feb. 18, 1736-7.* N^o. 447.
7. *An account of the Treatise of Fluxions*, January 27th, 1742-3. N^o. 467.
8. ——— *The same continued*, March 10th, 1742-3. N^o. 469.
9. *A rule for finding the meridional parts of a spheroid with the same exactness as of a sphere*, August 1741. N^o. 461.
10. *Of the bases of the cells wherein the bees deposit their honey*, Novem. 3, 1743. N^o. 471.

But the great work, on which he bestowed the most labour, and which will for ever do him honour, is his *Treatise of Fluxions*.

The occasion of it was related above, namely the objections of some ingenious men against the doctrine of fluxions, on account of the different modes of explication which had been used by different authors. Nor can it be denied, that the terms *infinite* and *infinitesimal* were become much too familiar to mathematicians, and had been abused both in *arithmetic* and *geometry*: At one time in-

roducing and palliating real absurdities, and, at others, giving these sciences an affected mysterious air which does not belong to them. To remedy this growing evil, and for ever take away the handle which it gave to cavilling, Mr. *Maclaurin* found it necessary, in demonstrating the principles of fluxions, to reject altogether those exceptionable terms, and to suppose no other than finite determinable quantities, such as *Euclid* treats of in his geometry; nor to use any other form of demonstration than what the antients had frequently used, and which had been allowed as strictly conclusive from the first rise of the science: by which means he has secured this admirable invention from all future attacks, and at the same time done justice to the accuracy of the great inventor. The work cost him infinite pains; but he did not grudge it: he thought that in proportion “as the general methods are valuable, it
 “is important that they be established above all exception,
 “and since they save us so much time and labour, we
 “may allow the more for illustrating the methods themselves*.”

To his demonstrations of this doctrine he has added many valuable improvements of it, and has happily applied it to so many curious and useful enquiries, that his work may be called a storehouse of mathematical learning, rather than a treatise on one branch of it. The particulars we need not enumerate, especially as there is printed in the *Philosophical Transactions*, N^o. 468, 469, a clear and methodical account of them; to which we refer the reader.

Throughout this whole work, though not equally perfect in all its parts, because of the infinite extent of the field into which he was led, there appears a very masterly genius, and an uncommon address.

An ordinary artist follows the first, not generally the best, road that presents itself, and arrives perhaps at the solution of his problem; but it will scarcely be either elegant or clear; one may see there is still something want-

* *Introd. to Fluxions*, at the end.

ing, the result being little more scientific than that of an arithmetical operation, where the given numbers and their relations have all disappeared. This was not the case of Mr. *Maclaurin*; he had a quick comprehensive view, taking in at once all the means of investigation; he could select the fittest for his purpose, and apply them with exquisite art and method. This is a faculty not to be acquired by exercise only; we ought rather to call it a species of that taste, the gift of nature, which in mathematics, as in other things, distinguishes excellence from mediocrity.

We have in all Mr. *Maclaurin*'s latter works, especially in his *Treatise of Fluxions*, numberless instances of this address: We need only instance in his reducing so many solutions which used to be managed by the higher orders of fluxions to those of an inferior order, and many of the questions concerning the *maxima* and *minima*, even some of the most difficult, to plane geometry.

These are all the writings which our author lived to publish; since his decease two volumes more have appeared, his treatise of *Algebra*, and this account of Sir *Isaac Newton*'s philosophy.

His *Algebra*, tho' it had not the advantage to be finished by his own hand and published under his eye, is yet allowed to be excellent in its kind; containing, in no large volume, a complete elementary treatise of that science, as far as it has hitherto been carried; all the most useful rules, which lie scattered in so many authors, being clearly laid down and demonstrated, and in the order which he had found to be the best in a long course of methodical teaching. He is more sparing, it is true, in the practical applications than most other writers, but this was designedly; he was of opinion that many of those applications deserve to be treated of apart; and to have taken too much of them into his plan, would have been like disfiguring the elements of *Euclid*, by mixing with them the rules of practical geometry. To this work is subjoined, as a proper appendix, his Latin tract *De Linearum Geometricarum proprietatibus generalibus*. It is carefully printed from a

manuscript all written and corrected by the author's own hand; and we need only add, that as it was among the last, so it appears to have been, in his own judgment, one of the best of his performances.

The account of Sir *Isaac Newton's* philosophy lies now before the reader; who, by casting his eye on the *table of contents*, may see the author's design and method; and in perusing the work itself will not, we hope, find himself disappointed.

One question however may be put, which it is proper for us to obviate. Why, in this account, Sir *Isaac Newton's* grand discoveries concerning light and colours, are but transiently and in general touched upon? To this it is answered, that our author's main design seems to have been to explain only those parts of Sir *Isaac's* philosophy that have been, and are still, controverted. But it is known, that, ever since the experiments, on which his doctrine of light and colours is founded, have been repeated with due care, this doctrine has suffered no contestation: Whereas his system of the world, his accounting for the celestial motions, and the other great appearances of nature, from gravity, is misunderstood and even ridiculed to this day: the weak charge of *occult qualities* has been frequently repeated; foreign professors still amuse themselves with imaginary triumphs; even the polite and ingenious Cardinal *de Polignac* is seduced to lend them the harmony of his numbers.

It was proper therefore that these gentlemen should once more be told (and by Mr. *Maclaurin*) that their objections are altogether out of season; that the spectres they are daily combating are a creation of their own, no more related to Sir *Isaac Newton's* doctrines than observation and experience are to occult qualities; that the followers of Sir *Isaac Newton* will for ever assert their right to stop where they find they can get no farther upon sure ground; and to make use of a principle firmly established in experience, adequate to all the purposes they apply it to, and in every application uniform and consistent with itself;

itself *; although they perhaps, despair of tracing the ulterior cause of that principle.

But besides that Sir *Isaac Newton's* treatise of optics wanted no defence, it may be said likewise, that it scarce admits of an explication; it is such an absolute masterpiece of philosophical writing, that it can as little be abridged as enlarged; and we had better take all his experiments, illustrations and proofs in the words in which he has delivered them, than risque the injuring them by a different dress. As for the hints which he could not further pursue, and which he proposes as queries; Mr. *Maclaurin* had too sound a judgment, and had too thoroughly imbibed the genius and spirit of his great Master, to run away with them as materials for rearing doubtful theories: He leaves them as he found them, till future discoveries can give them another name.

Besides his printed and more finished works, Mr. *Maclaurin* had by him a number of manuscript papers, and imperfect essays on mathematical and other subjects. These the increase of his distemper did not give him time to put in order, or to leave particular directions how they were to be disposed of; he therefore entrusted them all together to the care of three gentlemen, in whose hands he knew they would be perfectly safe: his honoured friend *Martin Folkes*, Esq; president of the Royal Society; *Andrew Mitchell*, Esq; member of parliament for the shire of *Aberdeen*, who, he knew, would spare no pains to do justice to the memory of a person whom he had so long, and so entirely, loved; and the reverend Mr. *John Hill*, chaplain to his Grace the Archbishop of *Canterbury*, with whom he had for some years cultivated a most intimate friendship. In consequence of this trust, these gentlemen immediately set about publishing what Mr. *Maclaurin* had designed and prepared for the press; his algebra, and the account of Sir *Isaac Newton's* philosophy: and because

* Of this we see a fresh instance in a *second* admirable discovery of Dr. *Bradley's*; of a small nutation of the earth's axis, from the motion of the nodes of the lunar orbit.

they could not take upon themselves the immediate care of these editions, they appointed, for that purpose, a person, whose regard for the Author's memory was a sure pledge of his utmost diligence. They likewise set on foot and solicited a subscription for the following work; which the situation of Mr. *Maclaurin's* family made necessary. For not to mention, that the thoughts of a philosopher are not much turned to the saving of money, nor is his curiosity to be gratified but at a considerable expence, Mr. *Maclaurin's* liberality was greater than his fortune could well bear: it was not advice and recommendation only that he furnished to young men, in whom he could discover a promising and virtuous disposition; he often supplied them with money till his recommendations could take place. This, however, will not, we hope, upon the whole, be any loss to his family; as it has been remembered, and rewarded by the generous manner in which many gentlemen of worth have promoted this subscription.

If we now look back upon the numerous writings of our author, and the deep researches he had been engaged in, his patience and assiduity will be equally astonishing with his genius. To endeavour to account for it to a person who has not himself tasted the pleasures of a contemplative mind, would be a vain attempt. Whoever has devoted himself to worldly views, or to the mere joys of sense and imagination, must be a stranger to the charms of *truth*, naked, unportioned, and unadorned; such as Mr. *Maclaurin* courted her, through his whole life, with a most faithful and persevering passion. Call his speculations but a kind of luxury; it is however a higher and more refined luxury than other pursuits can furnish: an exercise, in which the human faculties find themselves the most rationally employed, and the most sensibly strengthened and improved. At the same time, it best distinguishes the limits to which they are confined; inspiring that humility which belongs to man, and makes a principal part of true wisdom, *the knowledge of one's self.*

How great an example Mr. *Maclaurin* was of this virtue, those who had the happiness of his acquaintance can testify, and his writings abundantly shew. The farther he advanced in the knowledge of geometry and of nature, the greater his aversion grew to perfect systems, hypotheses, and dogmatizing; without peevishly despising the attainments we can arrive at, or the uses to which they serve, he saw there lay infinitely more beyond our reach; and used to call our highest discoveries but a dawn of knowledge, suited to our circumstances and wants in this life; which, however, we ought thankfully to acquiesce in for the present, in hopes that it will be improved in a happier and more perfect state.

In weak and unexperienced minds, it is true, the study of mathematics has often wrought quite different effects: sometimes an overweening and most ridiculous self-conceit, with a contempt of all other studies; at other times a rash confounding of the different kinds of evidence, and the different subjects to which they can be applied; sometimes, because demonstrative evidence is the most perfect, it has been taken for granted there is none other; or moral evidence, to bring it to the same level, has been disguised in an awkward and disadvantageous dress. But to oppose the single example of Mr. *Maclaurin* to such pretenders, will be a sufficient censure of their absurd conduct; and at the same time a sufficient answer to the unjust reproaches, which, on occasion of these abuses, have been thrown out against mathematicians.

It was not mental pleasure and improvement only, that Mr. *Maclaurin* sought in his favourite studies; he saw their great importance in all the arts of civil life, in assisting (as my Lord *Bacon* expresses it*) the powers of man, and extending his dominion in nature. Whosoever is the least acquainted with the history or the present state of trade and manufactures, is fully apprized that there

is nothing great or beautiful, nothing convenient or expeditious, nothing universally beneficial, but wants *their* direction: nor are even the hints which accident throws in our way, to be improved to any tolerable purpose, without the help of *Arithmetic* and *Geometry*.

To this view of general utility, Mr. *Maclaurin* had accommodated all his studies; and we find in many places of his works an application, even of the most abstruse theories, to the perfecting of mechanical arts. He had resolved, for the same purpose, to compose a course of practical mathematics, and to rescue several useful branches of the science, from the bad treatment they often meet with in less skilful hands. But all this his death has deprived us of; unless we would reckon as a part of his intended work, the translation of Dr. *David Gregory's* practical geometry, which he revised and published, with additions, in the year 1745.

In his life-time, however, he often had the pleasure to serve his friends and country by his superior skill. Whatever difficulty occurred concerning the construction or perfecting of machines, the working of mines, the improvement of manufactures, the conveying of water, or the execution of any other public work, Mr. *Maclaurin* was at hand to resolve it. He was likewise employed to terminate some disputes of consequence, that had arisen at *Glasgow* concerning the gauging of vessels; and for that purpose, presented to the commissioners of excise two elaborate memorials, containing rules by which the officers now act, with their demonstrations.

But what must have given him a higher satisfaction than any thing else of this kind, was the calculations he made, relative to that wise and humane provision, which is now established by law, for the children and widows of the *Scotch* clergy, and of the professors in the universities; entitling them to certain annuities and sums, upon the voluntary annual payment of a certain sum by the incumbent. In contriving and adjusting the scheme, Mr. *Maclaurin* had bestowed great labour; and the gentlemen

men who were appointed to solicit the affair at *London*, own that the authority of his name was of great use to them, for removing any doubts that were moved concerning the sufficiency of the proposed fund, or the due proportion of the sums and annuities.

To find himself thus eminently useful, even to late posterity, must have been a delightful enjoyment. But what still more endeared his studies to him, was the use they are of in demonstrating the Being and Attributes of the Almighty Creator, and establishing the principles of natural religion on a solid foundation; equally secure against the idle sophistry of *Epicureans*, and the dangerous refinements of *modern metaphysicians*. He agreed with the great Mr. Cotes *, in thinking that *the knowledge of nature will ever be the firmest bulwark against Atheism*, and consequently the surest foundation of true religion. This knowledge does more than excite mere *wondering*; it inspires love and adoration of the Creator, our reasonable *Service*: for it must be a superficial view of nature, indeed, that suggests no *relation*, or *duty*, to Him *in whom we live, move, and have our being*. The argument from final causes, from the order and design that evidently shews itself throughout the universe, Mr. *Maclaurin* held to be the shortest and simplest of all others; and consequently of most general use, and the best adapted to the human faculties: whereas metaphysical deductions are to be apprehended but by the few, and are ever liable to be perverted. So that altho' he could use them with as much subtlety and force as any man living, he chose rather, in his conversation as well as his writings, to bring the dispute to a short issue in his own way.

He was no less strenuous in the defence of revealed religion; which he would warmly undertake as often as it was attacked, either occasionally in conversation, or in those pernicious books which have brought the name of Free-thinker into disgrace, and have so much contributed to spoil our taste as well as our morals: and how firm his

* In Præfat. ad *Neut. Principia*.

own persuasion of it was, appeared from the support it afforded him in his last hours.

Such was the life of this eminent person ; spent in a course of laborious, yet not painful study ; in continually doing good to the utmost of his power : in improving curious and useful arts, and propagating truth, virtue, and religion amongst mankind. He was taken from us at an age when he was capable of doing much more ; but has left an example which, we hope, will be long admired and imitated : till the revolution of human affairs puts an end to learning in these parts of the world ; or the fickleness of men, and their satiety of the best things, have substituted for this philosophy some empty form of false science ; and, by the one or the other means, we are brought back to our original state of barbarism.

A N
A C C O U N T
O F
Sir ISAAC NEWTON's
Philosophical Discoveries.

B O O K I.

*Of the method of proceeding in natural philosophy,
and the various systems of philosophers.*

C H A P. I.

*A general view of Sir Isaac Newton's method, and of
his account of the system of the world.*

1. **T**O describe the *phenomena* of nature, to explain their causes, to trace the relations and dependencies of those causes, and to inquire into the whole constitution of the universe, is the business of natural philosophy. A strong curiosity has prompted men in all times to study nature; every useful art has some connexion with this science; and the unexhausted beauty and variety of things makes it ever agreeable, new, and surprising.

But natural philosophy is subservient to purposes of a higher kind, and is chiefly to be valued as it lays a sure foundation for natural religion and moral philosophy; by leading us, in a satisfactory manner, to the knowledge of the Author and Governor of the universe. To study nature is to search into his workmanship: every new discovery opens to us a new part of his scheme. And while we still meet, in our inquiries, with hints of greater things yet undiscovered, the mind is kept in a pleasing expectation of making a further progress; acquiring at the same time higher conceptions of that great Being, whose works are so various and hard to be comprehended.

Our views of Nature, however imperfect, serve to represent to us in the most sensible manner, that mighty power which prevails throughout, acting with a force and efficacy that appears to suffer no diminution from the greatest distances of space or intervals of time; and that wisdom which we see equally displayed in the exquisite structure and just motions of the greatest and subtilest parts. These, with perfect *goodness*, by which they are evidently directed, constitute the supreme object of the speculations of a philosopher; who, while he contemplates and admires so excellent a system, cannot but be himself excited and animated to correspond with the general harmony of nature.

In order to obtain those great purposes, we must not proceed hastily in our enquiries, but with the utmost caution. False schemes of natural philosophy may lead to atheism, or suggest opinions, concerning the Deity and the universe, of most dangerous consequence to mankind; and have been frequently employed to support such opinions. We have the more reason to be on our guard, because philosophers have, on many occasions, shown an unaccountable disposition to give into extravagant fictions in their accounts of nature. A considerable party adopted, of old, that monstrous system, which, excluding the influences of a Deity *, attempted to explain the formation of the universe from the accidental play of atoms, and derived the ineffable beauty of things, even life and thought itself, from a lucky hit in the blind uproar. An horror at the dire effects of superstition may have induced them to have recourse to a doctrine so opposite to common sense and reason; but we have not even this

* *Lucret. de rerum natura, lib. I. v. 63, &c.*

excuse to offer in defence of some modern philosophers of great name, who seem to have copied too much after those masters, in their mechanical accounts of the production of the material system.

While we guard against atheism and opinions that approach towards it, we ought likewise to beware of listening to superstition; which discourages inquiries into nature, lest, by having our views enlarged, we should escape from her bonds, and our discoveries should weaken some darling tenets. If those tenets are true, they will rather be confirmed by our inquiries; and if they are false, surely it is better they should be detected. We may pursue truth steadily, secure that it will be always found consistent with itself, and stands in no need of the jealousies and dark suspicions of the superstitious to support it; in whose hands truth itself is apt to suffer by the base alloy they mix with it, and by the detested means which they have too often employed to maintain so incongruous an union. The philosophers who have been devoted to so mean views, have never failed to expose themselves to just ridicule, without doing service to the cause which they espoused. *Cosmas Indopleustes* * of old, misled by an injudicious zeal, compiled a system of nature from some expressions in the sacred writings; which, against the constant and universal use of language, he would needs understand in the most literal and the very strictest sense.

The earth therefore, according to him, was not globular, but an immense plane of a greater length than breadth, environed by an unpassable ocean.

* *Fabrit. bibliotheca græca*, vol. II. p. 609, &c. where an account is given from *Photius* and others of this author, with a figure to illustrate his system.

He placed a huge mountain towards the north, around which the sun and stars performed their diurnal revolutions; and from the conical shape which he ascribed to it, with the oblique motion of the sun, he accounted for the inequality of the days and the variation of the seasons. The vault of heaven leaned upon the earth extended beyond the ocean, being likewise supported by two vast columns: beneath the arch, angels conducted the stars in their various motions. Above it were the celestial waters, and above all he placed the supreme heavens. However absurd the conceits of this author, who wrote in darker times, may appear, we have a more inexcusable instance, in the last century, of the same kind, in what *Kircher* calls his *Ecstatic Voyage to the Planets*; who, after many great discoveries had been made concerning the celestial bodies, produced nothing worthy * of so noble a subject, or of his own extensive learning and invention, having determined to make a sacrifice of both to certain decrees of the church of Rome: he descends even so low as to adopt the folly or rather impiety, of astrologers, in deriving the good or evil that happens to man from the propitious or malignant influences of planets. True religion requires no such sacrifices; nor are its interests advanced by feigning philosophical systems purposely to favour it: for when we afterwards find these to be ill-grounded, we may be in danger of falling into scepticism.

An entire liberty must be allowed in our enquiries, that natural philosophy may become subser-

* In the planet *Venus*, for example, he finds no other amusement but to admire the limpid waters and beautiful crystals he found there; and to ask the genie, his companion and guide, whether baptism with such water would be valid. The rest is of a piece with this.

vient to the most valuable purposes, and acquire all the certainty and perfection of which it is capable : but we ought not to abuse this liberty by *supposing* instead of *enquiring*, and by imagining systems, instead of learning from observation and experience the true constitution of things. Speculative men, by the force of genius, may invent systems that will perhaps be greatly admired for a time ; these, however, are phantoms which the force of truth will sooner or later dispel : and while we are pleased with the deceit, true philosophy, with all the arts and improvements that depend upon it, suffers. The real state of things escapes our observation : or, if it presents itself to us, we are apt either to reject it wholly as fiction, or, by new efforts of a vain ingenuity, to interweave it with our own conceits, and labour to make it tally with our favourite schemes. Thus, by blending together parts so ill suited, the whole comes forth an absurd composition of truth and error.

Of the many difficulties that have stood in the way of philosophy, this vanity perhaps has had the worst effects. The love of the marvellous, and the prejudices of sense, obstructed the progress of natural knowledge ; but experience and reflection soon taught men to examine and endeavour to correct these. Tho' philosophers met with great discouragements in the dark and superstitious ages, learning flourished with liberty, in better times. The disputes amongst the sects, more fond of victory than of truth, produced a talkative sort of philosophy, and a vain ostentation of learning, that prevailed for a long time ; but men could not be always diverted from pursuing after more real knowledge. These have not done near so much harm, as that pride and ambition, which has led philosophers to think it be-

neath them, to offer any thing less to the world than a complete and finished system of nature; and, in order to obtain this at once, to take the liberty of inventing certain principles and hypotheses, from which they pretend to explain all her mysteries.

2. Sir *Isaac Newton* saw how extravagant such attempts were, and therefore did not set out with any favourite principle or supposition, never proposing to himself the invention of a system. He saw that it was necessary to consult nature herself, to attend carefully to her manifest operations, and to extort her secrets from her by well-chosen and repeated experiments. He would admit no objections against plain experience from metaphysical considerations, which, he saw, had often misled philosophers, and had seldom been of real use in their enquiries. He avoided presumption, he had the necessary patience as well as genius; and having kept steadily to the right path, he therefore succeeded.

Experiments and observations, it is true, could not alone have carried him far in tracing the causes from their effects, and explaining the effects from their causes: a sublime geometry was his guide in this nice and difficult enquiry. This is the instrument, by which alone the machinery of a work, made with so much art, could be unfolded; and therefore he sought to carry it to the greatest height. Nor is it easy to discern, whether he has shewed greater skill, and been more successful, in improving and perfecting the instrument, or in applying it to use. He used to call his philosophy *experimental philosophy*, intimating, by the name, the essential difference there is betwixt it and those systems that are the product of genius and invention only. These could not
long

long subsist ; but his philosophy, being founded on experiment and demonstration, cannot fail till reason or the nature of things are changed.

In order to proceed with perfect security, and to put an end for ever to disputes, he proposed that, in our inquiries into nature, the methods of *analysis* and *synthesis* should be both employed in a proper order ; that we should begin with the phænomena, or effects, and from them investigate the powers or causes that operate in nature ; that, from particular causes, we should proceed to the more general ones, till the argument end in the most general : this is the method of *analysis*. Being once possess'd of these causes, we should then descend in a contrary order ; and from them, as established principles, explain all the phænomena that are their consequences, and prove our explications : and this is the *synthesis*. It is evident that, as in mathematics, so in natural philosophy, the investigation of difficult things by the method of *analysis* ought ever to precede the method of composition, or the *synthesis*. For in any other way, we can never be sure that we assume the principles which really obtain in nature ; and that our system, after we have composed it with great labour, is not mere dream and illusion.

By proceeding according to this method, he demonstrated from observations, analytically, that gravity is a general principle ; from which he afterwards explained the system of the world. By *analysis* he discovered new and wonderful properties of light, and, from these, accounted for many curious phænomena in a *synthetic* way. But while he was thus demonstrating a great number of truths, he could not but meet with hints of many other things, that his sagacity and diligent observation suggested to him,
which

which he was not able to establish with equal certainty : and as these were not to be neglected, but to be separated with care from the others, he therefore collected them together, and proposed them under the modest title of *queries*.

By distinguishing these so carefully from each other, he has done the greatest service to this part of learning, and has secured his philosophy against any hazard of being disproved or weakened by future discoveries. He has taken care to give nothing for demonstration but what must ever be found such; and having separated from this what he owns is not so certain, he has opened matter for the inquiries of future ages, which may confirm and enlarge his doctrines, but can never refute them. He knew where to stop when experiments were wanting, and when the subtilty of nature carried things out of his reach : nor would he abuse the great authority and reputation he had acquired, by delivering his opinion concerning these, otherwise than as matter of question. It was long before he could be prevailed on to propose his opinion or conjectures concerning the cause of gravity ; and what he has said of it, and of the other powers that act on the minute particles of matter, is delivered with a modesty and diffidence seldom to be met with amongst philosophers of a less name. Nor do they act in a conformity with the spirit of this philosophy who speak dogmatically on these subjects, till a clearer light from new observations and experiments brings them from the class of queries, and places them on the level of demonstration.

3. Such was the method of our incomparable philosopher, whose caution and modesty will ever do him the greatest honour in the opinion of the unprejudiced,

prejudiced. But this strict method of proceeding was not relished by those who had been accustomed to treat philosophy in a very different way, and who saw that, by following it, they must give up their favourite systems. His observations and reasonings were unexceptionable; so, finding nothing to object to these, they endeavoured to lessen the character of his philosophy by general indirect insinuations, and, sometimes, by unjust calumnies. They pretended to find a resemblance between his doctrines and the exploded tenets of the scholastic philosophy. They triumphed mightily in treating gravity as an occult quality, because he did not pretend to deduce this principle fully from its cause. His extending over all the system a power which is so well known to us on the earth, and explaining by it the motions and influences of the celestial bodies, in the most satisfactory manner; and his determining the measures of the various motions that are consequences of this power, by so skilful an application of geometry to nature; all these had no merit with such philosophers, because he did not assign the mechanical cause of gravity. I know not that ever it was made an objection to the circulation of the blood, that there is no small difficulty in accounting for it mechanically; for they who first extended gravity to air, vapour, and to all bodies round the earth, had their praise, though the cause of gravity was as obscure as before; or rather appeared more mysterious, after they had shewn that there was no body found near the earth, exempt from gravity, that might be supposed to be its cause. Why then were his admirable discoveries, by which this principle was extended over the universe, so ill relished by some philosophers? The truth is, he had, with great evidence, overthrown the boasted schemes by which they pretended to unravel all the mysteries of nature; and the philosophy he

introduced,

introduced, in place of them, carrying with it a sincere confession of our being far from a complete and perfect knowledge of it, could not please those who had been accustomed to imagine themselves possessed of the eternal reasons and primary causes of all things.

But to all such as have just notions of the great Author of the universe, and of his admirable workmanship, Sir *Isaac Newton's* caution and modesty will recommend his philosophy; and even the avowed imperfection of some parts of it will, to them, rather appear a consequence of its conformity with nature. To such, all complete and finished systems must appear very suspicious: they will not be surprized that refined speculations, or even the labours of a few ages, are not sufficient to unfold the whole constitution of things, and trace every phænomenon through all the chain of causes to the first cause. Is the admirable progress which has been made in this arduous pursuit to be despised or neglected, because more remains behind undiscovered? Surely we ought rather to rejoice that so much is opened to us of the consummate art by which all things were made, and ought to be afraid to intermix with it our own extravagant conceits.

The processes of nature lie so deep, that, after all the pains we can take, much, perhaps, will remain undiscovered beyond the reach of human art or skill. But this is no reason why we should give ourselves up to the belief of fictions, be they ever so ingenious, instead of hearkening to the unerring voice of nature; for she alone can guide us in her own labyrinths; and it is a consequence of her real beauty, that the least part of true philosophy is incomparably more beautiful than the most complete systems which
have

have been the product of invention. This is particularly true of Sir *Isaac Newton's* philosophy; and we may compare it in this respect with those celebrated pieces of *Apelles*, which, though they never received his last hand, were in greater admiration amongst the ancients, than the most finished pieces of other artists: and we with posterity may not find cause to say of this philosophy what the ancients said of those pieces,——*Ipsū defectum cessisse in gloriam artificis, nec qui succederet operi ad præscripta lineamenta inventum fuisse.* Plin.

4. It was, however, no new thing that this philosophy should meet with opposition. All the useful discoveries that were made in former times, and particularly in the last century, had to struggle with the prejudices of those who had accustomed themselves not so much as to think but in a certain systematic way; who could not be prevailed on to abandon their favourite schemes, while they were able to imagine the least pretext for continuing the dispute: every art and talent was displayed to support their falling cause; no aid seemed foreign to them that could in any manner annoy their adversary; and such often was their obstinacy, that truth was able to make little progress, till they were succeeded by younger persons who had not so strongly imbibed their prejudices.

Sir *Isaac Newton* had very early experience of this temper of philosophers, and appears to have been discouraged by it. He had a particular aversion to disputes, and was with difficulty induced to enter into any controversy. The warm opposition his admirable discoveries in optics met with, in his youth, deprived the world of a full account of them for many years, till there appeared a greater disposition
among

among the learned to receive them; and induced him to retain other important inventions by him, from an apprehension of the disputes in which a publication might involve him. He thus weighed the reasons of things impartially and coolly, before a publication of them can be suspected to have engaged him in their defence. It is well known how slow he was in publishing: and we cannot but observe that the temper and disposition of mind, as well as the abilities of this great man, fitted him in a particular manner for penetrating far into nature and unfolding her harmony.

Nor did his aversion to disputes proceed from the love of quiet only. Philosophy had been in high esteem of old, but had lost its antient lustre from the endless idle janglings that had arisen amongst the sects; and could never recover it while a faculty of inventing a system readily, and defending it obstinately, were the admired talents of a philosopher. While one age or sect overturned for the most part the laborious productions of another, many of the wiser sort despaired of acquiring certainty in natural knowledge, and chose rather to content themselves with the general view of things, open to all men, than attach themselves to schemes which produced no real fruit, and really led them farther from the truth. Our author therefore proposed that all prejudices should be laid aside, and the genuine method of treating natural philosophy, which we have described from him, should be closely followed. By his adhering to it himself, we are secure that truth and nature are on his side; and by following the excellent models which he has given us, we may be able to make farther advances.

Others have pretended to explain the whole constitution of things by what they call clear ideas, and
by

by mere abstracted speculations. They express a contempt * for that knowledge of causes which is derived from the contemplation of their effects, and are unwilling to condescend to any other science than that of effects from their causes. Therefore they set out from the *first cause*; and from their ideas of him pretend to unfold the whole chain, and to trace a complete scheme of his works. This is the philosophy that stands in opposition to our author's to this day. It flatters human vanity so much, and sets out in so pompous a manner, that they who attend not to the unexhaustible variety of nature, and consider not how unequal the human powers are to so arduous an undertaking, are deluded by its promises. It may be doubted if such a philosophy lies within the reach of any created being; and it seems to be very plain that it far surpasses the reach of men. But since many are devoted to this phantom, and use all their art to adorn, and recommend it to more admirers, it will be necessary for the service of truth, that, while we proceed, we have in view likewise the detection of this imposture.

5. The view of nature, which is the immediate object of sense, is very imperfect, and of a small extent; but by the assistance of art, and the help of our reason, is enlarged till it loses itself in an infinity on either hand. The immensity of things on

*. Perspicuum est optimam philosophandi viam nos sequuturos, si, ex ipsius Dei cognitione, rerum ab eo creatarum explicationem deducere conemur, ut ita scientiam perfectissimam, quæ est effectuum per causas, acquiramus. *Cartes Princip.* part II. § 22. Afterwards, having occasion to speak of the phænomena, he takes care to tell us, that he would not make use of them to prove any thing from them, because he wanted to derive the knowledge of effects from their causes, and not reciprocally that of the causes from their effects. *Princip.* part III. § 4, &c.

the one side, and their minuteness on the other, carry them equally out of our reach, and conceal from us the far greater and more noble part of physical operations. As magnitude of every sort, abstractly considered, is capable of being increased to infinity, and is also divisible without end; so we find that, in nature, the limits of the greatest and least dimensions of things are actually placed at an immense distance from each other. We can perceive no bounds of the vast expanse in which natural causes operate, and can fix no border or termination of the universe; and we are equally at a loss when we endeavour to trace things to their elements, and to discover the limits which conclude the subdivisions of matter. The objects which we commonly call great vanish when we contemplate the vast body of the earth; the terraqueous globe itself is soon lost in the solar system: in some parts it is seen as a distant star. In great part it is unknown, or visible only at rare times to vigilant observers, assisted, perhaps, with an art like to that by which *Galileo* was enabled to discover so many new parts of the system. The sun itself dwindles into a star; *Saturn's* vast orbit, and the orbits of all the comets, croud into a point, when viewed from numberless places between the earth and the nearest fixed stars. Other suns kindle light to illuminate other systems where our sun's rays are unperceived; but they also are swallowed up in the vast expanse. Even all the systems of the stars that sparkle in the clearest sky must possess a small corner only of that space over which such systems are dispersed, since more stars are discovered in one constellation, by the telescope, than the naked eye perceives in the whole heavens*.

* In the constellation of *Orion*, 2000 stars have been numbered by astronomers.

After we have risen so high, and left all definite measures so far behind us, we find ourselves no nearer to a term or limit; for all this is nothing to what may be displayed in the infinite expanse, beyond the remotest stars that ever have been discovered.

If we descend in the scale of nature, towards the other limit, we find a like gradation from minute objects to others incomparably more subtile, and are led as far below sensible measures as we were before carried above them, by similar steps that soon become hid to us in equal obscurity. We have ground to believe that these subdivisions of matter have a termination, and that the elementary particles of bodies are solid and uncompounded, so as to undergo no alteration in the various operation of nature or of art. But from microscopical observations that discover animals, thousands of which could scarce form a particle perceptible to the unassisted sense, each of which have their proper vessels, and fluids circulating in those vessels; from the propagation, nourishment, and growth of those animals; from the subtilty of the effluvia of bodies retaining their particular properties after so prodigious a rarefaction; from many astonishing experiments of chymists; and especially from the inconceivable minuteness of the particles of light, that find a passage equally in all directions through the pores of transparent bodies, and from the contrary properties of the different sides of the same ray†; it appears, that the subdivisions of the particles of bodies descend by a number of steps or degrees that surpasses all imagination, and that nature is inexhaustible by us on every side. Nor is it in the magnitude of bodies only that this endless gradation is to be observed. Of motions, some are

† *Newton's Optics.* Query 26.

performed in moments of time; others are finished in very long periods: some are too slow, and others too swift, to be perceptible by us. The tracing the chain of causes is the most noble pursuit of philosophy; but we meet with no cause but what is, itself, to be considered as an effect, and are able to number but few links of the chain. In every kind of magnitude, there is a degree or sort to which our sense is proportioned, the perception and knowledge of which is of greatest use to mankind. The same is the ground-work of philosophy*; for tho' all sorts and degrees are equally the object of philosophical speculation; yet it is from those which are proportioned to sense that a philosopher must set out in his enquiries, ascending or descending afterwards as his pursuits may require. He does well indeed to take

* If we were to examine more particularly the situation of man in nature, we should find reason to conclude, perhaps, that it is well adapted to one of his faculties and inclinations, for extending his knowledge, in such a manner as might be consistent with other duties incumbent upon him; and that they have not judged rightly who have compared him in this respect (*Spinoz. Epist. 15.*) with the animalcules in the blood discovered by microscopes. He must be allowed to be the first being that pertains to this globe, which, for any thing we know, may be as considerable (not in magnitude, but in more valuable respects) as any in the solar system, which is itself, perhaps, not inferior to any other system in these parts of the vast expanse. By occupying a lower place in nature, man might have more easily seen what passes amongst the minute particles of matter, but he would have lost more than he could have gained by this advantage. He would have been in no condition to institute an analysis of nature, in that case. On the other hand, we doubt not but there are excellent reasons why he should not have access to the distant parts of the system, and must be contented at present with a very imperfect knowledge of them. The duties incumbent upon him, as a member of society, might have suffered by too great an attention to them, or communication with them. Had he been indulged in a correspondence with the planets, he next would have desired to pry into the state of the fixed stars, and at length to comprehend infinite space.

his

his views from many points of sight, and supply the defects of sense by a well-regulated imagination ; nor is he to be confined by any limit in space or time : but as his knowledge of nature is founded on the observation of sensible things, he must begin with these, and must often return to them, to examine his progress by them. Here is his secure hold ; and as he sets out from thence, so if he likewise trace not often his steps backwards with caution, he will be in hazard of losing his way in the labyrinths of nature.

6. From this short view of nature, and of the situation of man, considered as a spectator of its phænomena and as an inquirer into its constitution, we may form some judgment of the project of those, who, in composing their systems, begin at the summit of the scale, and then, by clear ideas, pretend to descend through all its steps with great pomp and facility, so as in one view to explain all things. The processes in experimental philosophy are carried on in a different manner : the beginnings are less lofty, but the scheme improves as we arise from particular observations, to more general and more just views. It must be owned, indeed, that philosophy would be perfect, if our view of nature, from the common objects of sense, to the limits of the universe upwards, and to the elements of things downwards, was complete ; and the powers or causes that operate in the whole were known. But if we compare the extent of this scheme with the powers of mankind, we shall be obliged to allow the necessity of taking it in parts, and of proceeding with all the caution and care we are capable of, in enquiring into each part. When we perceive such wonders, as naturalists have discovered, in the minutest objects, shall we pretend to describe so

C 2

easily

easily the productions of infinite power in space, that is at the same time infinitely extended and infinitely divisible? Surely we may rather imagine, that in the whole, there will be matter for the inquiries and perpetual admiration of much more perfect beings.

It is not therefore the business of philosophy, in our present situation in the universe, to attempt to take in at once, in one view, the whole scheme of nature; but to extend with great care and circumspection, our knowledge, by just steps, from sensible things, as far as our observations or reasonings from them will carry us, in our inquiries concerning either the greater motions and operations of nature, or her more subtile and hidden works. In this way Sir ISAAC NEWTON proceeded in his discoveries: he established his account of the system of the world upon the best astronomical observations, on the one hand; and performed himself, on the other, with the greatest address, the experiments by which he was enabled to pry into the more secret operations of nature amongst the minute particles of matter. On either side he has extended our views very far, and has left valuable hints and intimations of what yet lies involved in obscurity.

For those purposes he has given us two incomparable treatises, the most perfect in their kind philosophy has to boast of; his mathematical *Principles* of Natural Philosophy, and his Treatise of *Optics*. In the first he describes the system of the world, and demonstrates the powers which govern the celestial motions, and produce their mutual influences. These are extended from the center of the sun to the utmost altitude of the highest comet, and probably to the farthest limits of the universe. Nor
are

are these new or abstruse principles, like to those which never had a being but in the imagination of philosophers, but the same which are most familiar to mankind, and in common use, farther extended and more accurately defined. In the second, he treats of light, which, though the most potent agent in nature, that is sensible to us, acts only at the least distances. His admirable discoveries, on this subject, led him to search into the motions that are amongst the minute particles of matter, the most abstruse of all natural phænomena.

In the first, he had the observations of astronomers for many ages to build on, with valuable consequences that had been derived from them, by the laborious calculations of diligent and ingenious men. The constancy and regularity of the celestial motions had contributed, with the observations of some thousands of years, to render astronomy the most exact part of the history of nature; the doctrine of comets only excepted. The vast distances of the great bodies which compose the system, from each other, rather favoured a just *analysis* of the powers by which they act on one another; since by the greatness of the distance, these must be reduced to a few simple principles, and be the more easily discovered. In the second treatise, he inquires into more hidden parts of nature, and had most of the phænomena themselves to trace, as well as their causes. The subject is rather more nice and difficult, because of the inconceivable minuteness of the agents, and the subtilty and quickness of the motions; and the principles combined in producing the phænomena being more various, it could not be expected that they should be so easily subjected to an *analysis*. Hence it is, that what he has delivered in the first (tho' still capable of improvement) is more complete

and finished in several respects ; while his discoveries of the second sort are more astonishing.

After having established the principle of the universal Gravitation of Matter in the first treatise, when he is not able to demonstrate the causes of the phænomena described in the second more evidently, he endeavours to judge of them, by *analogy*, from what he had found in the greater motions of the system ; a way of reasoning that is agreeable to the harmony of things, and to the old maxim ascribed to *Hermes* *, and approved by the observation and judgment of the best philosophers, “ That what passes in the heavens above is similar and analogous to what passes on earth below.” He had found that all bodies gravitated towards each other, by a power that acts on all their particles equally at equal distances, and increases according to a stated law when the distance is diminished. From a like principle, acting at less distances with greater vigour, and with more variety, but insensibly at larger distances, he suspected that the more abstruse phænomena of nature proceeded. It was a great matter in philosophy to be secure of one general principle ; and one was sufficient for carrying on the regular motions of the heavenly bodies. A greater variety was necessary for conducting the different operations of nature in particular parts ; and these being involved in some obscurity, till better light should appear, he could find no surer ground on which to found a judgment of them, than that principle he had already shewn to take place in nature. But be-

* A principle not unlike this is ascribed to the *Persian* and *Chaldean* magi, *συμπραθὴν εἶναι τὰ ἄνω τοῖς κατω*. *Psell.* Declaratio dogmat. Chaldaic. Tho' this, as other maxims, was much abused in progress of time, when philosophers degenerated from their first simplicity.

cause we often find that phænomena, which, at first sight, appear of a very different sort, flow nevertheless from the same cause, and several such causes are often resolved, on farther enquiry, into one more general principle; the whole constitution of nature (notwithstanding the variety of appearances) manifestly leading to one supreme cause; this great philosopher was hence induced, as well as from several observations he had made, to think that all these powers might proceed from one general instrument or agent, as various branches from one great stem, whose efficacy might be resolved more immediately into the direction or influences of the sovereign cause that rules the universe. But he speaks of this in the manner that became a philosopher who had so much studied nature, and knew how obscure those arduous parts of her scheme must be to us.

7. As the most obvious views of the creation suggest to all men the persuasion of the being and government of a Deity; so every discovery in natural philosophy enforces it: and with this improvement of his discoveries, this great man concludes both those treatises. Nor is his philosophy to be thought of little service for this purpose, though he has not been able to explain fully the primary causes themselves.

The great mysterious Being, who made and governs the whole system, has set a part of the chain of causes in our view; but we find that, as he himself is too high for our comprehension, so his more immediate instruments in the universe, are also involved in an obscurity that philosophy is not able to dissipate; and thus our veneration for the supreme Author is always increased, in proportion as we advance in the knowledge of his works. As we arise in philosophy

towards the first cause, we obtain more extensive views of the constitution of things, and see his inducement more plainly. We perceive that we are approaching to him, from the simplicity and generality of the powers or laws we discover; from the difficulty we find to account for them mechanically; from the more and more complete beauty and contrivance, that appears to us in the scheme of his works as we advance; and from the hints we obtain of greater things yet out of our reach: but still we find ourselves at a distance from Him, the great source of all motion, power, and efficacy; who, after all our enquiries, continues removed from us and veiled in darkness. He is not the object of sense; his nature and essence are unfathomable; the more immediate instruments of his power and energy are but obscurely known to us; the least part of nature, when we endeavour to comprehend it, perplexes us; even *place* and *time*, of which our ideas seem to be simple and clear, have enough in them to embarrass those who allow nothing to be beyond the reach of their faculties. These things, however, do not hinder but we may learn to form great and just conceptions of him from his sensible works, where an art and skill is expressed that is obvious to the most superficial spectator, surprizes the most experienced enquirer, and many times surpasses the comprehension of the profoundest philosopher. From what we are able to understand of nature, we may entertain the greater expectations of what will be discovered to us, if ever we shall be allowed to penetrate to the first cause himself, and see the whole scheme of his works as they are really derived from him, when our imperfect philosophy shall be completed.

C H A P. II.

Of the systems of the ancient philosophers.

I. **T**HOSE who have not imbibed the prejudices of philosophers, are easily convinced that natural knowledge is to be founded on experiment and observation. But there is a philosophy that intoxicates the mind, while it pretends to elevate and satisfy it, which teaches to despise the plain and sober way of truth. And it is no easy matter to deal with those who have lost themselves in the dark schemes of an inviolable and universal necessity, or with those who are ever dreaming themselves possessors of the eternal reasons and primary causes of things. The least shew of an argument in their own visionary way takes infinitely more with them, than the clearest evidence from fact or observation; and so fond they appear of such airy schemes, that they would chuse rather to go on disputing for ever, than condescend to acquiesce in certainty obtained in a lower way.

To an impartial enquirer, Sir *Isaac Newton's* method, described in the last chapter, approves itself; and some ingenious men have been sensible of the necessity of following it, in former times. But the general practice of philosophers has been very different; and systems founded on abstracted speculations still so much prevail, that it will be necessary for our purpose to shew, by a few observations on the history of learning, how vain and fruitless such attempts have always proved.

Theories of this kind have been invented, and amended again and again, with great labour and expence of thought; but still when they came to compare them with nature, how wide has been the difference!—*ibi omnis effusus labor*. If we look back into the state of philosophy in the different ages, we shall learn from the history of every period, that as far as philosophers consulted nature, and proceeded on observation, they made some progress in true knowledge; but as far as they pretended to carry on their schemes without this, they only multiplied disputes.

The beginnings of learning, as of other things, are uncertain and obscured with fables: we collect, however, from several testimonies, that the oldest and most celebrated philosophers of *Phœnicia* and *Greece* made a *vacuum* and *atoms*, and the *gravity* of atoms, the first principles of their philosophy*; whether these were suggested to them from their early observations of nature, before her plain appearances were obscured by the imaginary schemes and the disputes of speculative men, or were derived from some other origin. Afterwards various systems appeared, but some traces of those antient principles are for a long time to be discovered amongst the doctrines of succeeding philosophers, though interwoven with their own particular tenets;

* According to *Posidonius* the stoick, as cited by *Strabo* and *Sextus Empiricus*, the doctrine of Atoms was more ancient than the times of the Trojan war, having been taught by *Mochus* a Phœnician, the same probably meant by *Iamblichus*, when he tells us that *Pythagoras* conversed at *Sidon* with the prophets, the successors of *Mochus* the physiologer. In those early times the characters of lawgiver and philosopher were united, and this *Mochus* is supposed by many to have been the same with *Moses* the legislator of the Jews.

and what appears to be most uniform in the variety of their opinions seems to be derived from this source*. The more ancient atomists seem to have taught that there were living substances also, which pre-existed before the union of the systems of those elementary corpuscles, and continued to exist after their dissolution. They saw the necessity of admitting active as well as passive principles, life as well as mechanism, throughout the world†. But this entire and genuine philosophy was dismembered afterwards, and from an affectation of simplicity, or for other reasons, one sort of permanent substance was thought sufficient. One party retained the passive and sluggish matter only, and from the fortuitous concurrence of its corpuscles pretended to explain the formation of the universe. Others, more refined, ascribed reality and permanency to active incorporeal substances chiefly, or only. And so similar were their divisions and disputes to those of our own times, that a third sort seem to have rejected the reality of both, while they maintained that there was no stability of essence or knowledge any where to be found; that all being and knowledge was fantastical and relative only; that man was the measure of truth to himself in all things; and that every opinion or fancy of every one was true‡. While one sect thought that nothing was permanent, but that all things were in a continual flux or motion, and

* They taught that nothing was made out of nothing, that no substance is generated or destroyed, that colour and taste are not in the objects, &c. which seem to be the genuine doctrines of this atomical philosophy amongst the Greeks. See *Aristot. de anima, Lib. III. Cap. I.* who ascribes such opinions to most of the physiologists before his time.

† See Dr. *Cudworth's* intellectual system of the universe. Book I. Chap. I.

‡ This was the doctrine of *Protagoras* the Abderite. *Plat. Thætetus, &c.*

others, that all things consisted of one immoveable and infinite essence, it is no wonder that their successors own themselves at a loss to understand their meaning*.—Opposition to each other seems to have driven them to extremes, and both aimed at too general and extensive principles.

As to the particular tenets of *Thales*, and his successors of the *Ionic* school, the sum of what we learn from the imperfect accounts we have of them is, that each overthrew what his predecessor had advanced; and met with the same treatment himself from his successor. One of them is said to have made water the principle of all things; another chose air; a third fire; a fourth preferred earth; and some took them all in, and made these four the elements or principles of things. So early did the passion for systems begin, and disputes in consequence of such precipitancy were unavoidable.

2. In the time of this uncertainty amongst the physiologists (for such all the more antient philosophers were) *Socrates* appeared in the world. A sublimity of genius, a simplicity of manners, a particular talent of investigating truth and exposing error, distinguished this great man. In his youth he applied himself, as his predecessors had done, to natural knowledge, and endeavoured to reduce it to a method and principles. But after examining their schemes without receiving any satisfaction from them, he was too sincere a lover of truth, and too just to mankind, to attempt to invent one of his own, or to dissemble his ignorance of nature. He saw that imaginary knowledge was the greatest obstruction to true science, and made those who were puffed up with it very troublesome to the lovers of solid learn-

* *Plat. Theætet.*

ing. He therefore took every occasion to expose it, and had a happy talent in ridiculing the vanity of the sophists of those times, who pretended to know all things. The oracle on a certain occasion had declared him the wisest of men; and this preference he explained, with his usual modesty, to be owing to this only, that while others vainly imagined they knew what they were indeed ignorant of, he knew this one thing more than they, “that he knew nothing.”

After many other fruitless attempts he had made in his youth * to see into the causes of things, happening to hear that *Anaxagoras* taught that all things were governed by a supreme mind, and being mightily pleased with this principle, he had recourse to his writings; full of expectation to see the whole scheme of nature explained from the perfect wisdom of an all-governing mind, and to have all his doubts about the perfection of the universe satisfied. But he was much disappointed, when he found that *Anaxagoras* made no use of this sovereign mind in his explications of nature, and referred nothing to the order and perfection of the universe as its reason; but introduced certain aerial, æthereal and aqueous powers, and such incredible principles for the causes of things. Upon the whole, *Socrates* found that this account of nature was no more satisfactory, than if one who undertook to account for all the actions of *Socrates*, should begin with telling that *Socrates* was acted by a principle of thought and design; and pretending to explain how he came to be sitting in prison at that time, when he was condemned to die by the unjust and ungrateful *Athenians*, he should acquaint us that the body of *Socrates* consisted of bones and muscles, that the

* ἐγὼ γὰρ νέος ὄν.

bones were solid and had their articulations, while the muscles were capable of being contracted and extended, by which he was enabled to move his body, and put himself in a sitting posture; and after adding an explication of the nature of sound, and of the organs of his voice, he should boast at length that he had thus accounted for *Socrates's* sitting and conversing with his friends in prison; without taking notice of the decree of the *Athenians*, and that he himself thought it was more just and becoming to wait patiently for the execution of their sentence, than escape to *Megara* or *Thebes*, there to live in exile. “ ’Tis true, says he, that without bones and
 “ nerves I should not be able to perform any action
 “ in life, but it would be an unaccountable way of
 “ speaking to assign those for the reasons of my ac-
 “ tions, while my mind is influenced by the appear-
 “ ance of what is *best*.”

I have taken notice of this passage the rather, because it shews how essential the greatest and best philosophers have thought the consideration of final causes to be to true philosophy; without which it wants the greatest beauty, perfection, and use. It gave a particular pleasure to *Sir Isaac Newton* to see that his philosophy had contributed to promote an attention to them (as I have heard him observe) after *Des Cartes* and others had endeavoured to banish them. It is surprizing that this author should represent it as greater presumption in us * to aim at the

* *Princip.* Part I. § 28. Nullas unquam rationes circa res naturales a fine, quem Deus aut natura in iis faciendis sibi proposuit, desumemus; quia non tantum debemus nobis arrogare ut ejus consiliorum participes nos esse putemus; sed ipsum ut causam efficientem rerum omnium considerantes, videbimus quidnam, ex iis ejus attributis quorum nos nonnullam notitiam voluit habere, circa illos ejus effectus, qui sensibus nostris apparent, lumen naturale quod nobis indidit concludendum esse ostendat.
 knowledge

knowledge of final causes, than to attempt to derive a complete system of the universe from the nature of the Deity, considered as the supreme efficient cause, or, after discarding mental and final causality, to resolve all into mechanism and metaphysical or material necessity. Surely this is the sort of causes that is most clearly placed in our view ; and we cannot comprehend why it should be thought arrogant in us, to attend to the design and contrivance that is so evidently displayed in nature, and obvious to all men ; to maintain, for instance, that the eye was made for seeing, tho' we may not be able either to account mechanically for the refraction of light in the coats of the eye, or to explain how the image is propagated from the retina to the mind.

Socrates, finding all dark and uncertain in the various systems of his predecessors, was satisfied that it was better to rest contented with the general view of nature open to all, than adopt any one of them ; and having applied himself to promote the practice as well as the theory of moral philosophy amongst his fellow-citizens, by his example and precepts, he merited the highest esteem and admiration of mankind *. *Plato*, however, and his followers, being sensible of the influence which natural knowledge

* See *Aul. Gellius*, Lib. 6. ch. 10. where an extraordinary instance of this is given from *Taurus* a Platonic philosopher. The *Athenians*, upon some difference with the inhabitants of *Megara*, made it capital for any of them to enter *Athens*. *Euclid* of *Megara*, after this edict, used to disguise himself as a woman, and travel twenty miles in the night to hear *Socrates*. Whence *Taurus* takes occasion to lament how much philosophy was sunk in esteem in his time. Now, says he, we see philosophers run of their own accord to attend at the gates of the young and rich, and there sit waiting to noon till their disciples have slept out their last night's debauch. *Diogenes Laertius*, however, speaks of a stranger who came to *Athens* and found fault with *Socrates* in some things.

must have on the most important truths, returned to it. The beauty of the universe was the favourite subject of the Platonists; and they used to recommend the contemplation and imitation of its regular and constant motions, by the practice of virtue, as the best means to recover their antient conformity with it in a prior state, and to become worthy of returning to the same state again. While a sect of the Atomists resolved all things into the motions and modifications of matter, *Plato* strove to raise the thoughts of men above the objects of sense, and zealously maintained the pre-eminence of active incorporeal and intellectual beings. These, according to him, are the true substances, the other the shadows; which last only, those gross philosophers could perceive; as he who has his back towards the light sees it not, or the bodies placed betwixt him and it, but the images projected from them only*. He speaks, however, sometimes of the insensible particles of bodies, which can only be perceived by the mind and understanding, ascribing different figures to them in the style of the atomical philosophy†. If he carried his fondness for his ideas too far, we must own, at least, that he erred on the most innocent side of the question, in opposition to the dangerous doctrines of *Democritus* and others. But however laudable the views of this amiable philosopher may have been, surely the unintelligible mystical doctrines of some of his followers‡ ought to admonish us to be on our guard against excesses, even in a good cause.

* *Plato* de republica, Lib. 7 & 10.

† *Plat.* Timæus.

‡ It were unnecessary to cite here instances of the most profound mysticism from *Plotinus*, and other platonists.

3. In the mean time the followers of *Pythagoras* flourished in *Italy*, and taught a philosophy that does not appear to have been so much the result of their own observations, as to have been transplanted from the east by their great master; who spent two and twenty years in those parts, and scrupled not to comply with the customs * most peculiar to the eastern nations, in order to obtain the freer access to their learned men. And as he was a man of extraordinary qualities and at the most pains, so he seems to have been the most successful of the ancients in getting acquainted with their philosophy. We find that his followers taught the true account of the planetary motions, particularly that the earth moved daily on its own axis, and revolved annually round the sun; and gave the same account of the comets which is agreeable to modern discoveries †. They also taught that every star was a world ‡, and that each of them had something corresponding to our earth, air, and water, in the vast expanse. The moon particularly, according to them, was inhabited by larger and more beautiful animals than this globe. We find some hints concerning the gravitation of celestial bodies, in what is related of the doctrines of *Thales* and his successors: but *Pythagoras* seems to have been better acquainted with it, and is supposed to have had a view to it, in what he taught concerning the harmony of the spheres §.

* He was circumcised in *Egypt* after the manner of the priests of that country, and is said to have been the most graceful person of his time. *Clem. Alexandr. Strom. Lib. I.*

† *Aristot. Meteorol. Lib. I. cap. 6. Plutarch. de placitis philosoph. Lib. III. cap. 2.*

‡ *Ibid. cap. 13, & 30.*

§ *Plin. Lib. II. cap. 22. Macrob. in somnium Scip. Lib. II. cap. 1. See also Plutarch. de animal. procreatione, è Timæo. εἰτε παλαι θεολόγοι πρεσβύτατοι φιλοσόφων ὄργανα, μεσικὰ θεῶν, &c. to the end.*

A musical chord gives the same notes as one double in length, when the tension or force with which the latter is stretched is quadruple: and the gravity of a planet is quadruple of the gravity of a planet at a double distance. In general, that any musical chord may become unison to a lesser chord of the same kind, its tension must be increased in the same proportion as the square of its length is greater; and that the gravity of a planet may become equal to the gravity of another planet nearer to the sun, it must be increased in proportion as the square of its distance from the sun is greater. If therefore we should suppose musical chords extended from the sun to each planet, that all these chords might become unison, it would be requisite to increase or diminish their tensions in the same proportions as would be sufficient to render the gravities of the planets equal. And from the similitude of those proportions, the celebrated doctrine of the harmony of the spheres is supposed to have been derived.

As these doctrines of the *Pythagoreans*, concerning the diurnal and annual motions of the earth, the revolutions of the comets, the inhabitants of the moon and stars, and the harmony of the spheres, are very remote from the suggestions of sense, and opposite to vulgar prejudices; so we cannot but suppose that they who first discovered them must have made a very considerable progress in astronomy and natural philosophy. It is no easy matter to persuade a person unacquainted with the true theory of motion, that the earth, which of all things in nature appears to be most fixed and stable, is carried on in such a manner, and with so much rapidity, in the expanse. To be satisfied of these doctrines, so as to reckon the earth amongst the stars, and consider
the

the stars as so many worlds, one must have got over many difficulties from sense, as well as from the religious prejudices that prevailed in those days. When therefore we find the accounts of them given by the *Greeks* to be very imperfect, mixed with errors and misrepresentations, it seems reasonable to suppose that they had some hints of them only from some more knowing nations who had made greater advances in philosophy; and that they were able to describe them perhaps not much better than we may imagine an ingenious *Indian*, after passing some years in *Europe*, and having had some access to learned men, would represent our systems to his countrymen after his return. Hence it was that the *Pythagoreans* do not seem to have been in a condition to defend their doctrines, tho' true; and *Aristotle* refutes them with the appearance of reason on his side. What he says of their system shews that either it was not described rightly by them, or that he misunderstood them. We are told that they taught that there was an earth opposite to our earth, and several other bodies revolving about the sun which were concealed from us by the earth, and that from this, they explained why there were more eclipses of the moon than of the sun *. On this occasion he urges against them a complaint, for which philosophers have too often given ground, “ That instead of suiting their
 “ philosophy to nature, they had misrepresented the
 “ phænomena, that they might appear conformable
 “ to their own suppositions.” But had he been

* De coelo, lib. II. cap. 15. We may be the less surprized that the *Greeks* had so imperfect accounts of the eastern learning, if it be true that some of the most noted amongst their philosophers, travelled into *Egypt* from a very different view than acquiring their philosophy. *Plato's* chief view is said to have been to sell his oil.

better acquainted with the phænomena and this system, he had formed a better judgment of it.

At this time geometry was in high esteem. We have reason to think that the fondness of the *Pythagoreans* and *Platonists* for it sometimes misled them, by inducing them to derive the mysteries of nature from such analogies of figures and numbers as are not only unintelligible to us, but in some cases seem not capable of any just explication. The use they made of the five regular solids in philosophy is a remarkable instance of this, and must have been a very important part of their scheme, if we may depend upon the antient commentators on *Euclid*; who tell us that he was a platonic philosopher, and composed his excellent elements for the sake of this doctrine. But as it is a matter of pure speculation, we cannot conceive that there can be any analogy between it and the constitution of nature; and they have not been successful who have of late endeavoured to explain this analogy; as we shall have occasion to shew afterwards, when we come to give some account of *Kepler's* discoveries. Nor is this the only instance, where a pursuit of analogies and harmonies has led us into error, in philosophy. Geometry can be of little use in it till *data* are collected to build on, and Lord *Verulum* has justly observed, *Mathesin philosophiam naturalem terminare debere, non generare aut procreare.*

4. From *Aristotle's* philosophy we may learn, that the greatest penetration, without other helps, will ever be of less service in enquiries into nature, than in metaphysics and dialectics; where the force of genius may indeed achieve wonders. Instead of the more antient systems, he introduced *matter*, *form*, and *privation* as the principles of all things: but it
does

does not appear that this doctrine was of great use to him in natural philosophy. He surpassed all the other philosophers, in stating the divisions and definitions relating to his subjects, with peculiar accuracy; yet some of his doctrines are so obscurely expressed, according to the confession of his most devoted disciples, that tho' they took the utmost pains to discover his meaning (and some of them, as is reported, in a very extraordinary manner) they were not able to penetrate into it; and it is disputed to this day what were his sentiments on some of the most important subjects.

He was enabled by the liberality of his pupil *Alexander* to make vast collections relating to the history of nature, at an immense expence, which have been often copied by natural historians since*. But in his general and theoretical writings concerning nature, tho' his reasonings may appear acute and subtle, the conclusions are commonly such as are overthrown by later discoveries. How he described the *Pythagorean* doctrine concerning the two-fold motion of the earth, and endeavoured to refute it, we observed above: in one of the treatises that are ascribed to him †, the author pretends to demonstrate that the matter of the heavens is ungenerated, incorruptible, and subject to no alteration; and supposes the stars to be carried round the earth in solid orbs. In these doctrines he was generally followed, till *Tycho* by his observations, and *Galileo* by his arguments, exposed their fallacy. Some have complained, that there is less mention of a Deity, in his

* According to *Pliny*, *Aristotle* wrote fifty volumes concerning animals, and several thousand persons in *Greece*, and *Asia*, by *Alexander's* orders, assisted him in his enquiries. The expence is said to have amounted to eighty talents.

† De cœlo.

extensive and various works, than in most of the antient philosophers ; that ΠΕΡΙ ΚΟΣΜΟΥ, (or, as some say it ought to be entitled, ΠΕΡΙ ΠΑΝΤΟΣ) excepted ; which for this reason has been ascribed to another author. But there are many who judge this admirable piece to be *Aristotle's* ; and *Gassendus* is of opinion that he composed it towards the end of his life, as the result of his most serious thoughts *.

It may be observed in favour of this great philosopher, that perhaps he did not intend his discoveries should be well understood from his public writings ; for we are told, that when his pupil * complained of his publishing some of his treatises, he insinuated, by his answer, that they would be understood by philosophers only. Had we a more perfect account of his doctrines concerning forms and qualities, possibly they might appear in a better light : perhaps he meant only to assert, in opposition to that branch of the atomists who followed *Democritus*, that the phænomena of nature could not be accounted for from matter and motion only ; but that the qualities of bodies arise from hidden powers acting variously on different combinations of the particles of matter, according to the laws established. The conduct of *Callisthenes*, whom he recommended to *Alexander* to accompany him in his *Asiatic* conquests, does great honour to *Aristotle* : A prosecution however, carried on by the *Athenian* priests, obliged him to abandon their city, to avoid the fate of *Socrates*.

Aristotle was for a long time called the Prince of Philosophers ; and possessed the most absolute authority in the schools, not in *Europe* only, but even in

* De physiologia Epicuri.

Africa, amongst Mahometans as well as Christians. They had translations of his works in *Persia* and at *Samarcand*; and no philosopher ever acquired so universal or so high an esteem. His opinion was allowed to stand on a level with reason itself; nor was there any appeal from it admitted, the parties, in every dispute, being obliged to shew that their conclusions were no less conformable to *Aristotle's* doctrine than to truth. This, however, did not put an end to disputes, but rather served to multiply them; for neither was it easier to ascertain his meaning than to come at the truth, nor was his doctrine consistent with itself. It is not improper to have this slavish subjection of philosophers in remembrance; because an high esteem for great men is apt to make us devoted to their opinions even in doubtful matters, and sometimes in such as are foreign to philosophy.

5. We have already mentioned the Epicurean system, and shall have occasion frequently to make remarks upon it afterwards. Whoever considers the extravagant doctrines of this sect, and of the other Dogmatists, of whatever denomination, Peripatetics or Stoics, may admire some of them for their morality, and more for their eloquence, it having been their chief business to dispute for their schemes and declaim upon them; but cannot be greatly surprized that, as to what relates to natural knowledge, so many joined the sceptics; and either maintained that it was impossible to discover truth, with some of them; or with others, that men were only in pursuit, not in possession of it. The sects, and subdivisions of sects, at length became so numerous, and their systems so various, that almost every person of any note addicted himself in some degree to philosophy: for none could be at a loss to find a sect and doctrine suited to his taste and inclination. But it does not

appear that this great increase of philosophers contributed much to the advancement of the science, or did service to truth: such was their licentiousness, and so great the variety of their opinions, that there has hardly appeared any doctrine, in later times, but may be supported by the authority of one or other of them. It has been justly observed that we may learn something from the faults and mistakes of others, in every art; but we do not find that the errors of one sect in philosophy served to put others on their guard. The great masters we have mentioned had given an unhappy example; and their successors exceeded them in grafting one fiction upon another, to serve their purposes. Thus the Platonists became unintelligible mystics, and the Peripatetics unwearied disputants; while every sect had its tale or scheme, magnified by the party, but condemned by all the rest.

When the antients, however, applied themselves to consider the heavens, or to collect the history of nature, they did not lose their labour; their observations, sometimes, suggested to them imperfect views of the true causes which obtain in the universe: and we have reason to admire some hints of this kind that appear in several passages of their writings, and seem to be anticipations of some of the most valuable modern discoveries. But, generally speaking, they indulged themselves too much in abstruse fruitless disquisitions concerning the hidden essences of things, and sought after a knowledge that was not suited to the grounds they had to build on. As to their accounts of the system of the world, the *Pythagorean* doctrines were quite forgot, and the opinions of *Aristotle* and *Eudoxus* universally prevailed. In process of time great liberties were taken with nature, solid orbs and epicycles were multiplied,

multiplied, to answer every appearance, till the universe in their descriptions lost its native beauty, and seemed reduced to a chaos again by their unhappy labours.

It is not worth while, nor of use for our purpose, to trace the history of learning thro' its various revolutions in the later ages, when philosophy and philosophers fell into contempt; when they became more distinguished by their extravagant opinions, manners and temper*, than by any real knowledge and merit. How different they were, so early as in the times of the *Cæsars*, from the famous *Pythagorean* lawgivers, the incomparable *Socrates*, and others who adorned the first ages of philosophy, we may learn from the picture given of them by *Tacitus*.
 “ *Nero*, says this author, used to bestow some time
 “ after meals in hearing the reasonings of different
 “ philosophers, and while each maintained his own
 “ sect, and every one exprelly contradicted another,
 “ they all conspired to expose their endless variance
 “ and broils, as well as to display their peculiar
 “ and favourite opinions; nay, there were some of
 “ those solemn masters of wisdom, highly fond of
 “ being seen with their gloomy aspect and rigid ac-
 “ cent, amidst the royal excesses and recreations of
 “ *Nero* †.

* *Sapientiam capillis et habitu jactant*, says *Lactantius* speaking of them. See also the complaint of *Taurus* the philosopher, cited from *Aul. Gellius* above in the notes on § 2. of this chapter.

† *Tacit. annal. lib 14.* We have said nothing of the *Chinese*, for tho' no nation has applied to astronomy for so long a time, or with so much encouragement from the public, they seem to have made little progress, by the accounts we have of them; this may be ascribed, in part at least, to their neglect of geometry (without which it is impossible to make great advances in astronomy) and their having no correspondence with other nations.

But

But the state of learning proved still more deplorable in a later period; that ought to be remembered, because it discovers to us the most cruel enemy to true philosophy. 'Twas sometime after the fall of the *Roman empire*, when the majesty and policy of that people had given way to *Gothic* barbarity, that superstition reigned uncontrouled, liberty of enquiry was proscribed, and a savage zeal sought to root out the memory of antient learning, by destroying the records of it, the inestimable product of the labours of past times. The fatal scheme proved but too successful, for soon a thick cloud seems to have darkened the understandings of men, and to have almost extinguished their natural faculties; in so much that a part of the succeeding times obtained the appellation of the leaden ages, as worse than the iron age of the poets. Authority for a long time usurped the place of reason, and, under the abused pretence of making them more submissive to heaven, mankind were enslaved and degraded. Here and there some appeared worthy of better times; but these were obliged to conform to the genius of that barbarous age: if they applied to true philosophy, it was either in a private and mysterious manner, or their abilities and merit served only to provoke severe and cruel treatment from their bigotted cotemporaries. This was the fate of the famous *Roger Bacon*, who appears to have made surprising advances in natural knowledge, for those times, and seems to have been acquainted with some inventions that are most commonly supposed to be of a later date.

Learning, neglected and despised in *Europe*, found a sanctuary amongst the *Saracens*, to whom we are indebted for several inventions, as well as for the
pre-

preservation of some of the works of the antients. They had so great a value for these, that it was usual with them to demand copies of them, by particular articles, in their treaties with the *Greek* emperors; tho' they had destroyed an inestimable treasure of this kind, at *Alexandria*, in their first conquests. The caliph *Almaimon* is celebrated for encouraging astronomical learning, erecting a great number of observatories over his dominions, and providing them with instruments of a prodigious size. By his order, a degree of the circle of the earth, was, first, measured with exactness, as far as we know. But, at length, their philosophers seem to have devoted themselves absolutely to *Aristotle*, in no less slavish a manner than the *Europeans*; and to a talkative philosophy that served only to produce endless disputes.

The cloud was, at length, gradually dispell'd in *Europe*: the active genius of man could not be enslaved for ever. The love of knowledge revived, the remains of antient learning, that had escaped the wreck of the dark ages, were diligently sought after; the liberal arts and sciences were restored, and none of them has gained more by this happy revolution than natural philosophy.

C H A P. III.

Of the modern philosophers before Des Cartes.

1. **T**HE revolutions of learning were compared, by *Aristotle*, to the rising and setting of the stars; and *Pliny* speaks of four periods of it that preceded his time, the *Egyptian*, *Assyrian*, *Chaldean*, and *Grecian*. Learning, after it was once lost in those countries, has never revived again; and,
of

of the produce of three of those periods, there is little or nothing left. The western parts of *Europe* have been more happy. After a long interval, learning has returned to them ; and the period which commenced upon the revolution we have mentioned, has already continued some hundred years. It was ushered in by several inventions of the greatest use. If we may judge from these, from the valuable discoveries that have been made in its progress, and from those which learned men are still in pursuit of, (which afford matter for their enquiries, and at the same time keep up their curiosity and expectation) we may justly hope that it will be long ere it comes to an end : and if it should likewise have its termination, it cannot, however, but be ever memorable in the history of learning, in future times ; unless a general oblivion overwhelm all memory and record.

The invention of convex and concave glasses was as old as the thirteenth century, tho' no one thought of putting two of them together to make a telescope, till three hundred years later. Upon which it has been justly observed, that those things which we handle daily may have valuable properties altogether unknown to us, which chance, or future tryals, may discover. The polarity of the magnetic needle, which was made use of in navigation early in the fourteenth century (if not sooner) facilitated the correspondence between distinct nations, and conducted *Columbus* to the discovery of the new world. It is obvious how advantageous to learning the art of printing has proved, which we owe to the same century. These, with several other new and surprizing inventions, produced a great change in the affairs of the world ; and a spirit of reformation soon shewed
itself,

itself, in every thing that had any connexion with the arts and sciences.

2. *Peurbachius*, with his scholar *Regiomontanus* and others, revived astronomical learning, in the fourteenth century. The celebrated *Copernicus* (who was born at *Tborn* in *Prussia* in 1473) succeeded them, “ a man, says *Kepler* *, of a vast genius, and “ what is of great moment in these matters, of a “ free mind.” When he considered the form, disposition and motions of the system, as they were then represented after *Ptolemy*, he found the whole void of order, symmetry and proportion; like a piece (as he expresses himself) made up of parts copied from different originals, which not fitting each other, should rather represent a monster than a man. He therefore perused the writings of the antient philosophers, to see whether any more rational account had ever been proposed of the motions of the heavens. The first hint he had was from *Cicero*, who tells us, in his *academical questions* (book 4.) that *Nicetas* a *Syracusian* had taught that the earth turned round on its axis, which made the whole heavens to appear to a spectator on the earth to turn round it daily. Afterwards, from *Plutarch* †, he found that *Philolaus* the Pythagorean had taught that the earth moved annually round the sun. He immediately perceived that, by allowing these two motions, all the perplexity, disorder and confusion, he had complained of in the celestial motions, vanished, and that, instead of these, a simple regular disposition of the orbits, and a harmony of the motions appeared, worthy of the great author of the world.

* Prefatio ad *Paulum* III. pontif. max.

† De placitis philosophorum, lib. 3. cap. 13.

'Twas soon after the year 1500 he began to form this judgment of the system, in his own thoughts : but being sensible how ill it would be received by the generality of men, and even of the learned of that time, he could not be induced to publish his account of the celestial motions, for more than thirty years. He had a great inclination, as he tells us, to have followed the manner of the *Pythagoreans*, who would not publish their mysteries to the world, but chose rather to deliver them from hand to hand to posterity ; not that they envied others the knowledge of them, but that the beautiful discoveries of great men, the fruit of all their labours, might not become the sport of the presumptuous and ignorant. It was not without the greatest sollicitations, and much struggling on his part, that at length he gave his papers to his friends, with permission to publish them ; and he lived only to see a copy of his book in 1543, a few hours before his death.

In this treatise, he restores the antient *Pythagorean* system, and deduces the appearances of the celestial motions from it. Every age since has produced new arguments for it ; and, notwithstanding the opposition it met with, from the prejudices of sense against the earth's motion, the authority of *Aristotle* in the schools, the threats of ignorant bigots, and the terror of the inquisition, it has gradually prevailed. The chief argument that had induced *Aristotle*, and his followers, to consider the earth as the centre of the universe, was that all bodies have a tendency towards the centre of the earth. In answer to this, *Copernicus* * observed, that it was reasonable to think there

* Equidem existimo gravitatem non aliud esse quam appetentiam quandam naturalem, partibus inditam a divina providentia opificis

there was nothing peculiar to the earth in this principle of gravity; that the parts of the sun, moon, and stars, tended likewise to each other, and that their spherical figure was preserved in their various motions by this power. Thus every step in true knowledge gives a glimpse or faint view of what lies next beyond it, tho' yet unrevealed, in the scale of nature.

3. The restoration of the *Pythagorean* system was a step of the utmost importance in true philosophy, and paved the way for greater discoveries; but the minds of men were not sufficiently prepared for it, at that time. A just account of the theory of motion was wanting to make them sensible of its simplicity and beauty, and to enable them to resolve, in a satisfactory manner, the obvious arguments that appeared against it. According to *Copernicus*, the earth revolved on its axis, with a rapid motion, from west to east. It was objected, that such a motion could not but have sensible effects on many occasions; that a stone, for instance, drop'd from the summit of a tower, ought to strike the ground, not at the foot of the tower, but at a distance westward, according to this doctrine; the tower being carried, by the diurnal motion, towards the east, while the stone was falling. In answer to this, the motion of the earth was compared to the uniform progressive motion of a ship at sea; and it was affirmed, that a stone drop'd from the top of the mast would strike the deck at the foot of it, tho' the ship was under

opificis universorum, ut in unitatem integritatemque suam sese conferant, in formam globi coeuntes. Quam affectionem credibile est etiam soli, lunæ, cæterisque errantium fulgoribus, inesse, ut ejus efficaciam in eâ qua se representant rotunditate permaneant; quæ nihilominus multis modis suos efficiunt circuitus. *Nicol. Copernici* revol. lib. 1. cap. 9.

fail, and advanced at a great rate while the stone was falling. This experiment is now beyond all question: but some, who tried it without due care and attention, having reported to *Tycho Brahe* that it had not succeeded*, this, with a mistaken zeal for the sacred writings, and perhaps an ambition of being the inventor of a new system, induced him to reject the doctrine of *Copernicus*, and propose a middle scheme. *Tycho* was too well acquainted with the planetary motions to suppose their centre any where else than in the sun; but that the earth might be quiescent, he supposed the sun, with all the planets, to be carried annually around it, while these, by their proper motions, revolved about the sun in their several periods. Having rejected the diurnal rotation of the earth on its axis, he was obliged to retain the most shocking part of the *Ptolemaick* system, and to suppose that the whole universe, to its farthest visible limits, was carried, by the *primum mobile*, about the axis of the earth every day. In this, however, he was abandoned by some of his followers, who chose rather to save this immense labour to all the spheres, by ascribing the diurnal motion to the earth, with *Copernicus*; and therefore were called *Semi-Tychonics*.

Tho' this noble *Dane* was not happy in establishing a new system, he did great service, however, to astronomy, by his diligence and exactness in making observations, for a long series of years. He discovered the refraction of the air, and determined the places of a great number of the fixed stars, with an accuracy unknown to the astronomers of former times. He demonstrated that the comets were higher than the moon, from their having a very small parallax, against the opinion which then pre-

* *Gassend. in vita Tychoonis.*

vailed. He discovered what is called the *variation* in the motion of the moon ; and, from his series of observations on the other planets, the theories of their motions were afterwards corrected and improved. For these services he will be always celebrated by astronomers.

4. Towards the latter end of the sixteenth century, and about the beginning of the next, *Galileo* and *Kepler* distinguished themselves in the defence of the *Copernican* system, and by many new discoveries in the system of the world. The excellent *Galileo* was no less happy in his philosophical enquiries, than in the celebrated discoveries which he made in the heavens, by the telescope. To the admirable *Kepler* we owe the discovery of the true figure of the orbits, and the proportions of the motions of the solar system : but the philosophical improvement of these phænomena was reserved for *Sir Isaac Newton*.

Kepler had a particular passion for finding analogies and harmonies in nature, after the manner of the Pythagoreans and Platonists ; and to this disposition we owe such valuable discoveries as are more than sufficient to excuse his conceits. Three things, he tells us, he anxiously sought to find the reason of, from his early youth ; why the planets were six in number, why the dimensions of their orbits were such as *Copernicus* had described from observations, and what was the analogy or law of their revolutions. He sought for the reasons of the first two of these in the properties of numbers and plane figures, without success. But at length reflecting that while the plane regular figures may be infinite in number, the ordinate and regular solids are five only, as *Euclid* had long ago demonstrated ; he imagined

E. that

that certain mysteries in nature might correspond with this remarkable limitation inherent in the essences of things; the rather that he found the *Pythagoreans* had made great use of those five regular solids in their philosophy. He therefore endeavoured to find some relation between the dimensions of those solids and the intervals of the planetary spheres; and imagining that a cube inscribed in the sphere of *Saturn* would touch by its six planes the sphere of *Jupiter*, and that the other four regular solids in like manner fitted the intervals that are betwixt the spheres of the other planets, he became persuaded that this was the true reason why the primary planets were precisely six in number, and that the Author of the world had determined their distances from the sun, the center of the system, from a regard to this analogy. Being thus possessed, as he thought, of the grand secret of the *Pythagoreans*, and being mightily pleased with his discovery, he published it in 1596, under the title of *Mysterium Cosmographicum*.

Kepler sent a copy of this book to *Tycho Brahe*, who did not approve of those abstracted speculations concerning the system of the world, but wrote to *Kepler*, first to lay a solid foundation in observations, and then, by ascending from them, to strive to come at the causes of things. This excellent advice, to which we owe the more solid discoveries of *Kepler*, deserves to be copied from his own account of it *.

“ Argumentum literarum *Brackei* (says he) hoc
“ erat, uti suspensis speculationibus a priori de-
“ scendentibus, animum potius ad observationes,
“ quas simul offerebat, considerandas adjicerem.
“ Inque iis primo gradu facto, post demum, ad

* Notæ in editionem secundam *Mysterii Cosmographici*.

“causas ascenderem.” In this judgment the great men of different times have frequently conspired, but few have faithfully followed it.

Tycho however, pleased with his genius, prevailed with *Kepler* to reside with him near *Prague* (where he passed the last years of his life, after having left his native country on some ill usage) and to assist him in his astronomical labours. Soon after this *Tycho* died, but *Kepler* made many important discoveries from his observations: he found that astronomers had erred, from the first rise of the science, in ascribing always circular orbits and uniform motions to the planets; that each of them moves in an-ellipsis which has one of its *foci* in the center of the sun; that the motion of each is really unequable; and varies so, that a ray supposed to be always drawn from the planet to the sun describes equal areas in equal times.

It was some years later before he discovered the analogy there is between the distances of the several planets from the sun, and the periods in which they complete their revolutions. He easily saw that the higher planets not only moved in greater circles, but also more slowly than the nearer ones; so that, on a double account, their periodic times were greater; *Saturn*, for example, revolves at a distance from the sun nine times and a half greater than the earth's distance from it; and the circle described by *Saturn* is in the same proportion; and as the earth revolves in one year, so, if their velocities were equal, *Saturn* ought to revolve in nine years and a half; whereas the periodic time of *Saturn* is above twenty-nine years. The periodic times of the planets increase, therefore, in a greater proportion than their distances from the sun; but not in so great a pro-

portion as the squares of those distances ; for if that was the law of their motions (the square of $9\frac{1}{2}$ being $90\frac{1}{4}$) the periodic time of *Saturn* ought to be above 90 years. A mean proportion betwixt that of the distances of the planets, and that of the squares of those distances, is the true proportion of the periodic times ; as the mean betwixt $9\frac{1}{2}$ and its square $90\frac{1}{4}$ gives the periodic time of *Saturn* in years. *Kepler*, after having committed several mistakes in determining this analogy, hit upon it at last in 1618, *May* 15th, for he is so exact as to mention the precise day when he found, that “ The squares
“ of the periodic times were always in the same
“ proportion as the cubes of their mean distances
“ from the sun.” This is only a very brief and summary account of the fruits of his great labours for many years on the observations made by *Tycho* *.

When *Kepler* saw that his disposition of the five regular solids amongst the planetary spheres was not agreeable to the intervals between their orbits, according to better observations, he endeavoured to discover other schemes of harmony. For this purpose, he compared the motions of the same planet at its greatest and least distances, and of the different planets in their several orbits, as they would appear viewed from the sun ; and here he fancied that he found a similitude to the divisions of the octave in music. These were the dreams of this ingenious man, of which he was so fond, that, hearing of the discovery of four new planets (the satellites of *Jupiter*) by *Galileo*, he owns that his first reflections were from a concern how he could save his favourite scheme, which was threatned

* See his *Tabulæ Rudolphinæ*, and *Comment. de stellâ Martis*.

by this addition to the number of the planets *. The same attachment led him into a wrong judgment of the sphere of the fixed stars † : for being obliged, by his doctrine, to allow a vast superiority to the sun in the universe, he restrains the fixed stars within very narrow limits. Nor did he consider them as suns, placed in the centers of their several systems, having planets revolving round them; as the other followers of *Copernicus*, from their having light in themselves, their immense distances, and from the analogy of nature, have concluded them to be. Not contented with these harmonies, which he had learned from the observations of *Tycho*, he gave himself the liberty to imagine several other analogies, that have no foundation in nature, and are overthrown by the best observations. Thus from the opinions of *Kepler*, tho' most justly admired, we are taught the danger of espousing principles, or hypotheses, borrowed from abstracted sciences, and of applying them, with such liberty, to natural enquiries.

A more recent instance of this fondness, for discovering analogies between matters of abstracted speculation and the constitution of nature, we find in *Huygens*, one of the greatest geometers and astronomers any age has produced: when he had discovered that satellite of *Saturn*, which, from him, is still called the *Huygenian* satellite, this, with our moon, and the four satellites of *Jupiter*, completed the number of six secondary planets then discovered in the system: and, because the number of the primary planets is also six, and this number is called by mathematicians a perfect number, (being equal to

* Dissert. cum nuncio sidereo.

† Epitome Astronomiæ, lib. 4. part. 1.

the sum of its aliquot parts, 1, 2, and 3, *) *Huygens* was hence induced to believe that the number of the planets was complete, and that it was in vain to look for any more †. We do not mention this to lessen this great man, who never perhaps reasoned in such a manner on any other occasion; but only to shew, by another instance, how ill-grounded reasonings of this kind have always proved: for, not long after, the celebrated *Cassini* discovered four more satellites about *Saturn*; so that the number of secondary planets now known in the system is ten. The same *Cassini* having found that the analogy, discovered by *Kepler*, between the periodic times and the distances from the center, takes place in the lesser systems of *Jupiter* and *Saturn*, as well as in the great solar system; his observations overturned that groundless analogy which had been imagined between the number of the planets, both primary and secondary, and the number six; but established, at the same time, that harmony in their motions, which will, afterwards, appear to flow from one real principle extended over the universe.

5. But to return to *Kepler*, his great sagacity, and continual meditation on the planetary motions, suggested to him some views of the true principles from which these motions flow. In his preface to the commentaries concerning the planet *Mars*, he speaks of gravity as of a power that was mutual betwixt bodies, and tells us that the earth and moon tend towards each other, and would meet in a point so many times nearer to the earth than to the moon, as the earth is greater than the moon, if their motions did not hinder it. He adds, that the tides

* *Elem. Euclid.* lib. 7. defin. ult.

† See the dedication of his *Systema Saturnium*.

arise from the gravity of the waters towards the moon. But not having just enough notions of the laws of motion, he does not seem to have been able to make the best use of these thoughts; nor does he appear to have adhered to them steadily, since in his *epitome* of astronomy, published eleven years after, he proposes a physical account of the planetary motions, derived from different principles.

He supposes, in that treatise, that the motion of the sun on his axis is preserved by some inherent vital principle; that a certain virtue, or immaterial image of the sun, is diffused with his rays into the ambient spaces, and, revolving with the body of the sun on his axis, takes hold of the planets and carries them along with it in the same direction; as a load-stone turned round in the neighbourhood of a magnetic needle makes it turn round at the same time. The planet, according to him, by its *inertia* endeavours to continue in its place, and the action of the sun's image and this *inertia* are in a perpetual struggle. He adds, that this action of the sun, like to his light, decreases as the distance increases; and therefore moves the same planet with greater celerity when nearer the sun, than at a greater distance. To account for the planet's approaching towards the sun as it descends from the *aphelium* to the *perihelium*, and receding from the sun while it ascends to the *aphelium* again, he supposes that the sun attracts one part of each planet, and repels the opposite part; and that the part which is attracted is turned towards the sun in the descent, and that the other part is towards the sun in the ascent. By suppositions of this kind, he endeavoured to account for all the other varieties of the celestial motions.

Now the laws of motion are better known than in *Kepler's* time, it is easy to shew the fallacy of every part of this account of the planetary revolutions. The planet does not endeavour to stop in its place in consequence of its *inertia*, but to persevere in its motion in a right line. An attractive force makes it descend from the *aphelium* to the *perihelium* in a curve concave towards the sun: but the repelling force, which he supposed to begin at the *perihelium*, would cause it to ascend in a figure convex towards the sun. We shall have occasion to shew afterwards, from *Sir Isaac Newton*, how an attraction or gravitation towards the sun, alone, produces the effects, which, according to *Kepler*, required both an attractive and repelling force; and that the virtue which he ascribed to the sun's image, propagated into the planetary regions, is unnecessary, as it could be of no use for this effect tho' it were admitted. For now his own prophecy, with which he concludes his book *, is verified; where he tells us that “ the
 “ discovery of such things was reserved for the suc-
 “ ceeding age, when the Author of nature would
 “ be pleased to reveal those mysteries.”

6. In the mean time, *Galileo* made surprizing discoveries in the heavens by the telescope, an instrument invented in that time; and, by applying geometry to the doctrine of motion, began to establish natural philosophy on a sure foundation. He made the evidence of the *Copernican* system more sensible, when he shewed from the phases of *Venus*, like to the monthly phases of the moon, that *Venus* actually revolves about the sun. He proved the

* Hæc et cætera hujusmodi latent in pandectis ævi sequentis, non antea discenda quam librum hunc Deus arbiter seculorum recluserit mortalibus. *Epit. Astron.*

revolution of the sun on his axis, from his spots ; and thence the diurnal rotation of the earth became more credible. The four satellites that attend *Jupiter* in his revolution about the sun, represented, in *Jupiter's* lesser system, a just image of the great solar system ; and rendered it more easy to conceive how the moon might attend the earth, as a satellite, in her annual revolution. By discovering hills and cavities in the moon, and spots in the sun constantly varying, he shewed that there was not so great a difference between the celestial and sublunary bodies as the philosophers had vainly imagined *.

He did no less service by treating, in a clear and geometrical manner, the doctrine of motion, which has been justly called the key of nature. The rational part of mechanics had been so much neglected, that there was hardly any improvement made in it, from the time of the incomparable *Archimedes* to that of *Galileo* ; but this last named author has given us fully the theory of equable motions, and of such as are uniformly accelerated or retarded, and of these two compounded together. He, first, demonstrated, that the spaces described by heavy bodies from the beginning of their descent are as the squares of the times, and that a body, projected in any direction that is not perpendicular to the horizon, describes a parabola. These were the beginnings of the doctrine of the motion of heavy bodies, which

* *Galileo* observed something very extraordinary about *Saturn*, which he imagined to be two Satellites almost in contact with his body ; and *Des Cartes* fancied these two Satellites were quiescent in his vortex, because (as he supposed) *Saturn* did not turn round on his axis ; but *Huygens* shewed that this appearance proceeded from a ring that encompasses his body, without touching it, and accompanies him in his revolution about the sun.

has been since carried to so great a height by Sir *Isaac Newton*.

He also discovered the gravity of the air, and endeavoured to compare it with that of water; and opened up several other enquiries in natural philosophy. He was not esteem'd and followed by philosophers only, but was honoured by persons of the greatest distinction of all nations. *Des Cartes*; indeed,* after commending him for applying geometry to physics, complains that he had not examined things in order, but had enquired into the reasons of particular effects only; adding that, by his passing over the primary causes of nature, he had built without a foundation. He did not, 'tis true, take so high a flight as *Des Cartes*, or attempt so universal a system; but this complaint, I doubt, must turn out to *Galileo's* praise; while the censure of *Des Cartes* shews that he had the weakness to be vain of the worst part of his writings.

But all the merit of this excellent philosopher and elegant writer could not preserve him from persecution in his old age. Some pretended philosophers, who had imprudently objected against his new discoveries in the heavens, when they found themselves worsted and exposed to ridicule, turned their hatred and resentment against his person. He was obliged, by the rancour of the Jesuits (as 'tis said †) and the weakness of his protector, to go to *Rome*, and there solemnly renounce the doctrine of the motion of the

* Epistol. part 2. epist. 91.

† Vir in omni mathematicum parte summus *Galileus Galilei*, Jesuitarum in ipsum odio, ac principis Thufci sub quo vixit foecordi metu, coactus ire Romam, ideo quod terram movisset, non vetante vestro *Hortensio*, durè habitus, ut majus vitaret malum, quasi ab ecclesia edoctus, sua scita rescidit. *Hug. Grotius* in epistola ad *Vossium*, Lutet. 17. maii, 1635.

earth, which he had argued for with so much ingenuity and evidence *. After this cruel usage he was silent for some time, but not idle; for we have valuable pieces of his of a later date.

7. Sir *Francis Bacon* Lord *Verulam* †, who was cotemporary with *Galileo* and *Kepler*, is justly held amongst the restorers of true learning, but more especially the founder of *experimental philosophy*. When he was but sixteen years old, he began to dislike the vulgar physics and what was called *Aristotle's* philosophy. He saw there was a necessity for a thorough reformation in the way of treating natural knowledge, and that all theory was to be laid aside that was not founded on experiment. He proposed his plan in his *instauratio magna*, with so much strength of argument, and so just a zeal, as renders that admirable work the delight of all who have a taste for solid learning.

He considers natural philosophy as a vast pyramid, that ought to have the history of nature for its basis; an account of the powers and principles that operate in nature, which he calls the physical part, for its second stage; and the metaphysical part, that treats of the formal and final causes of things, for its third stage. But as for the summit of this pyramid, the supreme of nature, *opus quod operatur Deus a principio usque ad finem*, as he expresses it, he doubts if men can ever attain to the full knowledge of it. The philosophers who strive to erect these by the force of abstract speculation he compares to the

* He was besides condemned to a year's imprisonment in the inquisition, and the penance of repeating daily some penitential psalms.

† He was born in 1560, *Galileo* in 1564.

giants of old, who, according to the poets, endeavoured to throw mount *Ossa* upon *Pelion*, and *Olympus* upon *Ossa*.

An artist, says this noble author, would expose himself to the justest ridicule, who, in order to raise some vast obelisk, should attempt it by the force of his arms, instead of employing the proper machines; or if, after finding himself unequal to the task, he should call for the aid of more workmen in the same way. Would he appear less ridiculous if he should next set about chusing his men, and examining them carefully, that he might employ the vigorous and robust only? or if, after he found this was to no purpose, he should then apply himself to study the athletic art, and learn to compose curious ointments for strengthening their limbs, or consult learned physicians, who, by proper medicaments, should promote their health and vigour? Nor are they less absurd, in our noble author's judgment, who labour to interpret nature by the force and subtlety of genius only, tho' they should assume the aid of the acutest men in the same work, and carry the dialecticks, or the art of reasoning, to the greatest height for this purpose.

The *empirical* philosophers, those who have no higher view than to collect the history of nature, he compares to the ants, who gather the grain and lay it up as they find it (unless it be true, as is reported of them, that they first take care it should not germinate or become fruitful;) the *Sophists* to the spiders, who form their webs from their own bowels, to catch unwary insects in their aerial flights; while the bee that gathers the matter from the flowers of the field, from which with admirable skill she makes her honey, is the emblem of the true philosopher; who
neither

neither trusts wholly to his own understanding, nor contents himself with recording the matter with which he is furnished from natural history or mechanical experiments ; but, by reasoning skilfully from them, brings forth truth and science, the great and noble production of the human faculties. From the neglect of experiments it arose, that while nature was infinite, natural knowledge was at a stand for many ages, and that the various sects wandered in the dark, without kindling any light to guide them, or finding any path to conduct them in her mazes. But, from a happy conjunction of the experimental and rational faculties, Lord *Verulam* conceived the highest expectations. *Alexander*, he tells us, and *Cæsar* performed exploits that are truly greater than those reported of king *Arthur* or *Amadis de Gaul* ; tho' they acted by natural means, without the aid of magic or prodigy.

It was with great justice, and very seasonably, he reprehended those * who, “ upon a weak conceit of
“ sobriety, or ill-applied moderation, thought or
“ maintained that a man can search too far, or be
“ too well studied in the book of God’s word, or in
“ the book of God’s works. But rather, he adds,
“ let men awake themselves, and chearfully endea-
“ vour and pursue an endless progress and profi-
“ ciency in both ; only let them beware lest they
“ apply knowledge to pride, not to charity, to
“ ostentation, not to use ” He observes, that a superficial taste of philosophy may perchance incline the mind to atheism ; but a full draught thereof brings it back again to religion : in the entrance of philosophy, when the second causes most obvious to the senses offer themselves to the mind, we are apt

* *Bacon’s Advancement of Learning*, lib. 1.

to cleave unto them, and dwell too much upon them, so as to forget what is superior in nature. But when we pass further, and behold the dependency, continuation and confederacy of causes, and the works of providence, then, according to the allegory of the poets, we easily believe that the highest link of nature's chain must needs be tied to the foot of *Jupiter's* chair; or perceive "That philosophy, like "*Jacob's* vision, discovers to us a ladder, whose "top reaches up to the footstool of the throne of "God."

The *Aristotelian* philosophy appeared unsatisfactory to Lord *Bacon*, not from want of esteem for its author, whom he always used to extol; but because it seemed fit for disputes only, and incapable of producing real fruit. *Aristotle*, he said, had suited his physics to his logic, instead of giving such a kind of logic as might be of real use in physics. To supply this defect, he composed his *novum organum*; where his chief design is to shew how to make a good *induction*, as *Aristotle's* was to teach how to make a good *syllogism*. Had the philosophers, since Lord *Verulam's* time, adhered more closely to his plan, their success had been greater; and Sir *Isaac Newton's* philosophy had not found the learned so full of prejudices against it, in favour of some systems lately invented and mightily extolled by speculative men; that while all admired the sublime geometry which shone throughout his work, few for some time appeared to be disposed to hearken to his philosophy, or in a condition to judge of it impartially.

8. However, Lord *Bacon's* exhortations and example had a good effect; and experimental philosophy has been much more cultivated since his time than

than in any preceding period. Geometry and philosophy advanced together at a great pace, and gave mutual aid to each other. The evidence of geometry began to take place in philosophy, while all things were examined by number, weight, and measure; and the principles of the theory of motion, being now clearly understood, furnished excellent illustrations of the abstruse parts of geometry. *Galileo* had scholars worthy of so great a master, by whom the gravitation of the atmosphere was established fully, and its varying pressure accurately and conveniently measured, by the column of quicksilver of equal weight sustained by it in the barometrical tube. The elasticity of the air, by which it perpetually endeavours to expand itself, and, while it admits of condensation, resists in proportion to its density, was a phænomenon of a new kind (the common fluids having no such property) and of the utmost importance to philosophy. These principles opened up a vast field of new and useful knowledge, and explained a great variety of phænomena, which had been accounted for in an absurd manner before that time. It seem'd as if the air, the fluid in which men lived from the beginning, had been then first discovered. Philosophers were every where busy enquiring into its various properties and their effects; and valuable discoveries rewarded their industry. Of the great number who distinguished themselves on this occasion, we cannot but mention *Torricelli* in *Italy*, *Paschal* in *France*, *Otto Guericke* in *Germany*, and *Boyle* in *England*.

The views of philosophers began now to be mightily enlarged, not by their discoveries concerning the air only, but likewise by their enquiries into the more potent element fire and its effects, and into the chymical composition, resolution, and changes
of

of bodies. For about this time chymists began to speak more intelligibly concerning their art, and to connect it in some degree with natural philosophy, or to consider it, at least, as not quite foreign to it. This we owe in great measure to the honourable Mr. *Boyle*, whose favourite study chymistry is said to have been, and who was happy in an easy and familiar manner of describing the subjects which were treated by him.

It must be owned that none ever took so great pains to promote natural knowledge, in all its branches, or the best improvement that can be made of it, than this excellent person. It has been observed that he was born the same year that Lord *Bacon* died, as if he had been destin'd to carry on his plan. He spared no labour nor cost in collecting the history of nature, and making curious and useful experiments of all sorts. As Lord *Bacon*'s plan comprehended the whole compass of nature, so the variety of enquiries prosecuted by Mr. *Boyle*, with great care and attention, is very surprizing, and perhaps not to be parallel'd. Hydrostatics, tho' a most useful branch of mechanical philosophy, had been but ill understood, till he established its principles, and illustrated its paradoxes, by a number of plain experiments, in a satisfactory manner. The doctrine of the air afforded him an ample field; and, in all his researches, he shewed a genius happily turned for experimental philosophy, with a perfect candour, and a regular condescension in examining with patience, and refuting, without ostentation, the errors which philosophers had been led into from their prejudices, and the many artful subterfuges by which they strove to support them. The unexceptionable integrity, extensive charity, and singular piety of this excellent person did great honour to philosophy, and

and formed an eminent part of his character. The world he considered as the temple of God, and * “ man (to use his own words) as born the priest of “ nature, ordained (by being qualified) to celebrate “ divine service, not only *in* it but *for* it.” Not satisfied with having promoted the belief of a Deity and the evidence of true religion, to the utmost of his power, in the great number of volumes composed by him, on every occasion during the course of a laborious life, he has taken care, by his will, to perpetuate a succession of advocates for it, who should make the same improvement not of his discoveries only, or of those of former times, but of what should be produced by future ages. In this design, worthy of him, the success has been answerable to his intentions; and surely such a man, we must allow, was not an ornament to his own age and country only, but a publick benefit to all times and nations.

We are now arrived at the happy æra of experimental philosophy; when men, having got into the right path, prosecuted useful knowledge; when their views of nature did honour to them, and the arts received daily improvements; when not private men only, but societies of men, with united zeal, ingenuity and industry, prosecuted their enquiries into the secrets of nature, devoted to no sect or system. But we are obliged to abandon, at present, the agreeable task of following them in their discoveries, in this flourishing period of science, to give account of a most illusive scheme of speculative philosophy that prevailed amongst many at this very time, and, by misleading ingenious men, corrupted their notions and retarded their progress. It seems that, however

* Boyle's Usefulness of Natural Philosophy, part i. essay 3.

fertile this period was in new inventions, nature did not unveil herself readily enough to satisfy the impatience of some men, who could not be contented with those views of her which time and industry produced to them. Therefore they hearkened again to the vain promises of those who pretended to unravel all her mysteries at once, by the force of their abstracted speculations. The *Cartesian* system was the most extensive, and (according to many) the most exquisite in its contrivance, of any that have been imagined. The author of it was a bold philosopher, and doubtless of a subtle genius, to indulge which he retired from the world for many years. He valued himself on his clear ideas, and is allowed to have contributed to dissipate the darkness of that sort of science which prevailed in the schools. If we may believe some accounts, he rejected a *void* from a complaisance to the taste which then prevailed, against his own first sentiments; and amongst his familiar friends, used to call his system his philosophical romance. It had however great success; and his doctrines still prevail so much, that it is necessary for our purpose to give a short account of them.

C H A P. IV.

Of the philosophical principles of Des Cartes, the emendations of his followers, and the present controversies in natural philosophy.

DE S Cartes begins his *principia* by shewing the necessity of doubting first of every thing, in order to our obtaining certain knowledge; and recommends to his readers to consider his reasons for doubting of all things, not once only, but to employ weeks, or even months, on these alone, before he

he proceed farther. He first establishes the certainty of our own existence, and that of our ideas of which we are intimately conscious to ourselves; of the existence of which, however, after all he has said, it seems impossible for us to doubt for a moment. From our having the idea of a Being infinitely perfect and necessarily existing, he concludes that such a Being actually is; upon whose will he makes the certainty of self-evident propositions, or axioms *, as well as of all other necessary truths, to depend.

From the knowledge of the cause established in this manner, he pretends to deduce a complete knowledge of his effects, by necessary steps. It is clear, says he †, that we shall follow the best method in philosophy, if from our knowledge of the Deity himself, we endeavour to deduce an explication of all his works; that so we may acquire the most perfect kind of science, which is that of effects from their causes. As for final causes he rejected them from philosophy, as we observed above; and from these passages, which represent the genius of this author's philosophy, and from the manner in which he sets out, we may already form some judgment how hopeful his project was.

From the veracity of the Deity, he infers the reality of material objects, which are represented to us as existing without us. He places the essence of matter in extension; for this alone remains, he says,

* According to him, the Deity did not will that the three angles of a triangle should be equal to two right ones, because he knew that it could not be otherwise; but, because he would that the three angles of a triangle should necessarily be equal to two right ones, therefore this is true and can be no otherwise.

† See the passages cited above from his *Principia*, in the notes upon § 4, ch. 1.

when we reject hardness, colour, weight, heat and cold, and the other qualities which, we know, a body can be without. Hence he easily concludes that there can be no void, or extension without matter. He adds, however, immediately afterwards, as properties of matter, that its parts are separable and moveable; tho' these seem to imply more than mere extension.

He defines motion to be the translation of a body from the neighbourhood of other bodies that are in contact with it, and are considered as quiescent, to the neighbourhood of other bodies; and thus makes no distinction between absolute or real, and relative or apparent motions; both of which equally agree to this definition. The reason he gives why the same quantity of motion must be preserved for ever in the universe, without any augmentation or diminution in the whole, must appear concise, and very extraordinary. It is no other than that God must be supposed to act in the most constant and immutable manner. From the same property of the Deity, he infers that a body must continue in its state as to rest, motion, figure, &c. till some external influence produce a change; which is his first law of nature: that the direction of motion is naturally rectilinear, or that a body never changes its direction of itself; which is his second law: and that a body in motion, when it meets with another moving with a greater force, is reflected without losing any part of its first motion; but when it meets with a body moving with less force, it then carries this body along, and loses as much motion as is transferred to it; and this is his third law of nature. He accounts for the hardness of bodies from their parts being quiescent with respect to each other; and for fluidity, from their being moved perpetually in
all

all directions. He concludes the second part of his book with telling us, that these principles are sufficient for explaining all the phænomena of nature, and that no other ought to be admitted or even wished for.

He afterwards proceeds to shew how the universe might have assumed its present form, and may be for ever preserved, by mechanical principles. He supposes the particles of matter to have been angular, so as to replenish space without leaving any interstices between them; and to have been in perpetual agitations, by which the angular parts being broke off, the particles themselves became round, and formed what he calls the matter of the second element. The angular parts, being ground into the most subtile particles of all, became the matter of his first element, and served to fill all the pores of the other. But there being more of this first element than was necessary for that purpose, it became accumulated in the centers of the *vortices*, of which he imagined the universe to consist, and formed there the bodies of the sun and stars. The heavens were filled with the matter of the second element, the medium of light. But the planets and comets consisted of a third element grosser than the other two, the generation of which he traces at length through all its steps. According to him, the matter of the first element must have constantly flowed out through the interstices between the spherical particles of the second element, where the circular motion is greatest, and must have returned continually at the poles of this motion towards the centre of the vortex; where being apt to cohere together, they at length produced the grosser particles of the third; and when these came to adhere in a considerable quantity, they gave rise to the spots on the surfaces of

the suns or stars. Some being crufted over with fuch spots became planets or comets; and the force of their rotation becoming languid, their vortices were abforbed by fome more potent neighbouring vortex. In this manner the folar fyftem was formed, the *vortices* of the fecondary planets having been abforbed by the vortex of the primary, and all of them by that of the fun. He contends that the parts of the folar vortex increafe in density, but decreafe in celerity, to a certain diftance; beyond which he fupposes all the particles to be equal in magnitude, but to increafe in celerity as they are farther from the fun. In thofe upper regions of the vortex he places the comets; in the lower parts he ranges the planets; fupposing thofe that are more rare to be nearer the fun, that they may correspond to the density of the vortex where they are carried round.

He accounts for the gravity of terreftrial bodies from the centrifugal force of the æther revolving round the earth; which, he imagined, muft impell bodies downwards that have not fo great a centrifugal force, much in the fame manner as a fluid impells a body upwards that is immerged in it, and has a lefs fpecific gravity than it. He pretended to explain the phænomena of the magnet, and to account for every thing in nature, from the fame principles.

2. There never was, perhaps, a more extravagant undertaking than fuch an attempt, to deduce, by neceffary confequences, the whole fabric of nature, and a full explication of her phænomena, from any ideas we are able to form of an infinitely perfect Being. Was it not for the high reputation of the author, and of his fyftem, it would be hardly excufable to make any remarks upon fuch a rhapsody.

Should

Should we allow the principles he builds on, and his method, it must be obvious with how weak an evidence the consequences are connected with each other, in this visionary chain. How just a method he has taken to establish the existence and attributes of the Deity we shall not enquire, nor how far his making all truth and falsehood dependent on the will of the Deity tends to weaken all science and confound its principles. While he supposes extension to constitute the complete essence of matter, he neglects solidity, and the *inertia* by which it resists any change in its state of motion or rest; which distinguish body from space. If extension be understood to be the essence of matter, it is a trifling proposition to affirm that all space is full of matter, according to this definition. But still the question will remain, whether all space is full of that solid, moveable and resisting substance commonly called *body*. And as many parts of space appear to make no sensible resistance to motion, while others resist variously in proportion to the density of the medium diffused over them, we thence learn there is space void of what is commonly called *matter*. The comets which move with equal freedom in all directions with very rapid motions, and carry along with them tails of a prodigious size, consisting of some highly rarified matter, shew that the heavens are not replenished with dense fluids that admit no void. For it is evident in experimental philosophy that the resistance of fluids increases, *ceteris paribus*, with their density; so that all motion would soon languish in a fluid, which, having no pores, must far surpass quicksilver, or the heaviest solids, in density. Nothing is more evident, than that the force requisite to move two equal bodies with a given velocity, is double that which would produce the same celerity in either of them. When we compound greater

bodies from leffer, or when we resolve them into their parts, we find that the resistance or *inertia* increases or decreases in proportion to the quantity of matter. Therefore when the velocity is given, if a body moving in a denser fluid displaces more matter to make way for itself, the resistance which it meets with being equal to the motion communicated to the parts of the fluid, it must find a resistance proportionally greater.

It is not only from the free motions of the planets and comets that we learn the absurdity of the doctrine of an universal plenitude. The most common and plain phænomena of the motion of bodies, at or near the surface of the earth, are sufficient to overthrow it; for we find that they meet with no sensible resistance but from the air: whereas so dense a fluid as would replenish all space equally would necessarily produce a very great resistance.

It is objected *, that by supposing this dense fluid which replenishes space to penetrate the pores of bodies with the utmost freedom, (as light passes through transparent bodies, and the magnetic and electric *effluvia* through most kinds of bodies) its resistance will then be incomparably less than in proportion to its density; for then the resistance will not be measured by the density of the fluid, because the much greater part passes through the pores of the body in motion, freely without resistance. Supposing this to be admitted, it is, however, obvious that, even in this hypothesis, the resistance of a golden ball in a *plenum* would be still very great. For this subtle fluid, how penetrating soever it be, must resist the

* In a small piece published on this subject, a few years ago, by an ingenious gentleman.

solid parts of the ball; which cannot move in the fluid without displacing its parts, and losing as much motion as must be communicated to those parts; and this resistance depends on the quantity of solid parts in the ball: whereas the resistance which the same ball meets with in quick silver (which we suppose to have no passage through the ball) depends on the quantity of the solid parts in an equal bulk of the quick silver, which must be moved to make way for the ball. And this being less than the quantity of solid parts in an equal bulk of the golden ball, in proportion as the specific gravity of quick silver is less than that of gold, it follows that the resistance of a golden ball, moving in such a subtile penetrating *plenum*, would still be greater than its resistance in quick silver. To illustrate this farther, the specific gravity of gold being to that of quick silver nearly as 195 to 140, suppose a golden ball consisting of 195 solid particles to move in the *plenum* with a given velocity, and to describe a very small space; and then suppose the same ball to move in quick silver with the same velocity over the same space; in the former case, the solid parts of the ball displace a certain quantity of the *plenum*, suppose a quantity equal to the ball, or 195 parts; in the latter case, they displace an equal bulk of the quick silver, that is, 140 solid particles. But because it may be said for those who maintain an universal plenitude, that the golden ball meets with a resistance from the subtile fluid that replenishes space, while it moves in the quick silver, as well as from the quick silver itself; let this likewise be allowed, and let us even suppose it to meet with as much resistance from the *plenum*, while it moves in the quick silver, as when it moves in a space free from any gross fluid; yet it will still appear that the resistance of the golden ball in the *plenum* ought to bear at least as great a proportion

to

to its resistance in quick silver, as the density of gold is to the sum of the densities of gold and quick silver, or as 195 to 335, and consequently ought to be eight times greater than its resistance in water. This is the least resistance such a ball could meet with in a *plenum*, should we allow the suppositions that are most favourable in this doctrine; and this resistance would soon put an end to the motions of bodies. But it is evident that we allowed too much in favour of their doctrine, when we supposed the ball moving in the quick silver to meet with a resistance equal to the sum of the resistances that it would meet with from the *plenum* and quick silver separately. For, according to this supposition, its resistance in quick silver would be to its resistance in water, as the sum of the densities of gold and quick silver to the sum of the densities of gold and water, that is, as 335 to 205, or 67 to 41; so that the resistance of quick silver would not be double of that of water, or even double of that of air; than which nothing can be more contradictory to experiment.

It is of no importance to this argument how rare gold, quick silver, or the heaviest bodies, be supposed; since the resistance of quick silver in fact is known to be very great, and is not altered by such suppositions: neither is the proportion of the density of gold to that of quick silver (upon which proportion the argument is founded) affected by them. For it will always be found that the resistance of a golden ball in a *plenum* (how freely soever it pass through the pores of the ball, and how large or numerous soever these pores may be) must correspond to the solid matter in the ball; which is greater than the solid matter in any equal bulk of any of our fluids; upon which their resistance depends. The supposing the solid matter in the quick
silver

silver to occupy only the thousandth or millionth part of its bulk, has no other effect but that it supposes the *inertia* of a given quantity of solid matter to be increased in the same proportion with the rarity of the quick silver, whose *inertia* is in fact ascertained.

The resistance which arises from the tenacity or adhesion of the parts of fluids may be diminished; but still the resistance which arises from the *inertia* of the matter remains: if this could be taken away, as the matter would have no resistance, so it is not easy to conceive how it could have any activity or mechanical force to impel bodies, or to produce any of the effects which are attributed to the subtile matter of the *Cartesians*. For action and reaction are always equal, and we know of no force in bodies but what arises from their resistance to change their state, or their *inertia*. Without this there could be no centrifugal force, the favourite power by which those philosophers endeavour to explain the phænomena of nature.

They suppose the particles of those subtile fluids to move constantly and equally in all directions; and, by the favour of this hypothesis, they imagine that they may suppose them to act but not resist. But they have neither made this strange supposition probable, nor even credible, nor can they shew that it would answer their purpose. A motion of a fluid favours the motion of a body in it, only as far as it is in the same direction; and an intestine motion of the parts of the fluid, equal in all directions, cannot make the resistance less than if there was no motion of the parts. It is supposed by many that the particles of common fluids, water or air for example, are in a constant intestine motion; but this does
not

not hinder those fluids from resisting in proportion to their density.

We are told by some, that it is impossible to conceive a *vacuum*. But this surely must proceed from their having imbibed *Des Cartes's* doctrine, that the essence of body is constituted by extension; as it would be contradictory to suppose space without extension. To suppose that there are fluids penetrating all bodies and replenishing space, which neither resist nor act upon bodies, merely in order to avoid the admitting a *vacuum*, is feigning two sorts of matter, without any necessity or foundation; or is tacitly giving up the question. As for Mr. *Leibnitz's* arguments against a *vacuum*, we defer them till we come to consider the emendations that have been made to this system.

The same quantity of motion is not always preserved in the universe, as *Des Cartes* rashly concluded from the immutability of the Deity. The quantity of absolute motion is continually varying; it is diminished in the composition of motion, and, in many cases, in the collisions of bodies that have an imperfect elasticity; and it is increased in the resolution of motion, and, in some cases, in the collisions of elastic bodies. It requires an active principle to account for the hardness of bodies; and the particles being at rest is not sufficient for this purpose; for this would not hinder them to be separated from each other by the least force. There is hardly one article in this scheme but what is, in like manner, liable to insuperable difficulties.

After all, *Des Cartes* saw the necessity of having recourse to observation, tho' unwillingly; and he appears to be at a loss how to acknowledge it, after
having

having boasted so much of his principles. He tells us that he found these so extensive and fertile*, that many more things followed from them than we find in the visible world. Other philosophers have complained that they were able to account for too little of nature: *Des Cartes* finds that his principles were more than sufficient to account for all her phænomena, and seems only to fear lest he should account for too much. Therefore he has recourse to the phænomena, not because he would prove any thing from them; for he takes care that we should not have so mean an opinion of his philosophy, as to imagine he would establish it on facts; but that he might be able to determine his mind to consider some of those innumerable effects, which he judged might proceed from the same causes, rather than others. He likewise acknowledges †, that the same effect might be deduced, from his principles, many different ways; and that nothing perplexed him more than to know which of them obtained in nature. In those passages he magnifies his principles, in order to conceal the weakness of his system, with an affectation that only serves to make it more evident, and appear unworthy of so great a man.

3. *Des Cartes*, by placing the essence of matter in extension alone, gave occasion to others to draw

* He cites the effects, as he tells us, *Non quidem ut ipsis tanquam rationibus utamur ad aliquod probandum; cupimus enim rationes effectuum a causis, non autem e contrario causarum ab effectibus deducere; sed tantum ut ex innumeris effectibus, quos ab iisdem causis produci posse judicamus, ad unos potius quam alios considerandos mentem nostram determinemus.*

† Sed confiteri me etiam oportet potentiam naturæ esse adeo amplam, ut nullum fere amplius particularem effectum observem, quem statim *variis modis* ex iis principiis deduci posse non agnoscam: nihilque ordinario mihi difficilius videri, quam invenire quo ex his modis inde dependet. *De Methodo*, § 6.

consequences,

consequences, from this doctrine, of a dangerous nature; which undoubtedly he would have disowned, tho' 'tis not easy to see how he could have got rid of them. As we are not able to conceive that space can be annihilated, or that there ever was a time when space or expansion was not; so if we allow that extension alone constitutes the essence of matter, we cannot but ascribe infinity, eternity, and necessary existence to it. In this manner *Spinoza* reasons from the *Cartesian* principles, affirming that matter is not only infinite and necessary, but also that it is one and indivisible *. “ This, says he, cannot be denied by those who reject the possibility of a *vacuum*; “ for if matter could be so divided that its parts “ should be really distinct, why might not one part “ be annihilated, the rest remaining connected with “ each other as before? since of things which are “ really distinct from each other, the one can exist “ and remain in its state without the other.” In another place, he tells us, that if any one part of matter was annihilated, all extension would vanish with it †. This author appears to have been very con-

* Nam si substantia corporea ita posset dividi ut ejus partes realiter distinctæ essent, cur ergo una pars non posset annihilari manentibus reliquis, ut ante, inter se connexis? Et cur omnes ita aptari debent ne detur vacuum? Sane, rerum quæ realiter ab invicem distinctæ sunt, una sine aliâ esse & in suo statu manere potest. Cum igitur vacuum in naturâ non detur, sed omnes partes ita concurrere debent ut detur vacuum, sequitur hinc etiam easdem non posse realiter distingui; hoc est, substantiam corpoream, quatenus substantia est, non posse dividi. *Spinoz. Ethic. part 1. prop. 15. schol.*

† Si una pars materiæ annihilaretur, simul etiam tota extensio evanesceret. *Epist. 4. ad Herr. Oldenb.*

From these and other passages it appears, that this author was unhappily misled by the doctrine of *Des Cartes*, that the essence of matter is constituted by extension. It must be owned, however, that many of the *Cartesians* endeavoured to wrangle away the dreadful conclusion: but they had shortned their work, and

conversant in the writings of *Des Cartes* †, the two first parts of whose *principia* he reduced into the geometrical form. Mr. *Leibnitz* himself calls spinozism *un Cartesienisme outré*; and it is apparent that his method, and many of his doctrines, were derived from this source.

As *Des Cartes* had concluded, from the idea of an infinitely perfect necessarily-existing Being, that such a Being must exist; so *Spinoza*, from our having a true idea (that is a clear and distinct idea, as he himself explains it) of a substance, infers that it must necessarily exist*; or, to use his own words, that its existence as well as its essence must be an eternal truth. As *Des Cartes* pretended to deduce all the phænomena of nature from the nature and properties of the first cause; so *Spinoza* pretends, that all our knowledge is to be derived from true ideas (as he always calls them) and that those true ideas ought

and had proceeded on better grounds, if they had rejected the principle. Yet *Spinoza*, in his seventy-third letter, pretends to find fault with *Des Cartes* for defining matter by extension, which, according to him, ought to have been explained by an attribute that should express an *essential and infinite essence*.

† Quum ille summo sciendi amore arderet, quid in his ingenii vires valerent experiri decrevit. Ad hoc propositum urgendum scripta philosophica nobilissimi & summi philosophi *Renati Des Cartes* magno ei fuerunt adjumento. *Spinoz. oper. posth. præfat.*

* Si quis dixerit se claram & distinctam, hoc est veram, ideam substantiæ habere, & nihilominus dubitare num talis substantia existat, idem hercle esset ac si diceret se veram habere ideam, & nihilominus dubitare num falsa sit (ut satis attendenti sit manifestum:) vel si quis statuatur substantiam creari, simul statuit ideam falsam factam esse veram; quo sane nihil absurdius concipi potest: adeoque fatendum necessario est, substantiæ existentiam sicut ejus essentiam æternam esse veritatem. *Ethic. part 1. prop. 8. schol. 2.*

to be produced by the mind †, from that idea which represents the most perfect Being, the origin and fountain of nature. *Des Cartes* rejected the consideration of final causes from philosophy; and *Spinoza* tells us they are nothing but human fiction ‡, and laughs at those who imagine that the eyes were designed for seeing, or the sun for giving light. He derives our notions of good and evil, order and confusion, beauty and deformity, from the same source. As *Des Cartes* represented the universe as a machine that might have been produced at first, and may continue to exist for ever, by mechanical laws only, the same quantity of motion remaining always in it unalterable; so *Spinoza* represented it as infinite and necessary, endowed always with the same quantity of motion, or (to use his inaccurate expression *) having always the same proportion of motion to rest in it, and proceeding by an absolute natural necessity; without any self-mover or principle of liberty.

In all these, *Spinoza* has added largely, from his own imagination, to what he had learned from

† Ut mens nostra omnino referat naturæ exemplar, debet omnes suas ideas producere ab eâ quæ refert originem & fontem totius naturæ, ut ipsa etiam sit fons ceterarum idearum. *Spinoz. de emendatione intellect.*

‡ Ut jam ostendam naturam nullum sibi finem præfixum habere, & omnes causas finales nihil nisi humana esse figmenta, non opus est multis, &c. Hoc adhuc addam, nempe hanc de fine doctrinam naturam omnino evertere. *Append. prop. 36. part 1. Ethic.*

* Omnia corpora ab aliis circumcinguntur, & ab invicem determinantur ad existendum & operandum, certâ ac determinatâ ratione, servatâ semper in omnibus simul, hoc est in toto universo, eâdem ratione motus ad quietem, *Epist. 15.*——Corpus motum vel quiescens ad motum vel quietem determinari debuit ab alio corpore, quod etiam ad motum vel quietem determinatum fuit ab alio, & illud iterum ab alio, & sic in infinitum. *Ethic. part 2. prop. 13. lem. 3.*

Des Cartes. But from a comparison of their method and principles, we may beware of the danger of setting out in philosophy in so high and presumptuous a manner; while both pretend to deduce compleat systems from the clear or true ideas, which they imagined they had, of eternal essences and necessary causes. If we attend to the consequences of such principles, we shall the more willingly submit to experimental philosophy, as the only sort that is suited to our faculties. It were unreasonable to charge upon *Des Cartes* the impious consequences which *Spinoza* may have been led into from his principles: but we cannot but observe, to the honour of Sir *Isaac Newton's* philosophy, that it altogether overthrows the foundation of *Spinoza's* doctrine, by shewing that not only there may be, but that there actually is a *vacuum*; and that, instead of an infinite, necessary, and indivisible, plenitude, matter appears to occupy but a very small portion of space, and to have its parts actually divided and separated from each other.

It would be of no use to give a more particular account of the system of *Spinoza*; nor is it possible to describe fully, in an intelligible manner, so absurd a doctrine. It is allowed even by those who, on other occasions, have shewn a disposition towards scepticism, in relation to the foundations of natural religion, to be the most monstrous that can be imagined; and to be so opposite to the most evident notions we are able to form*, that no person of a right

* These are the words of Mr. *Bayle* in the article of *Spinoza*; where he exposes the absurdities of this system very clearly, and affirms that the weakest of its adversaries was able to have overturned it. Our view in giving some account of it, was not only to shew the absurd consequences to which *Des Cartes's* system leads,

right mind can be in hazard of giving into it. He pretends, indeed, to proceed in the geometrical method and style; but while he assumes a definition of substance and of its attributes at his pleasure, and passes from his definitions as true ideas (as he calls them) to the necessary existence of the thing defined, by a pretended immediate consequence, which he will not allow to be disputed, his whole superstructure appears a mere *petitio principii* or fiction. By his way of proceeding, any system whatsoever might be established. But it does not appear possible to invent another so absurd, while he maintains that there is but one substance in the universe, endowed with infinite attributes, (particularly, infinite extension and cogitation) that produces all other things, in itself, necessarily, as its own modifications; which alone is, in all things, cause and effect, agent and patient, in all respects physical and moral.

The *Cartesian* doctrine has been often altered, and variously mended, since it was first proposed by its author; and, for a hundred years together, many ingenious men have been making their utmost efforts to patch it up, and support its credit, by reforming first one part, and then new-modelling another part of this extensive system. But the foundation is so faulty, and the whole superstructure so erroneous, that it were much better to abandon the fabrick, and suffer the ruins to remain a memorial, in all time to come, of the folly of philosophical presumption and pride.

leads, but likewise to trace *Spinoza's* doctrine to its source (for the sake of some who may have been misled into a favourable opinion of it,) which is no other than the *Cartesian* fable; of which almost every article has been disproved by Sir *Isaac Newton*, or others.

Mr. *Leibnitz* retained the *Cartesian* subtle matter, with the universal plenitude and vortices; and represented the universe as a machine that should proceed for ever, by the laws of mechanism, in the most perfect state, by an absolute inviolable necessity; tho' in some things he differs from *Des Cartes*. After Sir *Isaac Newton*'s philosophy was published (in 1687,) he printed an essay on the celestial motions (*Act. Erudit.* 1689) where he admits of the circulation of the ether with *Des Cartes*, and of gravity with Sir *Isaac Newton*; but he never explained how these could be reconciled, and adjusted together, so as to account for the planetary revolutions; or how gravity arose from the impulse of this ether. Nor did he shew how his harmonical circulation of the ether could be reconciled with the law of the motions of the several planets, in their respective orbits; which is very different from the law of the motions of the same planet, at its various distances from the sun. The angular velocity of any one planet, decreases from the *perihelium* to the *aphelium*, in the same proportion as its distance from the sun increases, and this is what he calls the harmonical circulation. If this law took place likewise in the motions of the different planets compared together, throughout the system, this hypothesis, of their being carried along with a circulating ether, might appear more tolerable: but the velocities of the planets, at their mean distances, decrease in the same proportion as the square roots of the numbers which express those distances from the sun. Neither did he shew how to reconcile this circulating motion of the ether with the free motions of the comets in all directions, or with the obliquity of the planes in which the planets revolve to the equator of the sun and to one another;

ther; or resolve the other objections to which this hypothesis of a *plenum* and *vortices* is liable.

Afterwards however, on occasion of some disputes that had arisen concerning his title to the invention of the calculus of infinitesimals, or method of fluxions, he appeared with great warmth against Sir *Isaac Newton's* philosophy, and placed himself at the head of its opposers. It is needless to insist here on the passion and prejudices that his followers have expressed against it, and against those that have appeared in its defence. It is better to forget these, and to confine a philosophical dispute to philosophical matters.

Mr. *Leibnitz's* system has been the more acceptable to many, because, from the wisdom and goodness of the Deity, he concluded the universe, upon the whole, to be a perfect work, or the best that could possibly have been made. This doctrine was very agreeable in all times to the philosophers who acknowledged a supreme beneficent governor; but the origin of evil perplexed them. The solution of this was what *Socrates* expected from the writings of *Anaxagoras*, but was disappointed. The supreme Being, according to *Timæus Locrus*, was δαιμόργος τῷ βελτίονος. *Plato* taught that the supreme governor has disposed and complicated all things for the happiness and virtue of the whole, and that our complaints are groundless, arising from our narrow views of things. *Chrysippus* was of opinion * that it could

* Existimat *Chrysippus* non hoc fuisse naturæ principale consilium ut faceret homines morbis obnoxios, nunquam enim hoc convenisse naturæ auctori, parentique rerum omnium bonarum; sed quum multa atque magna gigneret, pareretque aptissima & utilissima, alia quoque simul agnata sunt incommoda iis ipsis quæ faciebat

could never have been the aim or first intention of the Author of nature, and parent of all good, to make men obnoxious to diseases; but that while he was producing many excellent things, and forming his work in the best manner, other things also arose, connected with them, that were incommodious; which were not made for their own sakes, but permitted as necessary consequences of what was best. Mr. *Leibnitz* has wrote at great length in defence of this doctrine, and has endeavoured to answer the objections that have been made against the perfection of the universe.

But this learned author's speculations, tho' they may perplex a cautious reader, cannot satisfy him. He proposes two principles as the foundation of all our knowledge; the first, that it is impossible for a thing to be, and not to be at the same time, which, he says, is the foundation of speculative truth. The other is, that nothing is without a *sufficient reason* why it should be so rather than otherwise; and by this principle, according to him, we make a transition from abstracted truths to natural philosophy. From this principle he concludes, that the mind is naturally determined, in its volitions or elections, by the greatest apparent good; and that it is impossible to make a choice between things perfectly like, which he calls *indiscernibles*; from whence he infers, that two things perfectly like could not have been produced even by the Deity. For this reason, and other metaphysical considerations, he rejects a *vacuum*, the parts of which must be supposed perfectly like to each other. For the same reason he also rejects atoms, and all similar particles of matter; to

faciebat cohærentia; eaque non per naturam, sed per sequelas quasdam necessarias, facta dicit, quod ipse appellat κατὰ παρακολληθῆσιν. *Aul. Gell.* lib. 6. cap. 1.

each of which, tho' divisible *in infinitum*, he ascribes * a *monad*, or active kind of principle, in which, says he, are as it were perception and appetites. The essence of substance he places in action or activity, or rather (as he expresses it) in something that is between acting and the faculty of acting. He affirms absolute rest to be impossible, and holds motion, or a sort of *nîsus*, to be essential to all material substances. Each *monad* he describes as representative of the whole universe from its point of sight; and, after all, in one of his letters tells us, that matter is not a substance, but a *substantiatum*, or *phenomene bien fondé*.

Such are the doctrines and expressions of a philosopher who valued himself upon his clear and adequate ideas, and ridiculed the metaphysics of the *English*, as narrow, and founded on unadequate notions. The *criterion* of truth is usually placed in clear and evident perception; but some philosophers seem to value doctrines in proportion as they are obscure. Who would imagine that, in natural philosophy, such arguments should be preferred to the plainest facts and experiments for determining the question concerning a *vacuum*? Let any man reflect on his own thoughts, from which only any notions we have of liberty (and consequently of the divine liberty) can be derived; and if he is satisfied that he could chuse between two desirable things that appear equally good, rather than want both, such arguments can have no force upon him. His difficulty seems still to remain against the particles of matter, after all the pains he had taken to distinguish them by his *monads*; for how shall we distinguish the *monads* themselves? or if that may be practicable,

* Acta Lipsiæ, 1698, p. 435.

how shall we distinguish the same *monad* from itself, in all the moments of its existence? If two things perfectly like to each other can exist in different times, surely they may exist in different places at the same time. This learned author appeared very averse to those doctrines which he imagined had a tendency to restore the exploded tenets of the scholastic philosophy; yet these *monads*, as far as he has condescended to describe them, appear to be as incomprehensible as their substantial forms, *entelecheia*, or most occult qualities.

He makes great use of a comparison between the effects of opposite motives on the mind, and of weights placed in the scales of a ballance, or of powers acting upon the same body with contrary directions. His learned antagonist denies that there is a similitude between a ballance moved by weights, and a mind acting upon the view of certain motives; because the one is entirely passive, and the other not only is acted upon, but acts also. The mind, he owns, is purely passive in receiving the impression of the motive, which is only a perception, and is not to be confounded with the power of acting after, or in consequence of, that perception. The difference between a man and a machine does not consist only in sensation, and intelligence; but in this power of acting also. The ballance for want of this power cannot move at all, when the weights are equal: but a free agent, says he, when there appear two perfectly alike reasonable ways of acting, has still within itself a power of chusing; and it may have strong and very good reasons not to forbear. It is evident that as it is from internal consciousness I know any thing of liberty, so no assertion contrary to what I am conscious of concerning it can be admitted; and it were better perhaps to treat of this

abstruse subject after the manner of experimental philosophy, than to fill a thousand pages with metaphysical discussions concerning it. But to leave this subject, the doctrine of liberty is so foreign to the questions concerning a *vacuum* and atoms, that it must appear a far-fetched uncommon stretch of metaphysics to pretend to determine them by it; and very unaccountable to refuse the Deity the power of producing, by one act of his will, all the matter in the universe at once, tho' it should be supposed perfectly similar and uniform.

5. From the same principle, Mr. *Leibnitz* concluded, that the material system is a machine absolutely perfect, that can never fall into disorder, or require to be set right; and that to imagine that God interposes in it, is to lessen the skill of the Author, and the perfection of his work.

But this is more than his own principles require. For tho' it should be allowed that nothing is limited without a sufficient reason; yet, upon the whole, it may be better that the Author of the world should act immediately in it, cherishing and governing his work, and sometimes changing or renewing it. Can the beauty and perfection of the universe be the worse for His acting in it, who must be supposed to act always with perfect wisdom? It was fit that there should be, in general, a regularity and constancy in the course of nature; not only for the sake of its greater beauty, but also for the sake of intelligent agents, who without this could have had no foresight, or occasion for choice and wisdom in judging of things by their consequences, and no proper exercise for their other faculties. But tho' the course of nature was to be regular, it was not necessary that it should be governed by those principles

ples only which arise from the various motions and modifications of unactive matter, by mechanical laws ; and it had been incomparably inferior to what it is, in beauty and perfection, if it had been left to them only.

Sir *Isaac Newton* was of opinion that the fabrick of the universe, and course of nature, could not continue for ever in its present state, but would require, in process of time, to be re-established or renewed by the same hand that formed it. Yet this philosophy was condemned by Mr. *Leibnitz* as leading to impiety ; and, which is very surprizing, this particular doctrine was excepted against as having such a tendency. He objected, that as a good artist made his workmanship as perfect as possible, so it argued a want of power or skill in the Author of the world, if it should ever require to be reformed or wound up again. But Sir *Isaac Newton* thought it altogether consistent with the notion of a most perfect Being, and even more agreeable to it, to suppose that he should form his work dependent upon himself, so as after proper periods to model it anew, according to his infinite wisdom. To exclude the Deity from acting in the universe, and governing it, is to exclude from it what is most perfect and best, the absence of which no mechanism can supply. Such a doctrine could not have been proposed by one of Mr. *Leibnitz*'s sentiments concerning the perfection of the universe, if he had not been misled by an excessive fondness for necessity and mechanism.

The capital doctrine of this philosophy that represents the universe as a perfect machine, such as may continue for ever by mechanical laws in its present state, is, that the same quantity of force and vigour remains

remains always in it, and passes from one portion of matter to another, without undergoing any change in the whole. *Des Cartes* maintained that the same quantity of motion is always preserved in the universe. *Spinoza* called it the same proportion of motion to rest. Mr. *Leibnitz* distinguished between the quantity of motion, and the force of bodies; he owns that the former varies, but maintains that the quantity of force is for ever the same in the universe: and yet there is no doctrine more repugnant to perpetual experience and common observation than this is, even tho' we should measure the forces of bodies by the squares of the velocities, according to his doctrine. If all bodies in the world had a perfect elasticity, there might be some pretence for maintaining this principle. But there never has been discovered as yet any one body, whose elasticity is perfect; and when any two bodies meet with equal motions, they rebound with less motions, and there is always force lost by their collision; and if the bodies are soft, they both stop, because of the impenetrability of their parts; or, to speak in this author's favourite style, because there can be no sufficient reason why one of them should prevail, rather than the other. In this case, their whole motion is lost; and the motion of the one being destroyed by the opposite motion of the other, it is without ground, and merely to save an hypothesis, that a fluid is imagined, which they feign to receive and retain the forces of those bodies. When liberty is taken to support one fiction by another, this by a third, and so on, any system may be maintained. According to our first views of matter and motion, from the plainest experiments, matter appears to be an unactive substance of no elasticity; yet they ascribe a perfect elasticity to all their subtile matter; and laws of motion are proposed by them as general,
which

which can hold of perfectly elastic bodies only, that is, of bodies not one of which has hitherto been found in nature. They have never been able to explain how this perfect elasticity arises from the laws of mechanism; yet, according to them, the world is a mechanical perpetual movement.

The genius of this kind of philosophy appears on no occasion so evidently, as from the arts which have been used to get rid of the insuperable objections against the *vortices*. To remove the difficulty a step farther, or to involve the question in obscurity, new *vortices* are introduced in every infinitely small particle of matter. From these, if there be occasion, they will descend into another order infinitely less; and so on; for they expressly pretend to take the same benefit from the infinite orders of infinitesimals, in philosophy*, that is claimed by some late geometers in the resolution of their problems. Thus (as we observed elsewhere†) an absurd philosophy is the natural product of a vitiated geometry. For tho' it follows from our notion of magnitude, that it always consists of parts, and is divisible without end, yet an actual division *in infinitum* is absurd, and an infinitely little quantity (even in Mr. Leibnitz's judgment‡) is a mere fiction. Philosophers may allow themselves to imagine likewise infinite orders of infinitely small particles of matter, and suffer themselves to be transported with the idea; but these illusions are not supported by sound geometry, nor agreeable to common sense. After all that has been said for the *vortices*, there is not one experiment to favour them; and some of the most

* Mem. de l'Academie Royale des Sciences, 1729.

† Treatise of *Fluxions* Introd. p. 47.

‡ Essay de *Theodicée*, § 70.

common and simple are against admitting such fluids and their motions.

We have another instance of the art by which they support their schemes, in the pretended demonstration they give against the possibility of atoms, or of any perfectly hard and inflexible bodies. According to what they call the law of *continuity*, all changes in nature are produced by insensible and infinitely small degrees; so that no body can, in any case, pass from motion to rest, or from rest to motion, without passing through all possible intermediate degrees of motion; from which they conclude that atoms, or any perfectly hard bodies, are impossible; because if two of them should meet with equal motions, in contrary directions, they would necessarily stop at once, in violation of the law of *continuity* *. But upon what grounds have they made this an universal law of nature? Tho' in common bodies (which are loosely compounded of particles that are themselves compounded of others of a lower order, and so on; so that we cannot arrive at the elements, or atoms, till after we know not how many resolutions) the parts yield in their collisions, we cannot affirm this of the atoms or ultimate elements themselves. This yielding is a consequence of the contexture of bodies, which have always much more of void interstices than of solid matter, and consist of particles that must be supposed to adhere to one another with a force incomparably less than that by which the matter of the elementary particles themselves holds together †. The

* Discours sur le Mouvement, *Paris* 1726.

† The author of the above cited discourse on motion, tells us, that if nature could pass from a state of motion to a state of rest at once, without passing through the intermediate degrees of motion,

The truth is, they found it necessary to reject bodies of a perfect hardness; because it was impossible to explain the effects of their collisions, in a manner consistent with the preservation of the same quantity of force in the universe, or with their new doctrine, *That the forces of bodies are as the squares of the velocities*; and therefore they had recourse to this new law of *continuity* to proscribe them. If such a body should strike another equal quiescent body, of the same kind, the velocity of the first would be equally divided by the stroke between them; but if we measure the force by the square of the velocity, each of them would have but one fourth part of the force of the first body; and both together would have but one half of its force; so that the other half would be necessarily lost, without producing any sort of effect. In order to get rid of objections of this kind, some of the favourers of the new doctrine, concerning the mensuration of the force of bodies, content themselves with observing, that no bodies of a perfect hardness have been found in nature; tho' there is the same objection against admitting and treating of bodies of a perfect elasticity. But others boldly reject such hard bodies as impossible, from those far-fetched metaphysical considerations we have described. How much they have endeavoured to perplex the theory of motion, in its plainest parts, from a zeal for the same doctrine, will appear afterwards.

motion, then one state would be destroyed before nature could know what new state she ought to determine herself to; and asks how she could then determine herself to any one state rather than another? In answer, we need only observe, that to cease to move is the same as to be at rest, and that when the equal atoms stop each other at once, there is no interval between the state of motion and that of rest; and that when motion is destroyed, rest necessarily ensues.

The power of mechanifm was never more magnified than by Mr. *Leibnitz*'s famous doctrine of a *pre-establiſhed harmony*, as he calls it. According to *Des Cartes*, the brutes were mere machines; and this doctrine, to many, appeared incredible. But this is nothing in comparifon to what Mr. *Leibnitz* would have us believe, when he tells us that the foul does not act on the body, nor the body on the foul; that both proceed by neceſſary laws, the foul in its perceptions and volitions, and the body in its motions, without affecting each other; but that each is to be confidered as a ſeparate independent machine. The volitions of the mind are followed inſtantly by the deſired motions of the body, not in conſequence of thoſe volitions in the leaſt, but of the nice and well adjusted machinery of the body. The impreſſions produced in the ſenſory have no effect on the mind, but the correſponding idea ariſes, at that precise time, in conſequence of a chain of cauſes of a different kind. Thus all that men do or ſay, is no more than the effect of exquisite machinery, according to him. But it is time for us to leave thoſe fictions, leſt the reader ſhould be tempted to think that all philoſophy is illuſion.

C H A P. V.

Concluſions from the foregoing obſervations.

1. **T**HE ſum of what we have obſerved is, that tho' theſe learned men may have ſhewn abundance of genius and invention in their writings; yet they, and all others who have followed a like method, have begun at the wrong end, in tracing the chain of cauſes, and have attempted to form a ſcheme of philoſophy that far ſurpaſſes the human facul-

faculties. The eternal reasons and primary causes of things, which they imagine they possess, rise infinitely above them ; while certain observation, and plain facts, perpetually appear in contradiction to their boasted speculations.

We are to endeavour to rise, from the effects thro' the intermediate causes, to the supreme cause. We are, from his works, to seek to know God, and not to pretend to mark out the scheme of his conduct, in nature, from the very deficient ideas we are able to form of that great mysterious Being. Thus natural philosophy may become a sure basis to natural religion, but it is very preposterous to deduce natural philosophy from any hypothesis, tho' invented to make us imagine ourselves possess of a more complete system of metaphysics, or contrived perhaps with a view to obviate more easily some difficulties in natural theology. We may, at length, rest satisfied, that in natural philosophy, truth is to be discovered by experiment and observation, with the aid of geometry, only ; and that it is necessary first to proceed by the method of *analysis*, before we presume to deliver any system *synthetically*.

We may also learn at length, from the bad success of so many fruitless attempts, to be less fond of perfect and finished schemes of natural philosophy ; to be willing to stop when we find we are not in a condition to proceed farther ; and to leave to posterity to make greater advances, as time and observation shall enable them. For we cannot doubt but that nature has discoveries in store for future times also, which may be retarded by our rash and ill-grounded anticipations. By proceeding with due care, every age will add to the common stock of knowledge ; the mysteries that still lie concealed in
nature

nature may be gradually opened, arts will flourish and increase, mankind will improve, and appear more worthy of their situation in the universe, as they approach more towards a perfect knowledge of nature.

2. 'Twas thus the speculative parts of the mathematics gradually arose, from small beginnings, by the conspiring labours of great men, in the distant ages of the world. The *Egyptians* began this science, the *Greeks* pursued it, the *Arabians* preserved it, when it was lost in *Europe*, and set a high value upon it while their empire flourished; and since the late memorable restoration of letters in *Europe*, its great progress has been the boast of modern learning.

The inundations of the *Nile* made it necessary for the *Egyptians* to invent some art by which they should be able to measure their land, and to this, we are told, geometry owes its origin and name. The priests of that country, abounding in leisure and genius, improved it into a science; and their kings wrote treatises upon it. *Thales* brought the principles of it into *Greece*, where it was so diligently cultivated that the elementary part was soon completed, and was so highly esteemed as to have the appellation of the *mathemata* in a manner appropriated to it. An oracle appointing the cubical altar of *Apollo* to be doubled was, we presume, of greater advantage to geometry than to the *Athenians* then afflicted with the plague; as it gave occasion to *Plato* to consider the famous problem of the duplication of the cube, and produced the *solid* geometry. It afterwards received great improvements from the incomparable *Archimedes*, who squared the area of the *parabola*, made some progress in the mensuration

tion of the circle, and enriched this science with many discoveries worthy of so excellent a genius.

It appears that it advanced but by degrees, and sometimes by very slow steps: one, we are told, discovered that the three angles of an equilateral triangle were equal to two right ones; another went farther, and shewed the same thing of those that have two sides equal and are called *isosceles* triangles; and it was a third who found that the theorem was general, and extended it to triangles of all sorts*. In like manner, when the science was farther advanced, and they came to treat of the conic sections, the plane of the section was always supposed perpendicular to the side of the cone; the *parabola* was the only section that was considered in the right-angled cone, the *ellipse* in the acute-angled cone, and the *hyperbola* in the obtuse-angled. From these three sorts of cones, the figures of the sections had their names, for a considerable time; till at length *Apollonius* shewed how they might be all cut out of any one cone, and by this discovery merited in those days the appellation of the *great geometrician*.

By such steps this science rose, in process of time, to that vast height for which it is admired. Problems that appeared of an insuperable difficulty in one age were resolved in another, and, in a third, were in a manner despised as too simple and easy; particular theorems were first investigated that led to more extensive discoveries; laborious methods were followed, till others were found that were more simple and general; but the greatest care was always taken of the certainty and evidence of the science, as it was carried on. There was indeed a long

* *Procli* Comment. in *Euclidem*.

interval of many ages, between the period when it flourished in *Greece*, and revived in *Europe* : but the antients, having founded it on unexceptionable grounds, and carried it on with the utmost accuracy, when learning was restored, their works served for a basis, as well as for models, to the modern inventors. Thus the gradual progress of mankind in this science appears similar, in some respects, to the advances of a man in vigour and knowledge. They first made essays of a weak and unexperienced strength, which by degrees acquired more and more force, till at length, after the successful labours of several ages, nothing seem'd too high for them.

3. From what we have observed concerning the history of natural philosophy, it may easily be understood why its progress has been so different ; and whence it proceeds that we seldom have found in it, as in geometry, that pleasing gradual rise from small beginnings to greater heights. Instead of searching into nature, men retired to contemplate their own thoughts ; instead of tracing her operations, they gave their imaginations full play : where they ought to have hesitated, they decided ; and where there was no difficulty, they doubted. What was simple they divided, and defined what was plain ; but in what was more intricate, the subterfuges of art were set up in opposition to nature, and captious science against common reason ; while one ill-grounded maxim was imagined, to support another, and fiction was grafted on fiction. Hypotheses were invented, not for reducing facts or observations of a complicated nature to rules and order, (for which purpose they may be of service) but as principles of science. These were of so great authority as not to be overturned by contradictory observations, or by the extravagant consequences that arose from them ; but the

the author, charm'd with his rhapsody, proceeded, without minding these, to the conclusion of his fable.

Thus one age or sect could not but destroy, for the most part, the labours of another. Sometimes the *numbers* and *harmony* of the *Pythagoreans* served for explaining what was most mysterious in nature; the *ideas* of *Plato*, the *matter* and *form* of *Aristotle* prevailed in their turn; but these were of use only to veil the ignorance of men. *Epicurus* employed his philosophy to overthrow the plain and evident dictates of sense and reason; yet disciples were not wanting to support and adorn so absurd a scheme. The *Sceptics* went into the opposite extreme, and became so fond of darkness that they would not see the light tho' never so clear; and some of them chose rather to doubt that they doubted, than to acknowledge any thing; yet they too had numerous followers. Afterwards philosophy was in no esteem but as far as it served, by a perplexed and false gloss, to promote the ends of superstition. Of late, the pretended clear ideas of *Des Cartes*, and metaphysical speculations of Mr. *Leibnitz*, have been received by many for true philosophy; not to mention the extravagancies of *Spinoza*, and a thousand crude notions that deserve no memory.

We have seen, in the foregoing account of the state of philosophy in different periods, that they who have indulged themselves in inventing systems and compleating them, tho' they have sometimes set out in a manner that has appeared plausible, yet, in pursuing those schemes, such consequences have arisen as could not fail to disgust all but such as were intoxicated with the deceit. Some, from their fondness to explain all things by mechanism, have been

led to exclude every thing but matter and motion out of the universe: others, from a contrary disposition, admit nothing but perceptions, and things which perceive; and some have pursued this way of reasoning, till they have admitted nothing but their own perceptions. Others, while they overlook the intermediate links in the chain of causes, and hastily resolve every principle into the immediate influence of the first cause, impair the beauty of nature, put an end to our enquiries into the most sublime part of philosophy, and hurt those very interests which they would promote. In framing those systems, he who has prosecuted each of them farthest has done this valuable service, that, while he vainly imagined he improved or compleated it, he really opened up the fallacy, and reduced it to an absurdity. Many who suffered themselves to be pleased with *Des Cartes's* fable, were put to a stand by *Spinoza's* impieties. Many went along with Mr. *Leibnitz's* scheme of absolute necessity, but demurred at his *monads* and *pre-established harmony*. And some, willing to give up the reality of matter, could not think of giving up their own and other minds.

The variety of opinions and perpetual disputes amongst philosophers has induced not a few, of late as well as in former times, to think that it was vain labour to endeavour to acquire certainty in natural knowledge, and to ascribe this to some unavoidable defect in the principles of the science. But it has appeared sufficiently, from the discoveries of those who have consulted nature and not their own imaginations, and particularly from what we learn from *Sir Isaac Newton*, that the fault has lain in the philosophers themselves, and not in philosophy. A complete system indeed was not to be expected from one man, or one age, or perhaps from the greatest number

number of ages ; could we have expected it from the abilities of any one man, we surely should have had it from Sir *Isaac Newton* : but he saw too far into nature to attempt it. How far he has carried this work, and what are the most important of his discoveries, we now proceed to consider.

B O O K II.

Of the theory of motion, or rational mechanics.

C H A P. I.

Of space, time, matter, and motion.

1. **A**S we are certain of our own existence, and of that of our ideas, by internal consciousness; so we are satisfied, by the same consciousness, that there are objects, powers, or causes without us, and that act upon us. For in many of our ideas, particularly those that are accompanied with pain, the mind must be passive, and receive the impressions (which are involuntary) from external causes or instruments, that depend not upon us. We easily distinguish these objects into two general classes. The first is of those which we perceive to have a spontaneity, or self-moving power, and several properties and affections similar to those of our own minds, such as reasoning, judging, willing, loving, hating, &c. The second general class is of those in which no such affections appear, but which are so far of a passive nature, that they never move of themselves, neither, when they are in motion, do they ever stop without some external influence. If one of these move out of its place, without the appearance of a mover, we immediately conclude that this is owing to some invisible agent; so much are we persuaded of its own *inertia*. If we lay up one of them in any place, we expect to find it there at
any

any distance of time, if no other powers have had access to it. This passive nature, or *inertia*, is what chiefly distinguishes the second class of external objects, which is called *body* or *matter*; as the former is called *mind* or *spirit*.

2. How external objects, of either class, act upon the mind, by producing so great a variety of impressions or ideas, is not our business at present to enquire: neither is it necessary for us to determine how exact or perfect the resemblance may be between our ideas and the objects or substances they represent. In our ideas which are repetitions of other ideas, we find very different degrees of resemblance between them and those of which they are repetitions. The idea we form in our imagination of a person, place, or figure which we have often seen, has a much more perfect resemblance to the impression we receive from sense, than the idea we are able in our imagination to form of pain, as to the sensation we have felt of it. And as it is no objection against the existence of the souls of other men, that they may be very different from the notion or conception we may have formed of them; so it is no just reason against admitting the existence of body, that its inward essence, or *substratum*, may be very different from any thing we know of it. It is, however, rating our ideas of external objects by much too low, to compare them to words or arbitrary signs, serving only to distinguish them from each other. For it is from our ideas of them that we learn their properties, relations, and their influences upon each other, and upon our minds and those of others, and acquire useful knowledge concerning them and ourselves. For example, by comparing and examining our ideas, we judge of order and confusion, beauty and deformity, fitness and

H 4

unfitness,

unfitness, in things. The ideas of number and proportion, upon which so useful and extensive sciences are founded, have the same origin.

3. The mind is intimately conscious of its own activity in reflecting upon its ideas, in examining and ranging them, in forming such as are complex from the more simple, in reasoning from them, and in its elections and determinations. From this, as well as from the influence of external objects upon the mind, and from the course of nature, it easily acquires the ideas of cause and effect. When a figure described upon a board produces a similar idea or impression on all those who see it, it is as natural to ascribe this to one cause, as, when we speak to a numerous audience, the effect of the discourse is to be ascribed to us; tho' we may be unable to explain how the impression of the figure is communicated to the several spectators, or the discourse to the hearers. It were easy to make many more remarks on the philosophy of those whose principles would lead them to maintain, that external objects vary with our perceptions, and that the object is always different when perceived by different minds, or by the same person at different times, or in different circumstances. It will not be expected from us that we should enter farther, in a treatise of this kind, into the examination of doctrines as fruitless as they are extravagant.

4. Body not only never changes its state of itself, in consequence of its passive nature or *inertia*, but it also resists when any such change is produced: when at rest, it is not put in motion without difficulty; and when in motion, it requires a certain force to stop it. This force with which it endeavours to persevere in its state, and resists any change, is called its *vis inertiae*; and arises from the *inertia* of
of

of its parts, being always proportional to the quantity of matter in the body ; infomuch that it is by this *inertia* only we are able to judge of the quantity of matter. And this judgment is well founded, because we constantly find that when we double or triple a body, or increase or diminish it in any proportion, we must double or triple the force that is requisite to move it with the same celerity, or increase or diminish it in the same proportion with the body. If the solid, uncompounded particles void of pores, of equal bulk, have their *inertia* equal, then this must be accurately true : but if matter be of kinds so different from each other, that the solid elementary particles of the one have a greater *inertia* than equal solid elementary particles of the other kind, then it is only when we compare those of the same kind, that we can affirm the *inertia* to be proportional to the quantity of matter. Such different kinds of matter may exist for ought we know ; but it is by diminishing or increasing the number or dimensions of the pores of bodies that they are condensed or rarified, according to our experience, and thereby the *inertia* of a given bulk increased or diminished.

5. Space is extended without limits, immoveable, uniform and similar in all its parts, and void of all resistance. It consists indeed of parts which may be distinguished into other parts, less and less, without end, but cannot be separated from each other, and have their situation and distances changed.

6. Body is extended in space, moveable, bounded by figure, solid, and impenetrable, resisting by its *inertia*, divisible into parts, less and less, without end, that may be separated from each other and have their situation or distances changed in any manner.

7. From

7. From the succession of our own ideas, and from the successive variations of external objects in the course of nature, we easily acquire the ideas of *duration* and *time*, and of their measures. We conceive true or absolute time, to flow uniformly in an unchangeable course, which alone serves to measure with exactness the changes of all other things. For unless we correct the vulgar measures of time, which are gross and inaccurate, by proper equations, (as in predicting the eclipses of the satellites of *Jupiter*, and most other astronomical phænomena) the conclusions are always found inaccurate and erroneous: and however various the flux of time may appear to different intellectual beings, it cannot, at least, be thought to depend upon the ideas of any created being. Time may be conceived to be divided into successive parts that may be less and less without end; tho', with respect to any one particular being, there may be a least sensible time, as well as a *minimum sensibile* in other magnitudes.

8. Motion is the change of place; that is, of the part of space which the body occupies, or in which it is extended. The motion is *real* or *absolute*, when the body changes its place in absolute space. It is called *relative*, when the body changes its place with relation only to ambient bodies; and it is *apparent* motion, when the body changes its situation with respect to other bodies that appear to us to be at rest. The parts of absolute space not being the objects of our senses, it is one of the great difficulties in philosophy to distinguish which motions are true and real, and which are apparent only. However, philosophers by proper care are often able to effect this, by arguing justly from the causes of the motion when known, or from their properties and effects. A real
circular

circular motion, for example, is always accompanied with a centrifugal force, arising from the tendency which a body always has to proceed in a right line. Thus, from the centrifugal force which, at the æquator, diminishes the gravity and retards the motion of the pendulum, so that it moves more slowly there than towards either pole, we have a proof of the earth's diurnal rotation on its axis. At the same time, the diurnal revolution of the heavenly bodies about the earth must be apparent only; since if it was real, an immense centrifugal force would thence arise, which could not but discover itself; because they move in free spaces, and the solid orbs have been exploded upon the most evident grounds.

9. I know that some metaphysicians of great character condemn the notion of absolute space, and accuse mathematicians in this of realizing too much their ideas: but if those philosophers would give due attention to the phænomena of motion, they would see how ill grounded their complaint is. From the observation of nature, we all know that there is motion; that a body in motion perseveres in that state, till by the action or influence of some power it be necessitated to change it; that it is not in relative or apparent motion in which it perseveres, in consequence of its *inertia*, but in real and absolute motion. Thus the apparent diurnal motion of the stars would cease, without the least power or force acting upon them, if the motion of the earth was stopt; and if the apparent motion of any star was destroyed by a contrary motion impressed upon it, the other celestial bodies would still appear to persevere in their course, the centrifugal force at the æquator would still subsist, with the spheroidical figure of the fluid ocean; the consequences of the real motion of the earth
upon

upon its axis. They who are not well acquainted with the theory of motion, more easily allow that a body at rest continues at rest, in consequence of its passive nature or *inertia*, than that when in motion it continues in motion: but this perseverance of a body in a state of rest can only take place with relation to absolute space, and can only be intelligible by admitting it. When a topp turns upon a small pivot, its circular motion will continue smooth for a long time, but any body placed upon its surface does not continue in that place, but immediately flies off. When a ship moves steadily, any body placed in the cabin continues in its place, as if the whole was at rest; but when the motion of the ship is stopt, the body flies off in the direction of its former motion; for, in consequence of its *inertia*, it endeavours to persevere, not in its state of rest in the ship, but in its state of motion or rest with regard to absolute space. It were easy to enlarge on this subject, and to shew that there is no explaining the phænomena of nature without allowing a real distinction between true, or real, and apparent motion, and between absolute and relative space. Whatever those philosophers may pretend, we have no clearer idea than of space; and tho' some puzzling disputes may arise in some of our enquiries concerning it, this is what we meet with in all our enquiries into nature; our knowledge of which we ought to take care to have as clear and well founded as possible, tho' it is in vain to pretend to make it complete and perfect; as we observed in the first book.

10. Body being distinguished from space by its *vis inertiae* or resistance, it is an obvious suggestion of common sense that all space is not equally full of matter; and it is the result of philosophical enquiries, that the solid matter in the densest bodies bears

a small

a small proportion to their whole bulk. The rays of light find a passage through a glass globe in all directions, which argues the great rarity of the globe, as well as the subtilty of light. The same is to be said of the magnetic and electric *effluvia*, and of the subtile matter that pervades the pores of bodies with great freedom in chymical experiments. As for those fluids which philosophers have invented, in order to replenish the pores of bodies, so as to exclude a void out of the universe, we made some observations upon them in the first book; and we may have occasion afterwards to shew how improper they are for accounting for the phænomena which have been ascribed to them.

11. Space and time serve to measure each other, reciprocally, by motion : time is in a perpetual flux and perishing; but a representation of it is preserved in the space described by the motion. When the space flows as the time, that is, when equal parts of space are described in any equal parts of the time, then the motion is uniform, and the velocity is constant or unvaried during the motion. When the parts of space, described in any equal successive parts of the time, continually increase, the motion is accelerated; and when those parts of space continually decrease, the motion is retarded. In general, the velocity of motion is always measured by the space that would be described by that motion continued uniformly for a given time. It is obvious that the space, described by an uniform motion, is in the compound proportion of the time and velocity of the motion : but in general, let AB , (*Fig. 1.*) the base of a figure, represent the time of a motion, and the ordinate or perpendicular PM , at any point P of the base, measure the velocity at the corresponding term of time, (that is, the space which would be described

described by the motion continued uniformly from that term for a given time) then the area of the figure ABD so formed will measure the space described by the motion, in the time represented by the base AB . Thus a rectangular parallelogram serves to measure the space described by an uniform motion, the time being represented by the base, and the constant velocity of the motion by the perpendicular. The space described by a motion which is uniformly accelerated (the velocity of which increases uniformly as the time, that is, receives equal augmentations in any equal successive parts of time) is represented by a triangle; the time being represented by the base, and the increasing velocity by the perpendicular, which increases in the same proportion as the base. Because the triangle is the half of a parallelogram of the same base and altitude, the space described by a motion uniformly accelerated, during any time, from the beginning of the motion, is one half of what would have been described if the motion had been uniform, and the velocity had been the same as is acquired at the end of that time. Because similar triangles are as the squares of their analogous sides, the spaces described by a motion uniformly accelerated, being measured by such triangles, are as the squares of the times from the beginning of the motion; or as the squares of the velocities acquired at the end of those times. The spaces described by motions uniformly retarded are measured in the same manner; only the times and velocities are to be taken in a contrary order, till the extinction of the motion. In other cases, the spaces are measured by curvilinear areas. And because there are areas whose ordinates decrease in such a manner, that tho' the figure be produced indefinitely, the area never amounts to a certain finite space; it appears that the velocities of a retarded motion
may

may decrease in such a manner, that, tho' the motion was continued ever so long, yet the space described by it should not exceed any certain given line. For example, if the velocity during the first hour be double of what it is in the second hour, and this be reduced to its half in the third hour, and so on for ever, then the space described by this motion, tho' it was to continue for the greatest number of ages, will never amount to the double of the line described in the first hour.

12. The quantity of motion in a body being the sum of the motions of its parts, is in the compounded ratio of its quantity of matter and of the velocity of the motion. If the body A, of a quantity of matter represented by 2, moves with a velocity represented by 5, and the body B, represented by 3, moves with a velocity represented by 4; then the quantity of motion of A, shall be to the quantity of motion of B, in the compounded ratio of 2 to 3 and of 5 to 4, that is as 2×5 to 3×4 , or as 10 to 12. There appears to be no ground for making a distinction between the *quantity of motion*, and the *force* of a body in motion; as all the power or activity of body arises from and depends upon its motion. We are not, however, to expect that all the effects of the motion of bodies should be proportional to the quantity of motion, unless a due regard be had to the time of the motion, and to the direction in which it acts, according to the true principles of mechanics. A body, in consequence of its uniform motion, describes a certain space in a certain time; but there is no space so great that may not be described by it, if the time be not limited. When a body acts upon another body, the effect is very different according to the direction in which it acts. How necessary it is to have regard to these,
in

n determining the effects of the motions and actions of bodies, will appear more fully in the next chapter.

13. When a body tends to move, but is hindered by some obstacle, this tendency is called *pressure*. It is not to be compared with the force of a body in motion, no more than a line is to be compared with the rectangle that is generated by it. Of this kind is the gravity of a body that rests and presses upon a table, or of water upon the bottom of a vessel, or of air upon the sails of a ship. When the obstacle is removed, the continual action of the pressure generates motion in the body, in any finite time. Thus gravity accelerates the motion of falling bodies, by acting incessantly upon them. When an orifice is opened in the bottom of a vessel, the pressure of the fluid accelerates the motion of the issuing water, and, in an exceeding little time, brings its velocity to a height. When the wind acts upon the sails of a ship, it accelerates her motion for some time, till the resistance of the water (which increases with the increasing velocity of the ship) ballances the action of the wind; after which her motion becomes uniform. In these, and all such other instances, the motion begins from nothing; and it is in consequence of the continual incessant action of the power or pressure, that the velocity, generated in any finite time, is finite. If we were to suppose that each action of the power produced a finite augmentation of velocity, the motion acquired in the least finite time would be infinite, or surpass any assignable velocity; as we have demonstrated elsewhere*.

* See the Treatise of *Fluxions*, § 44.

14. *Gravity* is the best known to us of all those powers or preffures. Because all bodies descend with equal velocity in a void, the gravity of bodies must be proportional to their quantity of matter; and depends not upon the figure or texture of the parts, but upon their solid matter only. This is evident by experiments of the motion of pendulums, made with the greatest exactness. For when the lengths of the pendulums are equal, bodies of very different bulks, and different internal and external texture, perform their vibrations in times exactly equal in equal arcs, keeping always pace together, and acquiring always equal velocities at the corresponding points of those arcs, unless so far as the resistance of the air acts upon them unequally. In the common business of life, the quantity of matter of bodies has been always measured out by their weight; tho' the influence of the air is various in its different states, and renders this mensuration somewhat unaccurate in things of great value. Though the gravity of bodies really arises from their gravitation towards the several parts of the earth (as will appear afterwards) yet, because this power acts around in all parts, and its direction is nearly towards the centre of the earth, it is therefore called a centripetal force. We shall, afterwards shew that similar centripetal forces tend to the sun and planets. These forces are of three kinds: the *absolute* force is measured by the motion that would be produced by it in a given body, at a given distance. For example, the absolute centripetal force tending towards the sun is to that which tends towards the earth, as the motion which would be produced by the force tending toward the sun in a given body, at a given distance without the sun's body, is to the motion which would be produced by the force tending towards the earth in an equal body, at an equal distance from it. As when we compare

the forces of two magnets, we must compare their effects at equal distances ; so when we compare the absolute forces which tend to the central bodies, the comparison cannot be just unless it be from effects produced when the circumstances are alike.

The second sort of centripetal force is the *accelerating* force, which is measured by the velocity generated by it in a given time, and is different at different distances from the same central body, but depends not on the quantity of matter of the body that gravitates, being equal in all sorts of bodies at equal distances from the centre. The third sort is the weight, or the *vis motrix*, and is measured by the quantity of motion that is generated in a heavy body in a given time ; and differs from the accelerating force in the same manner as motion differs from velocity.

15. Because the power of gravity is so well known to us, when we enquire into other powers, we endeavour to compare them with that of gravity, and to determine their proportion. We find a great variety of powers analogous to it in nature ; such as that by which the particles of fluids form themselves into drops ; that by which the parts of hard bodies cohere together ; that by which the rays of light, in entering into water or glass, or into any medium of a greater refractive power, are constantly bent towards the perpendicular, and when they are incident upon the farther surface of the glass, with a sufficient obliquity, are all turned back into the glass, though there be no sensible medium behind the glass to reflect it ; in the same manner as a heavy body projected obliquely upwards is bent into a curve, and brought back to the earth again by its gravity. These, and many other powers in nature, have an analogy

analogy to gravity, but extend to less distances, and observe laws somewhat different. It has been found very difficult to account for them mechanically. For this purpose, some have imagined certain *effluvia* to proceed from bodies, or atmospheres environing them; others have invented *vortices*; but all their attempts have hitherto proved unsatisfactory. That such powers take place in nature, and contribute to produce its chief phænomena, is most evident; but their causes are very obscure, and hardly accessible by us. In all the cases when bodies seem to act upon each other at a distance, and tend towards one another without any apparent cause impelling them, this force has been commonly called *attraction*; and this term is frequently used by Sir *Isaac Newton*. But he gives repeated cautions that he pretends not, by the use of this term, to define the nature of the power, or the manner in which it acts. Nor does he ever affirm, or insinuate, that a body can act upon another at a distance, but by the intervention of other bodies. It is of the utmost importance in philosophy to establish a few general powers in nature, upon unquestionable evidence, to determine their laws, and trace their consequences, however obscure the causes of those powers may be; and this he has done with great success.

16. But however commodious the term *attraction* may be, to avoid an useless and tedious circumlocution, yet because it was used by the school-men to cover their ignorance, the adversaries of Sir *Isaac Newton's* philosophy have taken an unjust handle from his use of this term, after all his precautions, to depreciate and even ridicule his doctrines; by which they only convince us that they neither understand them, nor have impartially and duly considered them. Mr. *Leibnitz* made use of this same

I 2

term,

term, in the same sense with Sir *Isaac Newton*, before he set up in opposition to him; and it is often to be met with in the writings of the most accurate philosophers, who have used it without always guarding against the abuse of it, as he has done. A term of art has been often employed by crafty men, with too much success, to raise a dislike against their opponents, and mislead the unwary, and to disgust them from enquiring into the truth; but such dissingenuity is unworthy of philosophers. No writer hath appeared against Sir *Isaac Newton*, of late, by whom this argument, tho' altogether groundless, is not insisted on at great length; and sometimes adorned with the embellishments of wit and humour; but if the reader will take the trouble to compare their descriptions with Sir *Isaac Newton*'s own account, he will easily perceive how little it was minded by them; and that the sum of all their art and skill amounts to this only, that they were able to expose a creature of their own imagination. Possibly some unskilful men may have fancied that bodies might attract each other by some charm or unknown virtue, without being impelled or acted upon by other bodies, or by any other powers of whatever kind; and some may have imagined that a mutual tendency may be essential to matter, tho' this is directly contrary to the *inertia* of body described above; but surely Sir *Isaac Newton* has given no ground for charging him with either of these opinions: he has plainly signified that he thought that those powers arose from the impulses of a subtile ætherial *medium* that is diffused over the universe, and penetrates the pores of grosser bodies. It appears from his letters to Mr. *Boyle* *, that this was his opinion early; and if

* See the life of Mr. *Boyle* premised to the late complete edition of his works.

he did not publish it sooner, it proceeded from hence only, that he found he was not able, from experiment and observation, to give a satisfactory account of this medium, and the manner of its operation, in producing the chief phænomena of nature. They who imagine that he has only introduced a new phrase or two into philosophy, without any real benefit, may be easily satisfied of their mistake, if they will but consider with what evidence he has resolved the chief phænomena of the system of the world from those powers; how he has computed the quantity of matter and density of the sun, and of several of the planets, from them; how nearly he has determined the motion of the *nodes* of the moon, from its cause; and explained many of her irregularities, and the other motions of the system. But we have insisted upon this perhaps at too great length; for as no philosopher scruples to say that the magnet attracts iron, and that electric bodies, when their virtue is raised by friction, attract light substances; it must be allowed to be at least as justifiable an expression, or even more unexceptionable, to say that the earth attracts heavy bodies towards it; since all of them descend towards it with forces proportional to their quantity of matter, at equal distances from it; and this power extends to all distances, varying according to a certain known law.

C H A P II.

Of the laws of motion, and their general corollaries.

1. **T**HE first law of motion is, “ That a body
 “ always perseveres in its state of rest, or
 “ of uniform motion in a right line, till by some
 “ external influence it be made to change its state.”
 That a body of itself perseveres in its state of rest, is matter of most common and general observation, and is what suggests to us the passive nature of body: but that it likewise, of itself, perseveres in its state of motion, as well as of rest, is not altogether so obvious, and was not understood, for some time, by philosophers themselves, when they demanded the cause of the continuation of motion. It is easy, however, to see that this last is as general and constant a law of nature as the first. Any motions we produce, here on the earth, soon languish and at length vanish; whence it is a vulgar notion, that in general, motion diminishes and tends always toward rest. But this is owing to the various resistances which bodies here meet with in their motion, especially from friction, or their rubbing upon other bodies in their progress, by which their motion is chiefly consumed. For when, by any contrivance, this friction is much diminished, we always find that the motion continues for a long time. Thus, when the friction of the axis is lessened by friction wheels applied to it, and turning round with it, the great wheel will sometimes continue to revolve for half an hour. And when a brass topp moves on a very small pivot on a glass plane, it will continue in motion very smoothly for a great number of minutes.

A pendulum, suspended in an advantageous manner, will vibrate for a great while, notwithstanding the resistance of the air. Upon the whole, it appears, that, if the friction and other resistances could be taken quite away, the motions would be perpetual. But what sets this in the clearest light, is, that a body placed on the deck, or in the cabin of a ship, continues there at rest while the motion of the ship remains uniform and steady; and the same holds of a body that is carried along in any space that has, itself, an uniform motion in a right line. For if a body in motion tended to rest, that which is in the cabin of a ship ought to fall back towards the stern, which would appear as surprizing, when the motion of the ship is uniform and steady, as if the body should, of itself, move towards the stern when the ship is at rest. It is for this reason that the uniform motion of the earth upon its axis has no effect on the motion of bodies at the surface; that the motion of a ship carried away with a current is insensible to those in the ship, unless they have an opportunity to discover it by objects which they know to be fixed, as the shores, and the bottom of the sea, or by astronomical observations; and that the motions of the planets and comets, in the free celestial spaces, require no new impulses to perpetuate them.

2. It is a part of the same law, that a body never changes the direction of its motion, of itself, but by some external influence only; and it is as natural a consequence of the passive nature of body, as that it never changes its velocity of itself. As body has no self-motive power, or spontaneity, if it was to change its direction, how could it determine itself to any one direction rather than to another? This part of the law is likewise confirmed by constant experience. If upon any smooth plane a globe of an

uniform texture be projected, it proceeds always in a right line, without turning to either side, till its motion be extinguished by the friction of the plane and resistance of the air. It is true, that, in certain cases, a ball proceeds upon a billiard table first in a right line, and afterwards, returns of itself a little way in the same right line; but this arises from the ball's having a motion upon its axis, with a direction contrary to that of its progressive motion on the table which when the progressive motion is destroyed by the friction, brings the ball back again, till this motion is likewise destroyed by the same friction. When a ball is projected in the air, its gravity indeed bends its motion into a curve, but it continues to move in the plane of its first projection perpendicular to the horizon, without turning to either side of that plane; unless in some cases, when, because of its motion upon its axis, the reaction of the air makes it deviate somewhat from it. If bodies changed the direction of their motion of themselves, they could not continue at rest in a space that is carried uniformly forward in a right line; as they are always found to do. As body, therefore, is passive in receiving its motion and the direction of its motion, so it retains them or perseveres in them, without any change, till it be acted upon by something external. This law is now generally received upon the best evidence, but was not clearly understood even so lately as in *Kepler's* time, as appears by the account we gave of his doctrines in the first book. From this law it appears, why we enquire not, in philosophy, concerning the cause of the continuation of the rest of bodies, or of their uniform motion in a right line. But if a motion begin, or if a motion already produced is either accelerated or retarded, or if the direction of the motion is altered, an enquiry into the power or cause that produces this

this change is a proper subject of philosophy : the chief business of which (as Sir *Isaac Newton* observes) is to discover the powers that produce any given motions ; or, when the powers are given, to trace the motions that are produced by them.

3. The second general law of motion is, “ that
“ the change of motion is proportional to the force
“ impressed, and is produced in the right line in
“ which that force acts.” Thus when a motion is accelerated, as that of a heavy body descending in the vertical line, the acceleration is proportional to the power that acts upon the body. If a body descend along an inclined plane, the acceleration of the motion along the plane is proportional not to the total force of gravity, but to that part only which acts in the direction of the plane, as will better appear when we come to treat of the resolution of motion. When a fluid acts upon a body, as water or air upon the vanes of a mill, or wind upon the sails of a ship, the acceleration of the motion is not proportional to the whole force of those fluids, but to that part only which is impressed upon the vanes or sails, which depends upon the excess of the velocity of the fluid above the velocity which the vane or sail has already acquired : for if the velocity of the fluid be only equal to the velocity of the vane or sail, it just keeps up with it, but has no effect either to advance or retard its motion.

It is, at the same time, of the utmost importance to have regard to the direction in which the force is impressed, in order to determine the change of motion produced by it. It would be very erroneous to suppose that the acceleration of the motion of the ship, in the direction in which she sails, is proportional to the force impressed when it acts obliquely
upon

upon the sail, or when the position of the sail is oblique to the direction in which the ship moves. The change of her motion is first to be estimated in the direction of the force impressed, and thence by a proper application of mechanical and geometrical principles, the change of the motion of the ship in her own direction is to be derived. When gravity or any centripetal force acts upon a body, moving with a direction oblique to the right line drawn from it to the center, the change of its motion is not proportional to the whole centripetal force which acts upon it, but to that part only, which, after a just resolution of the force, is found to act in the direction of its motion. It appears from these instances, of how extensive an use these general laws are in the doctrine of motion.

4. The third general law of motion is, “ that
“ *action* and *reaction* are equal with opposite direc-
“ tions, and are to be estimated always in the same
“ right line.” Body not only never changes its state
of itself, but resists, by its *inertia*, against every
action that produces a change in its motion. When
two bodies meet, each endeavours to persevere in its
state and resists any change; and because the change
which is produced in either may be equally measured
by the action which it exerts upon the other, or by
the resistance which it meets with from it, it follows
that the changes produced in the motions of each
are equal, but are made in contrary directions.
The one acquires no new force but what the other
loses in the same direction; nor does this last lose any
force but what the other acquires; and, hence, tho’
by their collisions, motion passes from the one to the
other, yet the sum of their motions, estimated in a
given direction, is preserved the same, and is unal-
terable by their mutual actions upon each other. In
col-

collecting this sum, motions that have contrary directions are to be affected with contrary signs; a motion eastward is contrary to a motion westward; so that if the motions are summed up as having a western direction, a motion eastward is to be considered as negative, or to be subducted from the rest. In this manner, this law serves to render the first law more general, and to extend it to any number of bodies; for as, by the first law, a body perseveres in its state of rest, or of uniform rectilinear motion, till some external influence affect it; so it follows from this law, “ that the sum of the motions of
“ any number of bodies, estimated in a given di-
“ rection, perseveres the same in their mutual ac-
“ tions and collisions, till some external influence
“ disturb them.”

5. The truth of this third law appears from manifold experiments, in the collisions of bodies of all kinds. But the meaning of it seems to have been mistaken, in several instances, by ingenious men; which it is necessary for us to guard against. They who maintain the new opinion concerning the forces of bodies, measuring them by the compounded proportion of the quantity of matter and the square of the velocity, found it impossible to explain the actions and collisions of bodies of a perfect hardness, void of all elasticity, consistently with this doctrine. Therefore, in order to get rid of them, some pretended that it is absolutely impossible such bodies should exist, upon grounds the weakness of which was shewn in the first book; while others contented themselves with observing that they knew of no such bodies in nature, and thought this a sufficient excuse for giving no account of their collisions; tho' at the same time they treated largely of bodies of a perfect elasticity, none of which are to be met with

with in nature ; and we have much better reason to conclude that there are bodies of a perfect hardness, than of a perfect elasticity ; because we cannot but suppose the ultimate elementary particles of bodies that are void of all pores, or atoms, to be perfectly hard or inflexible, so as not to yield in the ordinary actions and collisions of bodies. But after all this art in screening their favourite opinion, the difficulty still recurred in explaining the collisions of soft bodies ; and some farther new invention was requisite to reconcile the phænomena with their doctrine. For if a soft body, with the velocity u , strikes another equal quiescent soft body, they will proceed as in one mass with the velocity $\frac{1}{2}u$, dividing the motion of the first body equally between them, in consequence of the third general law of motion. According to the new opinion, the force of the first body before the stroke was uu , the force of each of them after the stroke is $\frac{1}{2}u \times \frac{1}{2}u$ or $\frac{1}{4}uu$; and the sum of their forces after the stroke is $\frac{1}{2}uu$; so that the sum of the forces, after the stroke, is only one half of what it was before the stroke, while the quantity of motion is preserved the same as it was, without any change. Now the difficulty was, how to account for the loss of one half of the force of the first body in the stroke : for this purpose, they advanced, without any other proof, this new doctrine, that when the parts of soft bodies yield without restoring themselves, being void of elasticity, a certain quantity of force is lost in the compression of their parts by the collision ; whereas we know no way by which force is lost in one body, but by its being communicated to another. The parts of soft bodies are indeed moved out of their places, in the collision, and some motion is lost in the first body by being communicated, in this manner, to the parts of the second ; but these parts cannot lose this motion

tion otherwise than by communicating it to other parts, or by its accruing to the whole body ; so that there is no just reason for supposing that any motion or force is lost in flattening or hollowing of soft bodies, in their collisions ; and this new tenet is invented merely to serve a particular purpose.

6. The most learned and skilful advocate for this new doctrine appears to have greatly mistaken this third law of motion, when he tells us that the preservation of the sum of the absolute motions of bodies, in their collisions, is so immediate a consequence of the equality of *action* and *reaction*, that to endeavour to prove it would only render it more obscure, the augmentation or diminution of the force of the one (says he) being the necessary consequence of the diminution or augmentation of the force of the other. Now it is plain that this third law of motion is general, extending to bodies of all kinds ; and it is well known that when soft bodies meet in opposite directions, the sum of their absolute motions or forces is diminished ; and when the bodies are equal, and their velocities likewise equal, it is totally destroyed by their collision. It is not the sum of the absolute motions or forces of bodies, but this sum estimated in a given direction, that is preserved unaltered in their collisions, in consequence of this third law of motion : nor can the preservation of the sum of the absolute forces of any sort of bodies be considered as an immediate consequence of it. On the contrary, the sum of the absolute motions of even perfectly elastic bodies is sometimes increased, and in some cases diminished, by their collisions ; so that a proof was necessary that the sum of their absolute forces (in whatever manner those forces are measured) is preserved unalterable, in their collision ; especially since this sum, according to his
own

own doctrine, undergoes an infinite variety of changes, during the small time in which the bodies act upon each other, while the parts first yield and then restore themselves to their former situations.

7. The same philosophers mistake this third law, or a most essential part of it, when they measure action and reaction on different right lines. In a celebrated argument which they advance for their new doctrine concerning the forces of bodies, and which is much applauded by those who favour it, they shew that a body with a velocity as 2, is able to bend and overcome the resistance of four springs, one of which alone is equivalent to the force of the same body moving with a velocity as 1; from which they infer that, in the former case, the force is quadruple, tho' the velocity be only double of what it is in the latter case. In like manner, because a body moving with a velocity proportional to the diagonal of the rectangle is able to ballance the resistance of two springs proportional to the sides of the same rectangle, they thence infer that the force of a body moving with a velocity as the diagonal is equal to the sum of the forces of two bodies moving with velocities proportional to the sides of the rectangle; and, because the square of the diagonal is equal to the sum of the squares of the two sides, they thence infer that the forces of equal bodies are as the squares of their velocities. But in all these arguments (which are the most plausible of any that have been offered for their new doctrine, and are most apt to mislead their readers) they do not consider that the force which one body loses, in acting upon another, is not equal to that which it produces or destroys in the other, estimated in any direction at pleasure, but in that only in which the first body acts; and that body, in consequence of its *inertia*, not only resists
any

any change in its quantity of motion, but likewise any change in the direction of its motion. If any planet revolves in a circle, the gravity of it towards the centre is employed, during the whole revolution, in changing the direction of its motion only, without producing the least augmentation or diminution of the motion itself. But these things will more easily appear after we have treated of the composition and resolution of motion : we only observe here, that, in order to support their favourite doctrine, they embarrass the plain, simple and beautiful theory of motion, in some cases by neglecting the time, and in others by confounding the directions in which bodies act upon each other, or upon springs ; while all the valuable consequences which they pretend to draw from this doctrine follow more naturally, and in a satisfactory manner only, from the laws of motion rightly understood and applied.

8. Our author's first corollary, from the laws of motion, is, that when a body is acted upon by two forces at the same time, it will describe the diagonal, by the motion resulting from their composition, in the same time that it would describe the sides of the parallelogram by those forces acting separately. Let the body A (*Fig. 2.*) have a motion in the direction A B, represented by the right line A B, at the same time let another motion be communicated to it in the direction A D, represented by the right line A D ; complete the parallelogram A B C D ; and the body will proceed in the diagonal A C, and describe it in the same time that it would have described the side A B by the first motion, or the side A D by the second. To understand our author's demonstration of this corollary, we must premise this obvious principle, that when a body is acted upon by a motion or power parallel to a right line given in position,

this

this power or motion has no effect to cause the body to approach towards that right line or recede from it, but to move in a line parallel to that right line only; as appears from the second law of motion. Therefore AD being parallel to BC , the motion in the direction AD has no effect in promoting or retarding the approach of the body A towards the line BC ; consequently it will arrive at this line BC in the same time as if the first motion AB only had been imprest upon it. In like manner, because AB is parallel to DC , the motion AB has no effect in promoting or retarding the approach of the body A towards the line DC ; consequently it will arrive at the line DC in the same time as if the motion AD only had been impressed upon it. Therefore the body A will arrive at both the lines BC and DC in the same time, that, by the first motion alone, it would have described AB , or, by the second alone, it would have described AD . But it can arrive at both the lines BC and DC no other way than by coming to their intersection C : therefore, when the two motions AB and AD are imprest upon it at once, it moves from A to C , and describes the diagonal AC , in the same time that, by these motions acting separately, it would have described the sides AB and AD .

9. Because this corollary is of very extensive use, it may be worth while to illustrate it farther. Suppose (*Fig. 3.*) the space $EFGH$ to be carried uniformly forward in the direction AB , and with a velocity represented by AB . Let a motion in the direction AD , and measured by the right line AD , be imprest upon the body A in the space $EFGH$. To those who are in this space, the body A will appear to move in the right line AD ; but its real or absolute motion will be in the diagonal AC of the paral-

parallelogram $A B C D$; and it will describe $A C$ in the same time that the space by its uniform motion, or any point of it, is carried over a right line equal to $A B$, or that the body A , by its motion across the space, describes $A D$. For it is manifest that the line $A D$, in consequence of the motion of the space, is carried into the situation $B C$, and the point D to C ; so that the body A really moves in the diagonal $A C$.

10. The converse of this corollary is, that the motion in the diagonal $A C$ may be resolved into the motions in the sides of the parallelogram $A B$ and $A D$. For it is manifest, that if (*Fig. 4.*) $A K$ be taken equal to $A D$ with an opposite direction, and the parallelogram $A K B C$ be completed, the right line $A B$ shall be the diagonal of this parallelogram; consequently, by the two last articles, the motion $A C$ compounded with the motion $A K$ equal and opposite to the motion $A D$, produces the motion $A B$; that is, if from the motion $A C$, in the diagonal, you subduct the motion $A D$ in one of the sides, there will remain the motion $A B$ in the other side of the parallelogram $A B C D$.

11. This doctrine will receive farther illustration by resolving each of the motions $A B$ and $A D$ into two motions, one in the direction of the diagonal $A C$ and the other in the direction perpendicular to it; that is, by resolving (*Fig. 5.*) the motion $A B$ into the motions $A M$ and $A N$, and the motion $A D$ into the motions $A K$ and $A L$. For the triangles $A D K$ and $B C M$ being equal and similar, $D K$ is equal to $B M$, or $A L$ to $A N$; so that the motions $A L$ and $A N$, being equal and opposite, they destroy each others effect: and it being an obvious and general principle, that the motion of a body in a right line is no way

K

affected

affected by any two equal powers or motions that act in directions perpendicular to that line, and opposite to each other, it thus appears how the body *A* is determined to move in the diagonal *A c*; and, because *A k* is equal to *m c*, it appears how the remaining motions *A m* and *A k* are accumulated in the direction *A c*, so as to produce a motion measured by *A c*. It appears likewise how absolute motion is lost in the composition of motion; for the parts of the motions *A B* and *A D* that are represented by *A n* and *A l*, being equal and opposite, destroy each other's effect, and the other parts *A m* and *A k*, only, remain in the direction of the compounded motion *A c*: while, on the contrary, in the resolution of motion, the quantity of absolute motion is increased, the sum of the motions *A B* and *A D*, or *B c*, being greater than the motion *A c*. But the sum of the motions, estimated in a given direction, is no way affected by the composition or resolution of motion, or indeed by any actions or influences of bodies upon each other, that are equal and mutual and have opposite directions.

For suppose that (*Fig. 6.*) the motions are to be estimated in the direction *A P*; let *C P*, *B R*, *D Q*, be perpendicular to this direction in the points *P*, *R* and *Q*; then the motions *A c*, *A B*, *A D*, reduced to the direction *A P*, are to be estimated by *A P*, *A R* and *A Q* respectively, the parts which are perpendicular to *A P* having no effect in that direction. Let *A P* meet *B c* in *s*; then because *R P* is to *s P*, as *B c* (or *A D*) to *C s*, that is, as *A Q* to *s P*, it follows that *A Q* is equal to *R P*, and that *A R* + *A Q* is equal to *A P*; that is, that the sum of the motions *A B* and *A D*, reduced to any given direction *A P*, is equal to the compounded motion *A c* reduced to the same direction. From which it is obvious, that, in general,

ral, when any number of motions are compounded together, or are resolved according to this general corollary, the sum of their motions continues invariably the same, till some foreign influence affects them.

12. The usefulness of the same corollary has induced authors to invent other demonstrations for the farther illustration of it. We shall only add a proof of the simplest case, when the motions $A B$ and $A D$ are equal, and the angle $B A D$ is a right one; in this case $A B C D$ (*Fig. 7, 8.*) is a square, and the diagonal $A C$ bisects the angle $B A D$; and, because the powers and motions of $A D$ and $A B$ are equal, and there can be no reason why the direction of the compounded power or motion should incline to one of these more than to the other, it is evident that its direction must be in the diagonal $A C$; and that the compounded power or motion is measured by $A C$ appears in the following manner. If it is not measured by $A C$, first let it be measured by any right line $A E$ less than $A C$; join $B D$ intersecting $A C$ in K , upon $A C$ take $A M$ greater than $A K$, in the same proportion that $A C$ is greater than $A E$; thro' the point M draw the right line $F G$ parallel to $B D$, meeting $A D$ in G and $A B$ in F ; complete the parallelograms $A M G H$ and $A M F N$: then because these parallelograms are squares as well as $A B C D$, and $A D$ is to $A G$, as $A K$ to $A M$, that is as $A E$ to $A C$; and $A B$ to $A F$ in the same proportion; and because $A E$ is supposed to be the power or motion compounded from $A B$ and $A D$, it follows that the power or motion $A D$ may be supposed to be compounded from the powers or motions $A M$ and $A H$, and $A B$ from $A M$ and $A N$. But $A H$ and $A N$, acting equally with opposite directions, destroy each other's effect; so that it would follow that the remaining

maining powers or motions $AM + AM$ (*i. e.* $2AM$) which are accumulated in the direction of the diagonal AC , ought to be equal to AE ; which is absurd, for AM is greater than AK by the construction, and $2AM$ greater than $2AK$ or AC , which is supposed to be greater than AE . In like manner, it is shewn (*Fig. 8.*) that the compounded power or motion, in the diagonal AC , is not measured by a right line greater than AC ; and therefore it is measured precisely by the diagonal AC itself.

13. The state of any system of bodies, as to motion or rest, is judged by that of their centre of gravity, in the most simple and convenient manner. In a regular body of a homogeneous texture, the centre of gravity is the same with the centre of magnitude; and, in general, it is that point of an heavy body, which being sustained, the body is in consequence itself sustained. In two equal bodies it is in a right line joining their centres, at equal distances from both: when the bodies are unequal, it is nearer to the greater body, in proportion as it is greater than the other; or its distances from their centres are inversely as the bodies. Let A (*Fig. 9.*) be greater than B , join AB , upon which take the point C , so that CA may be to CB , as the body B is to the body A , or that $A \times CA$ may be equal to $B \times CB$, then is C the centre of gravity of the bodies A and B ; and we shall afterwards shew, that if A and B be joined by an inflexible rod AB void of gravity, and the point C be sustained, then the bodies A and B shall be in *æquilibrio*. If the centre of gravity of three bodies be required, first find c the centre of gravity of A and B , and supposing a body to be placed there equal to the sum of A and B , find G the centre of gravity of it and D ; then shall G be the centre of gravity of the three bodies A , B , and

and D: in like manner, the centre of gravity of any number of bodies is determined.

14. The sum of the products that arise by multiplying the bodies by their respective distances from a right line, or plane, given in position, is equal to the product of the sum of the bodies multiplied by the distance of their centre of gravity from the same right line or plane, when all the bodies are on the same side of it: but when some of them are on the opposite side, their products when multiplied by their respective distances from it are to be considered as negative, or to be subducted. Let IL (*Fig. 10.*) be the right line given in position, c the centre of gravity of the bodies A and B , Aa , Bb , cc perpendiculars to IL in the points a , b , c ; then if the bodies A and B be on the same side of IL , we shall find $A \times Aa + B \times Bb = A + B \times cc$. For drawing thro' c the right line MN parallel to IL , meeting Aa in M , and Bb in N , we have A to B , as BC to AC , by the property of the centre of gravity; and consequently A to B , as BN to AM , or $A \times AM = B \times BN$; but $A \times Aa + B \times Bb = A \times cc + A \times AM + B \times cc - B \times BN = A \times cc + B \times cc = A + B \times cc$.

When (*Fig. 11.*) B is on the other side of the right line IL , and c on the same side with A , then $A \times Aa - B \times Bb = A \times cc + A \times AM - B \times BN + B \times cc = A + B \times cc$: and when the sum of the products of the bodies on one side of IL multiplied by their distances from it, is equal to the sum of the products of the bodies multiplied by their distances on the other side of IL , then cc vanishes, or the common centre of gravity of all the bodies falls on this right line IL .

15. Suppose now the bodies A and B to proceed in the right lines AD and BE , (*Fig. 12.*) and when

they come to D and E their common centre of gravity to be found in G : let Dd , Ee , Gg be perpendiculars to IL , in d , e , g ; let DM , EN , GK , parallel to IL , meet Aa , Bb , Cc , respectively, in the points M , N , K . By the last article, $A \times Dd + B \times Ee = A + B \times Gg$; and subtracting this from the equation in the preceding article, *viz.* $A \times Aa + B \times Bb = A + B \times Cc$, then $A \times AM + B \times BN = A + B \times CK$. By proceeding in the same manner it will appear that $A \times DM + B \times EN = A + B \times GK$. The motions of A and B being supposed uniform, the right lines AM and BN will increase uniformly, so as to become double in double the time; consequently CK will also increase uniformly, or in the same proportion as the time. And because DM , EN , increase uniformly, it follows, that GK also increases uniformly; and that CK is to KG in the constant ratio of $A \times AM + B \times BN$ to $A \times DM + B \times EN$. Hence it appears, that when any number of bodies move in right lines with uniform motions, their common centre of gravity moves likewise in a right line with an uniform motion; and that the sum of their motions, estimated in any given direction, is precisely the same as if all the bodies, in one mass, were carried on with the direction and motion of their common centre of gravity. Because the sum of the motions of the bodies, estimated in any given direction, is preserved invariably the same in their collisions, without being affected by their actions upon each other, that are equal and mutual and have contrary directions; it follows, that the state of their centre of gravity is no way affected by their collisions, or any such actions; and that it perseveres in its state of rest or uniform motion, in the same manner as by the first law of motion any one body perseveres in its state, till some external influence disturb it. These propositions represent to us the theory of

motion in a plain and beautiful light ; and enable us to judge the motions of a system of bodies, with almost the same facility as of those of one body.

16. The motions and actions of bodies upon each other, in a space that is carried uniformly forward, are the same as if that space was at rest ; and any powers or motions that act upon all the bodies, so as to produce equal velocities in them in the same or in parallel right lines, have no effect on their mutual actions or relative motions. Thus the motion of bodies aboard a ship, that is carried steadily and uniformly forward, are performed in the same manner as if the ship was at rest. When a fleet of ships is carried away by an uniform current, their relative motions are no way affected by the current, but are the same as if the sea was at rest. The motion of the earth and air round its axis has no effect on the actions of bodies and agents at its surface, but so far as it is not uniform and rectilinear. In general, the actions of bodies upon each other depend not upon their *absolute* but *relative* motion ; which is the difference of their *absolute* motions when they have the same direction, but their sum when they are moved in opposite directions.

17. No principle being more universally allowed than this, or more evidently established upon common experience, we deduced the following argument from it against the new doctrine concerning the forces of bodies in motion, in a piece that obtained the prize of the *royal academy* of sciences at *Paris*, in 1724 ; which, because of its plainness and simplicity we shall describe here again. Let A and B (*Fig. 13.*) be two equal bodies that are separated from each other by springs interposed between them

(or in any other equivalent manner) in a space $EFGH$, which in the mean time proceeds uniformly in the direction BA (in which line the springs act) with a velocity as 1 ; and suppose that the springs impress on the equal bodies A and B equal velocities, in opposite directions, that are each as 1 . Then the absolute velocity of A (which was as 1) will be now as 2 ; and according to the new doctrine its force as 4 ; whereas the absolute velocity and the force of B (which was as 1) will be now destroyed; so that the action of the springs adds to A a force as 3 , and subducts from the equal body B a force as 1 only; and yet it seems manifest, that the actions of the springs, on these equal bodies, ought to be equal; (and Mr. *Bernoulli* expressly owns them to be so): that is, equal actions of the same springs upon equal bodies would produce very unequal effects, the one being triple of the other according to the new doctrine; than which hardly any thing more absurd can be advanced in philosophy or mechanics. In general, if m represent the velocity of the space $EFGH$ in the direction BA , n the velocity added to that of A and subducted from that of B , by the action of the springs, then the absolute velocities of A and B will be represented by $m+n$ and $m-n$ respectively, the force added to A by the springs will be $2mn + nn$, and the force taken from B will be $2mn - nn$, which differ by $2nn$. Farther, it is allowed that the actions of bodies upon one another are the same in a space that proceeds with an uniform motion as if the space was at rest: but if the space $EFGH$ was at rest, it is allowed that the forces communicated by the springs to A and B had been equal; and, according to the new doctrine, the force of each had been represented by nn ; whereas the force communicated to A by the springs in the space $EFGH$ is represented by $2mn + nn$, and the force

taken

taken from B will be $2\ m\ n - n\ n$. These arguments are simple and obvious, and seem, on that account, to be the more proper in treating of this question. They who maintain the new doctrine may define force in such a manner, as to make the dispute appear to relate merely to words; but, as the terms *action* and *force* seem to be very nearly allied to each other, it surely tends to confound our notions and language, to maintain that equal actions generate or produce unequal forces in the same time. But what evidently shews that the authors on the side of this new opinion did not understand what they taught, is, their telling us, that the quantity of absolute force is unalterable by the collisions of bodies, and that this follows so evidently from the equality of *action* and *reaction*, that to endeavour to demonstrate it would only render it more obscure. For hence it appears, that they understood equal changes to be produced in the forces of bodies in consequence of the equality of *action* and *reaction*; and yet it is evident from what we have shewn, that the changes produced in the forces of bodies must be very unequal, according to this new doctrine, though the *action* and *reaction* by which they are produced be equal. It seems to have been by a mistake, that Mr. *Leibnitz* first found himself engaged to maintain this new doctrine, in 1686; and in like manner, some of his disciples seem to have rashly adopted the same, without having attended to the consequences.

18. In the theory of motion, rightly understood, the same laws that serve for comparing, compounding, or resolving motions, are observed likewise by pressures; that is, the powers that generate motion, or tend to produce it: for forces are nothing else but the sums of such pressures accumulated in the body,

body, in consequence of the continued action of the powers for a finite time ; and pressures are considered as infinitely small forces, or as the elements from which the forces are produced : and it adds no small beauty and evidence to this theory of motion, that both observe the same laws. When a force is generated in any body, by the accumulation of other forces or impulses, that which is generated, in any direction, must be equal to the sum of those which are all employed and consumed, in that direction, in producing it ; and if the force is produced by a continual successive action, the motion generated must be equal to the sum of the pressures that are exerted in producing it. In like manner, if motion is destroyed by the resistance of any opposite power, it must be equal to the sum of all the actions by which it is totally destroyed. On the other hand, the intensity of the power that generates motion in any body, is proportional to the augment of force which it generates in a given time, and the intensity of the power that resists or destroys motion, is measured by the decrement of force produced in a given time ; since the augment in the first case, and decrement of motion in the second case, are the adequate effects of the power ; which is supposed to be of such a nature as to be renewed every moment, and exert all its influence at once. In general, the intensity of any power that generates or destroys motion is the greater, in proportion as the change of velocity produced by it in the direction of that power is greater, and the less the time is in which that change is produced, if the intensity of the power continues uniform during that time : but if the power varies, its intensity, at any given term of the time, is to be measured by the change of velocity which would have been produced, in a given time, by the power continued uniformly for that time.

19. The pressure or power that generates motion in a body is in the compounded ratio of the quantity of matter in the body, and of the velocity which it would generate in it in a given time, if it was continued uniform for that time; and those pressures are equal in any two bodies, when their quantities of matter are reciprocally as those velocities, that is, when the intensity of the power that acts upon the greater body A, is less than the intensity of that which acts upon the lesser body B, in the same proportion as B is less than A. If two bodies that are acted upon by such powers, with opposite directions, be in contact, neither of the powers will prevail, and no motion will be produced. In the same manner, if two bodies, moving with velocities inversely proportional to their quantities of matter, meet with opposite directions, their motions will destroy each other, if they are soft bodies; or if they are so perfectly hard as that their parts are quite inflexible, they will both stop after the stroke: but if they have any elasticity, they will be reflected after the stroke with equal motions. Thus there is a perfect harmony between the laws of pressures, or powers, and the laws of motions or forces produced by those powers; as, in general, there must be an analogy between the powers that generate or produce any effect, and the effects themselves which are generated. But this harmony is quite lost, as to the forces of bodies, according to the new opinion concerning their mensuration; for, according to this opinion, when the velocity is finite, how small soever it may be, the force is measured by the square of the velocity; but when the velocity is infinitely little (as it is, according to the favourers of the new opinion) in consequence of the first impulse of the power that generates the motion, the force is simply

simply as the velocity ; and we cannot but observe, that this sudden change of the law does not appear to be consistent with the favourite principle of *continuity*, so zealously maintained by the same philosophers. According to the same opinion, forces that sustain each other, with opposite directions, and destroy each other's effect, may be unequal in any given ratio ; and when bodies meet with equal forces in opposite directions, they do not therefore sustain each other, but that which has the greater velocity carries it against the other. Let v denote the velocity of A , and v the velocity of B ; then $A \times v$ will denote the motion or force of A , and $B \times v$ the motion or force of B ; so that these motions are equal when $A \times v = B \times v$, that is, when v is to v , as B is to A : and this is the case wherein constant experience teaches us that the motions sustain each other, provided their directions be opposite. But, according to the new opinion, the force of A is measured by $A \times v v$, and the force of B by $B \times v v$, which are to each other in the same proportion as v to v , in the present case, because we suppose $A \times v = B \times v$. These forces, therefore, according to the new opinion, are so far from being equal, that the force of A is less than the force of B , in proportion as v is less than v , or B less than A ; so that, according to this doctrine, a force might sustain, or even overcome, a force 1000 times greater than itself, or greater than itself in any assignable proportion. According to the same doctrine, the forces of A and B are equal, when $A \times v v = B \times v v$, that is, for example, when A being quadruple of B , the velocity of B is double of the velocity of A ; in which case the quantity of motion, or *momentum*, of A is double of that of B ; and the motion of A appears, from experience, to be more than sufficient to sustain the motion of B . It has cost the favourers of the new

opinion

opinion a great deal of pains to compose their accounts, by which they endeavoured to reconcile their theory with experience; and how unsatisfactory their accounts have proved, will easily appear to the reader who will take the trouble to examine them.

20. Let the bodies A and B (*Fig. 14.*) by moving towards each other, compress equal and similar springs placed between them, till by the reaction of those springs their motions be destroyed. Mr. *Bernoulli* expressly owns, that the actions of the springs on those bodies are constantly equal to each other, and yet maintains that they destroy a force in B greater than the force of A, in the same proportion as the body A is greater than B, or (c being the centre of gravity of A and B) as c B is greater than c A. He therefore maintains, that equal pressures or actions of springs generate, in the same time, forces that may be unequal in any assignable ratio; which is repugnant to the plainest notions we are able to form of action and force, and serves only to introduce mysterious and obscure conceptions into the theory of motion, without any necessity. If we suppose the body A to compress the springs from A to c, then the body B will compress all the springs from B to c, in the same degree, and in the same time; and thence he infers, that the force of A is to the force of B, in the same proportion as the number of springs from c to A, to the number of springs from c to B. But since the motion, force, or effect of any kind, produced or destroyed in A or B, depends upon the immediate action which produces the effect, and upon it only; and since, in this case, the actions of the springs upon the bodies A and B are those which destroy their motions; and since it is allowed by him that the actions of the springs upon these bodies are equal, is it not evident that the forces

forces destroyed by them in the same time must be equal? And is it not manifest, that the forces which are produced or destroyed in bodies, are to be measured by the efforts which the springs exert upon the bodies in producing this effect, and not by the number of springs? It is the last spring only, which is in contact with the body, that acts upon it, the rest serving only for sustaining it in its action; so that any change produced in the body, by whatever name it be called, ought to be determined from the action of this last spring only, and in just reasoning ought to be computed from it alone. Had he defined force by the number of equal and similar springs, that, by a given degree of expansion or compression, produce or destroy it, just exceptions might have been made against the propriety and convenience of such new and unnecessary expressions, as tending to perplex and darken this most useful theory of motion, which was before very clear and evident: but then this controversy would have appeared to relate chiefly to words and terms of art, and there would not have been so much danger of mistakes arising from their doctrines. But he does not give this for the definition of force.

21. When a body descends by its gravity, the motion generated may be considered as the sum of the uniform and continual impulses accumulated in the body, during the time of its falling. And when a body is projected perpendicularly upwards, its motion may be considered as equivalent to the sum of the impulses of the same power till they extinguish it. When the body is projected upwards with a double velocity, these uniform impulses must be continued for a double time, to be able to destroy the motion of the body; and hence it arises, that the body, by setting out with a double velocity, and ascend-

ascending for a double time, must arise to a quadruple height, before its motion is exhausted. But this proves that a body with a double velocity moves with a double force, since it is produced or destroyed by the same uniform power continued for a double time, and not with a quadruple force, though it arise to a quadruple height. This, however, was the argument upon which Mr. *Leibnitz* first built this doctrine; and those which have been since derived from the indentings or hollows produced in soft bodies by others falling into them, are much of the same kind and force. Causes are not to be measured by any effects produced by them, taken without any choice, or judgment, or regard to their circumstances. Motions and forces are not to be measured by the effects produced, without regard to the times and directions of the motions, according to the principles of geometry and mechanics. In geometry, we judge of wholes by comparing their parts, or the elements from which they are generated; and, in mechanics, we can have no better method of judging of motions, or forces, than from the powers that produce them. The motion, or force of a body has a much more simple and plain analogy to the power that produces it, than to the space described by it in soft clay or any other resisting medium.

22. The principle, “that the cause is to be measured by its effect,” is one of those that will be very apt to lead us into error, both in metaphysics and natural philosophy, if applied in a vague and indistinct manner, without sufficient precautions. Force is defined to be that power of acting in a body which must be measured by its whole effect till its motion be destroyed, by those who favour the new opinion, or some of them at least, and by some who

who would represent this dispute as merely about words. But the same authors tell us likewise, that force is proportional to the number of springs which it can bend before it be destroyed; and this they propose, without any proof, as a definition or axiom. Did they content themselves with the latter of these only, we should allow the dispute to be of very little moment, farther than as such liberties tend to confound our notions of the action and motion of bodies, as we observed above. But while they pretend that force, defined by them at their pleasure, is to be considered as the cause of the effects produced by motion, and is to be measured by those effects, the dispute appears no longer to be about words only. Sir *Isaac Newton*, in his second law of motion, points out to us that the impressed force being considered as the cause, the change of motion produced by it is the effect that measures the cause; and not the space described by it against the action of an uniform gravity, nor the hollows produced by the body falling into clay. This law of motion is the surest guide we can follow, in determining effects from their causes, or conversely the causes from their effects.

23. The harmony between the laws of pressures, or powers, that generate motion, and the laws of these motions themselves, appears in a fuller light when we attend to their composition and resolution. Powers acting in the directions AB and AD (*Fig. 4.*) proportional to those right lines, compound a power that acts in the direction of the diagonal AC , and is measured by AC . Because AC is less than $AB + AD$, the power compounded from AB and AD is always less than those powers themselves; and this is fully accounted for by resolving the power AB into AM and AN , (*Fig. 5.*) and the power AD into AK and

A L; of which A N and A L are opposite and equal and destroy each other's effect, so that there remains $AM + AK$, or A C, the measure of the compounded power. The favourers of the new opinion agree with us in arguing in this manner, concerning powers and pressures; but in a manner quite inconsistent with this, in the composition and resolution of forces. When the angle B A D is right, the compounded force is equal to the sum of the forces A B and A D, according to them; and no force is lost, notwithstanding the opposite directions of the forces A L and A N; tho' it is not easy to conceive how this should not have an effect in the composition of forces, as well as of powers and pressures. When the angle B A D (*Fig. 15.*) is acute, the square of the diagonal A C exceeding the sum of the squares of A D and D C, (*Euclid, 12. 2.*) or of A D and A B, the two forces in the directions A D and A B, must, according to the new doctrine, compound a force A C greater than their sum. Now this appears directly contradictory to the metaphysical principle so much insisted on by them, that the effect is proportional to the cause which produces it; for in this case, the effect is greater than the cause; and this seems to be as absurd, in mechanics, as that two quantities collected together should produce a greater quantity than their sum, in geometry. When this was objected, the answer * given to it deserves to be copied, for a specimen of their way of getting over difficulties: it is no more but that “no absurdity follows from the
 “ new opinion, which by measuring forces, not by
 “ *momenta*, but, by the square of the velocities,
 “ concludes that on account of the angle D A B its
 “ being acute, the square of A C (which is the force

* See *Desagulier's* course of experimental philosophy, vol. 2, in the note at the bottom of page 72.

“ compounded) is greater than the squares of AB
 “ and AD , the sum of what they call the compound-
 “ ing forces.”

24. To illustrate this farther, suppose that the elastic body A (*Fig. 16.*) receives its force, in the direction AB , from the equal elastic body H , and its force, in the direction AD , from the equal elastic body G , at the same time. According to the patrons of the new doctrine, the forces of H and G are communicated to A by infinitely small degrees, or by an uninterrupted succession of pressures, and the whole force communicated to A is the sum of the effects of these pressures. Now in every instant the pressure, or infinitely small force impressed on A , is less than the sum of the pressures exerted in that instant by H and G , in proportion as AC is less than $AB + AD$, as is allowed on all sides. Therefore the sum of all the pressures, or the force impressed on A , must be less than the sum of all the pressures, or the sum of the forces exerted by H and G , in the same proportion of AC to $AB + AD$; that is, the forces of A , H , and G , must be as the lines AC , AB and AD , and not as their squares. It is not possible to conceive that while the force in A arises from the accumulation of the pressures, or infinitely small forces, which it receives every moment from the actions of H and G , and each of these pressures, or infinitely small forces, is less than the sum of the actions of H and G that produce them; yet the whole force of A should nevertheless exceed the sum of the whole actions or forces of H and G . I speak here of infinitely small forces, to comply as much as possible with the style of the favourers of this new opinion. To * this they gave no other answer than that what we call

* *Ibid.* p. 73, in the last note.

forces here ought to be called *momenta*. But they pretend not to explain how the infinitely small forces impressed upon A, in the direction AC, come to produce a finite force far greater than their sum total; or how the effect should be so far from corresponding to the cause; the metaphysical principle which they seem to use, or reject, just as it serves their turn. If we suppose the angle B A D to be infinitely acute, the same forces (according to the new opinion) generate a force in A which exceeds their sum as much as the square of $AB + AD$ exceeds the sum of the squares of AB and AD; so that if AD be equal to AB, they will in that case generate at A a force double of their sum, for then the square of $AB + AD$ will be equal to the square of $2 AB$, that is to $4 AB^2$; tho' the two equal forces which are supposed to produce this, taken together, amount only to $2 AB^2$, according to their own computation; so that, in this case, a cause produces an effect of the same kind double of itself. To this it has been † answered, that, according to the new opinion, a double *momentum* may produce a quadruple effect, if the velocity is double. But surely the author who gave this answer did not attend to the objection; for what we have proved, is not that a double *momentum* produces a quadruple effect, but that a double force, according to their own notion and computation, produces a quadruple force, according to the same notion and computation. And indeed the sum of the answers they have made to the absurdities which have been deduced from their favourite opinion amounts to this, *viz.* that they are no absurdities, because their new opinion obliges them to admit them.

† Ibid. p. 74, in the notes.

25. The resolution of powers, or pressures, is a necessary consequence of their composition. As motion is lost in the composition, so it is necessarily gained in the resolution of motion; and as this is allowed of motions, and of the powers that generate motion, there can be no good reason given why it ought not to be allowed of the effects of those powers, or of the force of bodies. The same reasons that argue for an increase in the one case, prove, with the same evidence, that an increase of the other ought likewise to be allowed. Let the body c (*Fig. 17.*) moving in the direction dc , the diagonal of the parallelogram $cldk$, strike the equal body A obliquely, so as to impel it in the direction ca the continuation of ck , and at the same time the equal body B , in the direction cb the continuation of cl ; the body A will proceed in the right line ca , and the body B will proceed in the direction cb the continuation of cl , and c having communicated all its force to them will stop. It will not appear strange that the motions and forces of A and B exceed the motion or force of c , if we consider that c communicates the whole motion or force ck to A , and the whole motion or force cl to B , that the resistance or *inertia* of A reacting upon c , not in the direction of its motion cd , but in the direction ck oblique to it, the absolute motion or force of c , in the direction dc , is not so much diminished by this reaction as if it was directly opposite to the motion of c ; for no power, or resistance, can produce so great an effect in any direction as in that wherein it acts. In like manner the reaction of B destroys the motion or force lc in the body c , in the direction in which B reacts; but not so great a motion or force in the direction dc to which it is oblique; and thus it appears, that the motion or force of c , in the direction

dc ,

DC , must necessarily be less than the sum of the motions or forces of the bodies A and B in their respective directions. If it be objected, that, in this case, the motion of c , in the direction DC , is the cause of the motions of A and B , in the directions CA and CB ; so that a cause produces effects whose sum is greater than itself; in answer to this, we have already observed, that as this is allowed on all hands of motions and pressures, it cannot be absurd to extend it to forces, but must obtain in them for the same reasons. But farther, we are to observe, that, in consequence of the *inertia* of body, it not only resists any change of its motion, but likewise any change in the direction of its motion; and that when the action of bodies upon each other is not in a right line, both these are to be taken into the account. Suppose the body c first to strike upon A , then the reaction of A has a twofold effect; it subducts somewhat from the motion or force of c , and at the same time it produces a change in the direction of c ; and the reaction of A (to which the motion or force produced in it is equal) is not to be estimated by one of those effects only, but by both conjointly. After the body c has struck A , it proceeds in the right line CB with a motion or force as CL , and, impinging upon B directly, it communicates its whole motion or force to B which reacts directly against it. We have supposed the bodies c , A , and B to be perfectly elastic, in conformity to the suppositions of our opponents, some of whom confine themselves in their enquiries to these only.

26. If we substitute springs in place of the bodies A and B , and their resistances be measured by CK and CL , it will appear, in the same manner, that the resistances of those springs are not the proper measures of the force of the body c , but that taken

together they must exceed it; for the spring *A* acts at a disadvantage against the motion or force of *c*. It has its whole effect in the direction *ck* in which it resists; but not so great an effect in the direction *co*, which is oblique to that in which it acts. If the spring *A* acted with the same advantage as *B*, they would together produce a greater effect than in the situation they have in the figure; and therefore the greatest resistances which they are able to exert taken together, must exceed the force of the body *c*. Thus it appears that this argument, instead of overthrowing our doctrine, confirms it, and that they who advanced it supposed those forces to be equal, which, according to the known principles of mechanics, are unequal. If it is asked what becomes of the excess of the force of the spring *A*, above what is subducted from the force of *c*? It may be answered, that it is not without its effect: for the direction of the body is changed from the line *bc* into the right line *cb*; and no principle, either in metaphysics or mechanics, teaches us that this effect is to be neglected, in comparing the cause and effects together on this occasion. On the contrary, many instances might be given where a force is employed in producing a change in the direction of a motion of a body only, without either accelerating or retarding it. The force that is sufficient to carry a body upwards in the perpendicular to the horizon, to a double distance from the centre of the earth, is equal to that which, impressed in a horizontal direction, would carry it in a circle about the earth for ever, abstracting from the resistance of the air; as appears from the theory of gravity: and yet the first would overcome the resistance arising from the gravity of the body for a certain time only; whereas the other would overcome that resistance for ever, without any diminution of motion. In the first case,

case, the gravity of the body would act directly against its force; in the second, it would act in a line perpendicular to the direction of its motion: in the first case, the action of gravity is entirely employed in consuming the force of the body; in the other, in changing its direction only. The arguments drawn in favour of the new opinion from the resolution of motion, seem, at first sight, the most plausible of any that have been offered for it; but, from the considerations which we have suggested, it may appear to an impartial reader, that instead of overthrowing the common doctrine, they rather confirm it. As, in other instances, Mr. *Leibnitz*'s followers neglect the consideration of time, in reasoning concerning the forces of bodies; so here we find that they have not due regard to the directions of motions and forces, in estimating and comparing their effects; which, however, in mechanical enquiries, are of no less importance than the motions or forces themselves.

27. We have insisted on these observations, because they set the theory of motion in a plain and just light. We often obtain this advantage from disputes concerning the elementary propositions of any science, that they are the more carefully enquired into, and when found just, are illustrated and the better understood for having been disputed. We cannot, however, leave this subject without mentioning an experiment, made by the ingenious and accurate Mr. *Graham*, to whom the mechanical sciences are so much indebted. He prepared a pendulous body with a cavity in it capable to receive another body of an equal weight, at the lowest point of its vibration; and when the body was drop'd into it, he found, by the subsequent vibration, that the velocity of the double mass was precisely one half of

what the velocity of the pendulum was before ; from which it appears, that the same force produces in a double quantity of matter one half of the velocity only ; which is agreeable to the common doctrine, but directly repugnant to the new one, concerning the forces of bodies in motion. Many ingenious pieces have been writ against this new doctrine by learned men, to which we refer the reader who desires to see more on the subject *. It is pretended, that by this new doctrine we are enabled to resolve problems in an easy manner, which are otherwise of great difficulty ; but by the rejecting hard and inflexible bodies, there is more lost than gained in this respect, as we have shewn elsewhere, and as will appear afterwards, when we come to determine more particularly the effects of the collisions of bodies.

28. It is because *action* and *reaction* are always equal, that the mutual actions of bodies upon one another have no effect upon the motion of the common centre of gravity of the system to which they appertain. If there was any *action* in the system that had not a contrary and equal *reaction* always corresponding to it, it would affect the state of the centre of gravity of the system, and disturb its motion : and, conversely, if it be allowed that the state of the centre of gravity of a system is not disturbed by the actions of bodies upon one another that are its parts, we may conclude that their actions are mutual, equal, and have contrary directions. It will therefore be found agreeable to the course of things, and to perpetual experience, that the third law of motion be extended generally to all sorts of

* As a piece of Mr. de Mairan, in the *memoires de l'academie royale des sciences* 1728. Several pieces of Dr. Jurin, *Philosophical Transactions*, &c.

powers that take place in nature, those of *attraction* and *repulsion* as well as others, (and not to be a supposition arbitrarily introduced by Sir *Isaac Newton*;) when those powers are found to depend upon the bodies that are said to *attract* or *repel*, as well as upon those that are *attracted* or *repelled*. We find the loadstone attracts iron, and that iron attracts the loadstone with equal force; and because they attract each other equally, they remain at rest when they come into contact. If a mountain, by its gravity pressed upon the earth, and the earth did not react equally on the mountain; then the mountain would necessarily carry the earth before it, by its pressure, with a motion accelerated *in infinitum*. The same is to be said of a stone, or the least part of the earth, as well as of a mountain. Bodies act upon light in proportion to their density, *cæteris paribus*, by refracting it when it enters into them; and conversely, light acts upon bodies by heating them and putting their parts in motion. This equality of *action* and *reaction* obtains so generally, that when any new motion is produced by any power or agent in nature, there is always a corresponding equal and opposite motion produced by its *reaction* at the same time, or some equal motion in the same direction destroyed. When from an engine a weight is thrown, the engine reacts with an equal force on the earth or air. If it was not for this law, the state of the centre of gravity of the earth would be affected by every action or impulse of every power or agent upon it. But by virtue of this law, the state of the centre of gravity of the earth, and the general course of things, is preserved independent of any motions that can be produced at or near its surface, or within its bowels. By the same law, the state of the lesser systems of the planets, and the

the repose of the general system, is preserved, without any disturbance from the actions of whatever agents there may be in them. We must therefore allow, that in the *attracting* and *repelling* powers which obtain in nature, from whatever sort of cause they may arise, *action* and *reaction* are always equal ; and since this law obtains in all sorts of motions that arise from impulse, we may be the more surprized if we should find the philosophers that explain those powers from impulse call it in question. Even in the motions produced by voluntary and intelligent agents, we find the same law take place ; for though the principle of motion, in them, be above mechanism, yet the instruments which they are obliged to employ in their actions are so far subject to it as this law requires. When a person throws a stone, for example, in the air, he at the same time reacts upon the earth with an equal force ; by which means the centre of gravity of the earth and stone perseveres in the same state as before. And the necessity of this law, for preserving the regularity and uniformity of nature, well deserved the attention of those who have wrote so fully and usefully of *final causes*, if they had attended to it.

C H A P. III.

Of the mechanical powers.

1. **T**H E knowledge of mechanics is one of those things that contribute most to distinguish civilized nations from barbarians : the works of art derive their chief beauty and value from it ; and without it we can make very little progress in the knowledge of the works of nature. It is by this science that the utmost improvement is made of every

every power and force in nature, and the motions of the elements, water, air, and fire, are made subservient to the purposes of life, when industry, with materials for the necessary instruments, are not wanting. However weak the force of man appears to be, when unassisted by this art, yet with its aid, there is hardly any thing above his reach. It is a science, that admits of the strictest evidence; and certainly it is worth while to establish it on its just principles, and to cultivate it with the greatest diligence.

It is distinguished by Sir *Isaac Newton* into *practical* and *rational* mechanics; the former treats of the mechanical powers, *viz.* the *lever*, the *axis*, and *wheel*, the *pulley*, the *wedge*, and the *screw*, to which the *inclined plane* is to be added; and of their various combinations together. Rational mechanics comprehends the whole theory of motion; and shews, when the powers or forces are given, how to determine the motions that are produced by them; and, conversely, when the phænomena of the motions are given, how to trace the powers or forces from which they arise. Thus it appears that the whole of natural philosophy, besides the describing the phænomena of nature, is little more than the proper application of rational mechanics to those phænomena; in tracing the powers that operate in nature from the phænomena, we proceed by *analysis*; and in deducing the phænomena from the powers or causes that produce them, we proceed by *synthesis*. But in either case, in order to proceed with certainty, and make the greatest advances, it is necessary that the principles of this art should be premised and clearly established, being the grounds of our whole work. We have already considered the *inertia* or passive nature of body, according to which it perseveres in its state of motion or rest, receives motion
in

in proportion to the force impreſt, and reſiſts as much as it is reſiſted; which is the ſum of the three general laws of motion: from which, and their general corollaries, demonſtrated in the laſt chapter, we are now to deduce the principles of mechanics. As theſe laws and their corollaries take place, tho' the cauſes of the motions, the nature of the impreſt force, or of the reſiſtance, be unknown or obſcurely underſtood; ſo the obſcurity of the nature and cauſe of the power that produces the motions, does not hinder us from tracing its effects in mechanics with ſufficient evidence, provided we can ſubject its action to a juſt menſuration: and, in fact, we know that excellent contrivances have been invented for raiſing weights, and overcoming their reſiſtances, by ſuch as gave themſelves no trouble to enquire into the cauſe of gravity.

2. In treating of the mechanical engines, we always conſider a *weight* that is to be raiſed, the *power* by which it is to be raiſed, and the *instrument* or engine by which this effect is to be produced. There are two principal problems that ought to be reſolved in treating of each of them. The firſt is, “to determine the proportion which the power and weight ought to have to each other, that they may juſt ſuſtain one another, or be in *æquilibrio*.” The ſecond is, “to determine what ought to be the proportion of the power and weight to each other, in a given engine, that it may produce *the greateſt effect poſſible in a given time*.” All the writers on mechanics treat of the firſt of theſe problems, but few have conſidered the ſecond; tho' in practice it be equally uſeful as the other. As to the firſt, there is a general uniform rule that holds in all the powers, is founded on the laws of motion, and is another inſtance of the beauty

beauty and harmony that results from the simplicity of the theory of motion described in the last chapter. Suppose the engine to move, and reduce the velocities of the power and weight to their respective directions in which they act; find the proportion of those velocities; then if the power be to the weight, as the velocity of the weight is to the velocity of the power, or, (which amounts to the same thing) if the power multiplied by its velocity give the same product as the weight multiplied by its velocity, this is the case wherein the power and weight sustain each other and are in *æquilibrio*: so that in this case, the one would not prevail over the other, if the engine was at rest; and, if it is in motion, it would continue to proceed uniformly, if it was not for the friction of its parts, and other resistances. This principle has a plain analogy to that by which the equality of the motions, or forces, of bodies was determined in general, in chap. 2. § 19. For, as the motions of bodies are equal, and destroy each other's effect, if their directions are contrary, when the first is to the second, as the velocity of the second is to the velocity of the first, the greater velocity of the lesser body just compensating its deficiency in quantity of matter; so the actions of the power and weight are equal, and destroy each other's effect upon the engine, when the power is to the weight, as the velocity of the weight is to the velocity of the power. But tho' it is useful and agreeable to observe how uniformly this principle prevails in engines of every sort, throughout the whole mechanics, in all cases where an *æquilibrium* takes place; yet it would not be right to rest the evidence of so important a doctrine upon a proof of this kind only. Therefore we shall demonstrate the law of the *æquilibrium* in the *lever* or *velis* (which is the foundation of all the other propositions of this kind in mechanics)

nics) by a new method, that seems to us to be founded on the plainest and most evident principles; to which we shall subjoin the demonstration given by Sir *Isaac Newton* of the same law, and that which is ascribed to *Archimedes*.

3. In the *first* place it is evident, that if equal powers act at equal distances on different sides of the prop, or centre of motion, with directions opposite and parallel to each other, they will have the same effect. Thus, *A B* (*Fig. 18.*) being bisected in *c*, if a power *A* act upon the lever in the direction *A F*, and an equal power *B* act upon it with an opposite and parallel direction *B E*, then the effects of those powers, to move the lever about the centre *c*, will be precisely equal; so that the one may be always substituted for the other. A *second* principle is, that, gravity being supposed to act in parallel lines, if the prop *c* (*Fig. 19. n. 1.*) be between the bodies *A* and *B*, it must bear the sum of their weights; because the lever being loaded with those weights, it must give way if the prop does not sustain their sum; but that when the powers *A* and *B* are on the same side of the prop or fulcrum *c*, (*Fig. 19. n. 2.*) in which case one of them, as *A*, must pull upwards, while the other *B* pulls downwards, that there may be an *æquilibrium*, it is then only loaded with the difference of the powers *A* and *B*. The one of those cases always follows from the other, if we consider, that in the case of the *æquilibrium*, any one of the three powers that act at *A*, *B*, and *c*, may be considered as that of the prop, and the other two as endeavouring to turn the lever about it. From these principles we deduce the law of the *æquilibrium* in the lever, in the following manner.

4. Supposing

4. Supposing first two equal powers, A and B (*Fig. 20.*) acting in the directions A F, B H, to carry a body c, upon the lever A B, placed at c at equal distances from them ; it is evident, that, in this case, each of the powers A and B sustains one half of the weight c, by dividing it equally between them. Imagine now that the power A is taken away, and that, instead of resting upon it, the end A of the lever rests upon a prop at A ; it is manifest that the power B, and the prop at A, sustain, as before, each one half of the weight c ; the prop now acting, in every respect, as the power at A before ; and, the *equilibrium* continuing, it appears, that, in this case, a power B equal to one half of the weight c sustains and ballances it, when the distance of c from the prop A is one half of the distance of B from the same ; that is, when B is to c, as c A to B A, or $B \times B A = c \times c A$. From this simple instance we see, that powers act upon a lever not by their absolute force only, but that their effect necessarily depends upon the distance of the point where they act from the prop, or centre of motion ; and particularly, that a power ballances a double power which acts at half its distance from the prop, on the same side of it, with an opposite direction.

The case when the two powers act on different sides of the prop, follows from this, by the principles laid down in the last article. For let B H and c G (*Fig. 21.*) represent the directions and forces with which the powers B and c act upon the lever ; upon B A produced take A E equal to A C, or $\frac{1}{2}$ A B, and in place of the power c G substitute an equal power E K at E, with an opposite direction ; and, by the first of those principles, this power E K will have the same effect as c G, only the prop, or centre of motion,

motion, A will now sustain the sum of the forces $E K$ and $B H$, by the second principle in the last article. But the *equilibrium* between the powers $B H$ and $E K$ will continue as it was before between $B H$ and $C G$; so that the powers $B H$ and $E K$ will be *in equilibrium*, when the power $B H$ is one half of $E K$, and the distance of $E K$ from the prop A is one half of the distance of $B H$ from the same; that is, when the power at B is to the power at E , as $A E$ to $A B$, or $B \times B A = E \times E A$. In this case the prop A being loaded with both the powers B and E which act with the same direction, its reaction must be equal to their sum $E K + B H = 3 B H$, and must be in the opposite direction $A F$. In place of this reaction, let us now (*Fig. 22.*) substitute a power $A F$ at A , equal to thrice $B H$; and in place of the power $E K$, let us substitute a prop at E , sustaining that end of the lever $B E$; and since the *equilibrium* continues as before, it follows, that the prop, or centre of motion, being at E , the power $B H$ sustains the power $A F$ which is triple of $B H$, when the distance of $B H$ from the prop E is triple of the distance of the power $A F$ from the same, that is, when $B H \times B E = A F \times A E$.

If we suppose the power $E K$ to remain (*Fig. 23.*) but the end B of the lever $E B$ to rest upon a prop, then the powers $A F$ and $E K$ will sustain and ballance each other, the prop at B now coming in place of the power $B H$; in which, $A F = 3 B H$, and $E K = 2 B H$; so that $A F$ is to $E K$ as 3 to 2; and the distances $E B$ and $A B$ being in the same proportion, it appears that when two powers in the proportion of 3 to 2 act upon a lever on the same side of the prop, or centre of motion, with opposite directions, at distances in the proportion of 2 to 3, they then sustain each other. We have demonstrated therefore,

fore, that when the powers are in the proportion either of 2 to 1, or of 3 to 1, or of 3 to 2, and the distances of their application from the centre of motion are in the inverse proportion, then those powers ballance each other, or are in *æquilibrio*.

5. Upon BE produced (*Fig. 25. n. 1.*) take $EL = EA$; and in place of the power AF substitute a power $LM = AF$, but with a contrary direction; this power LM will have the same effect to turn the lever round the center of motion E as AF had, by the first principle in § 3; consequently it will be in *æquilibrio* with the power BH , as AF was. Therefore when two powers LM and BH , in the proportion of 3 to 1, act upon a lever with the same direction, they are in *æquilibrio*, if their distances from the centre of motion LE and EB be in the ratio of 1 to 3; that is, when $LM \times LE = BH \times BE$. In this case, the powers LM and BH acting with the same direction, the prop E must sustain their sum $LM + BH = 4BH$, by the second principle of § 3. Therefore a power at L as 3, and a power acting at B with the same direction as 1, are sustained by a power acting at E , with a contrary direction, as 4. From which it follows, by substituting in the place of the power LM a prop at L , that a power at B as 1 sustains a power at E as 4, acting with a contrary direction, when BL is to EL as 4 to 1; that is, when the powers are inversely as their distances from the prop, or centre of motion. By substituting the prop at B in the place of the power BH , it appears that a power LM at L , as 3, sustains a power, acting with an opposite direction, at E , as 4, when their distances LB and EB from the prop B , are to each other as 4 to 3, or when $LM \times LB = EK \times EB$. By taking upon LB produced $Be = BE$, (*Fig. 24. n. 2.*) and in place of the power at E , substituting an equal power at e with a

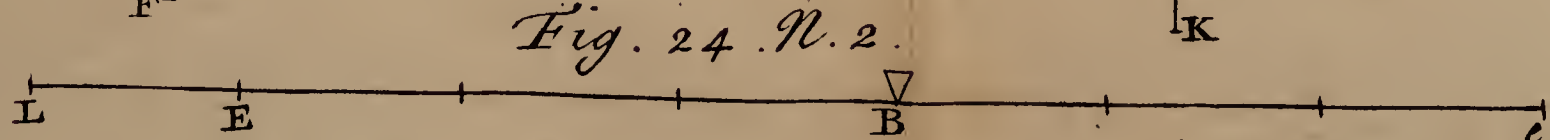
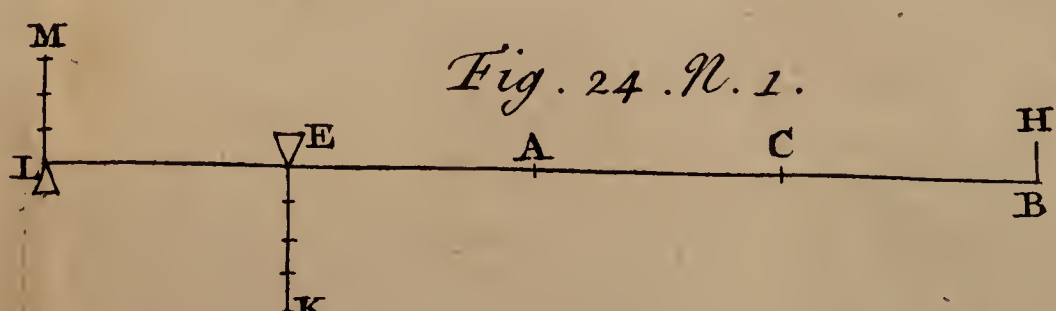
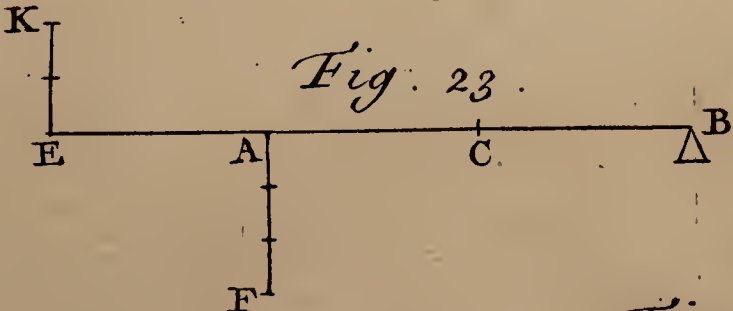
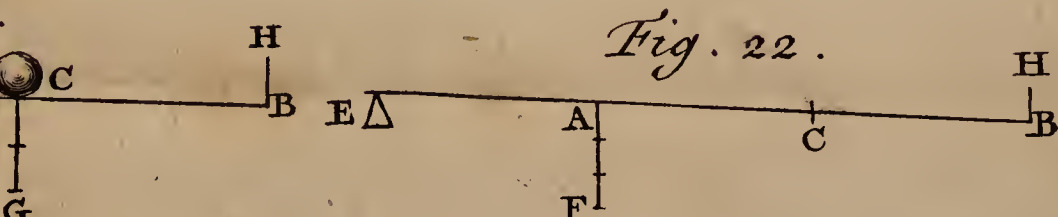
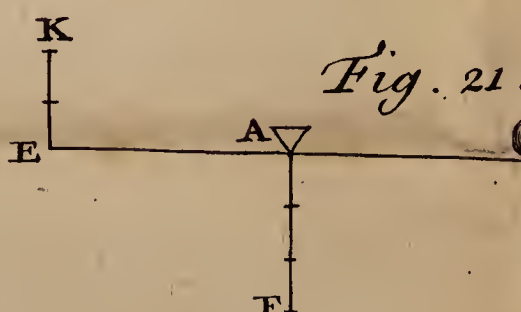
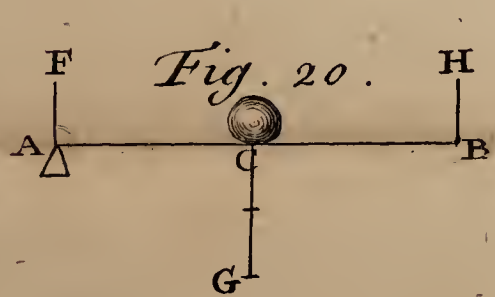
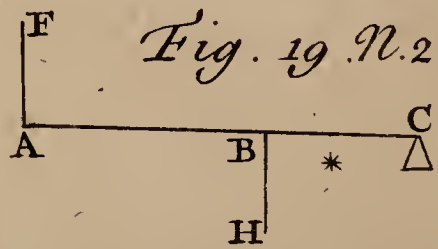
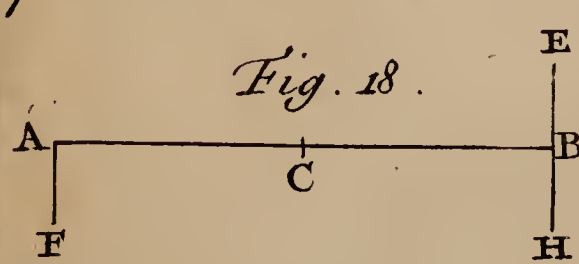
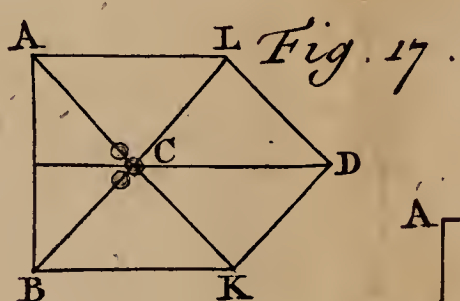
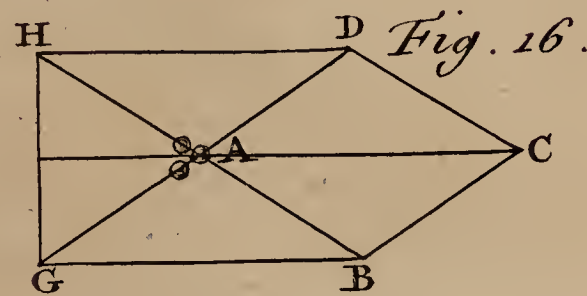
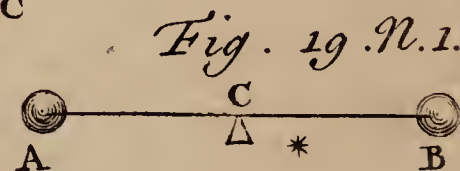
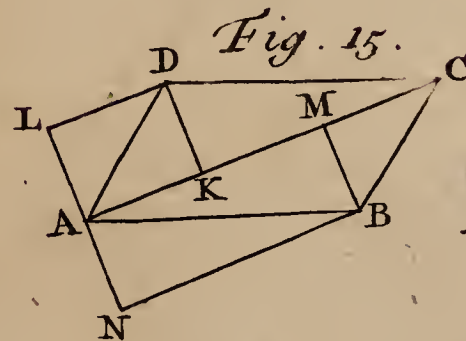
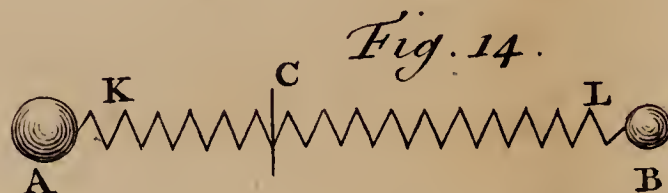
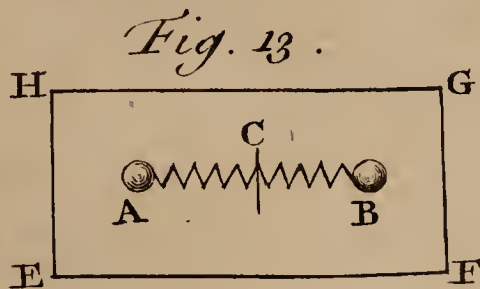
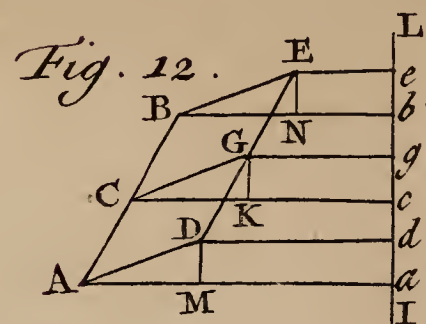
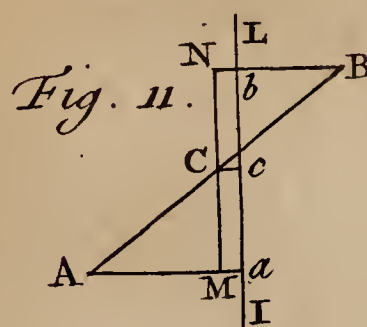
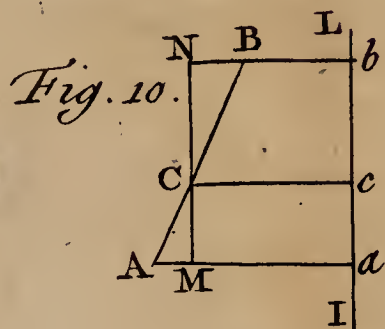
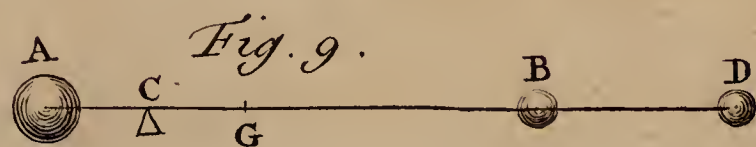
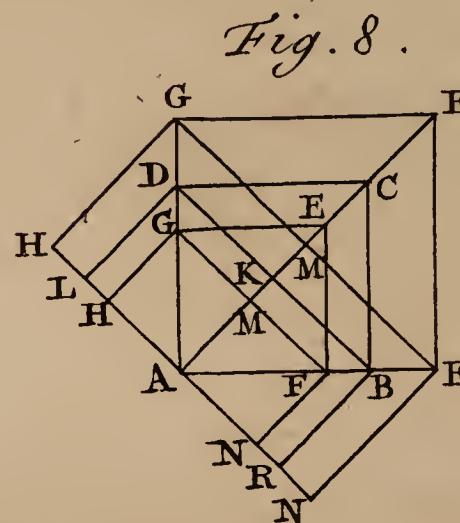
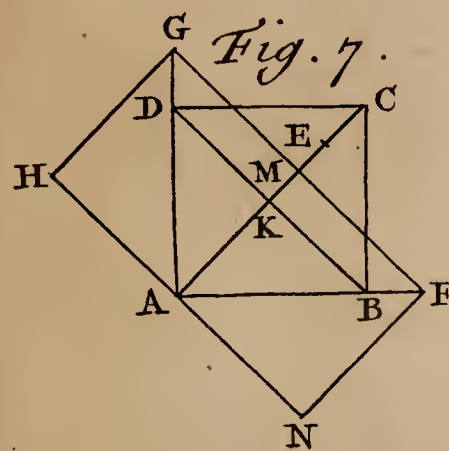
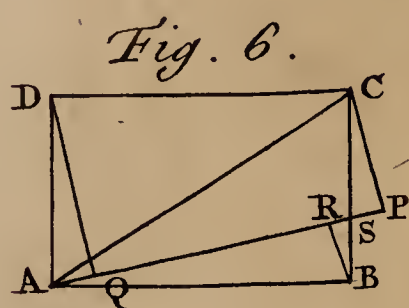
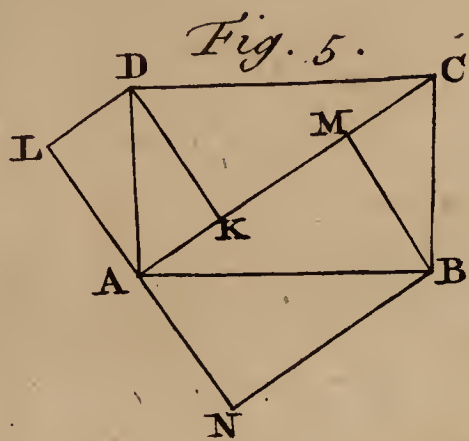
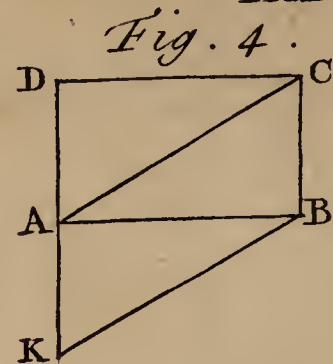
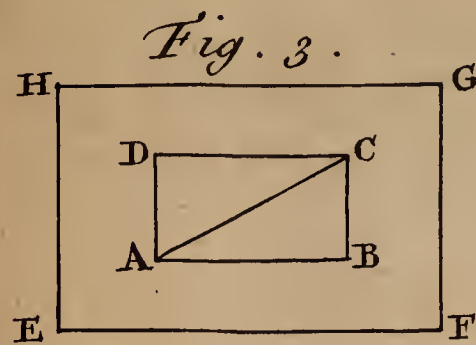
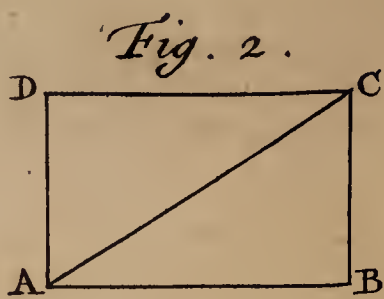
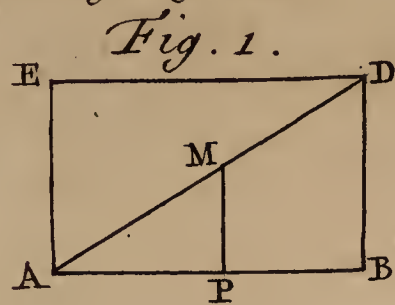
M

contrary

contrary direction, it appears, by the first principle in § 3. that a power at L as 3 sustains a power acting at e , with the same direction, as 4, when the distance LB is to the distance eB , as 4 to 3. In this case, the prop at B sustains the sum of the powers acting at L and e , that is, a power equal to seven times BH . From which it follows, by substituting a prop at L , or e , in place of the powers that act there, that a power at e as 4 sustains a power at B as 7, about the centre of motion L , when their distances from it eL , BL are to each other as 7 to 4: and that a power at L as 3 sustains the power at B as 7, about the centre of motion e , when their distances from it, Le and Be , are to each other as 7 to 3.

6. By proceeding in this manner, it appears, that when the powers are to each other as number to number, and when their distances from the centre of motion are in the inverse ratio of the same numbers, then the powers sustain each other, or are in *æquilibrio*. From which it is easy to shew, in general, that when the powers are to each other in any ratio, tho' incommensurable, and the distances of their application from the centre of motion in the same inverse ratio, then they are in *æquilibrio*; because the ratio of incommensurable quantities may be always limited, to any degree of exactness at pleasure, between a greater and a lesser ratio of number to number. And this I take to be the most direct and natural proof of the law of *æquilibrium* in the lever, the fundamental proposition of mechanics.

7. When the centre of motion c is between the bodies A and B , it is the same point which was called their centre of gravity, chap. 2. § 13. And hence it appears, that when the two bodies are supposed
to



to be joined by an inflexible rod void of gravity, if the centre of gravity be sustained, then the bodies shall be sustained. .

If two powers or weights, B and D , (*Fig. 25.*) act upon a lever at the distances BC and DC from the centre of motion, the forces with which they act upon the lever shall be in the same proportion of $B \times BC$ to $D \times DC$; that is, in the ratio compounded of the ratio of the powers, or weights, and that of their distances from the centre of motion. For the effort of B is sustained by A , if $A \times AC$ be equal to $B \times BC$; and the effort of the power D is sustained by K applied at the distance CA , if $K \times AC = D \times DC$. But the efforts of the powers or weights, B and D , upon the lever, are in the same proportion to each other as the powers A and K , which, applied at the same distance CA from the centre of motion, sustain them, or as $A \times AC$ to $K \times AC$, and therefore as $B \times BC$ to $D \times DC$. From this it appears, that when any number of powers act upon a lever, if the sum of the products that arise by multiplying each power by its respective distance from the centre of motion, on one side of it, be equal to the sum of the products that arise by multiplying each power on the other side of the centre of motion by its respective distance from it, then these powers sustain each other, and the lever is in *æquilibrio*. But by what was shewn in § 13. chap. 2. the centre of motion coincides, in this case, with what was there called the centre of gravity. Therefore if any number of powers or weights act upon a lever, and, their centre of gravity being determined by the construction in that article, if the prop or *fulcrum* be applied at this centre, the lever shall be in *æquilibrio*. In the same manner, if any number of powers or weights be applied upon a plane that rests upon a given right

line IL , (*Fig. 26.*) and the centre of gravity of all the powers or weights fall upon that line, the plane shall be in *æquilibrio*: for, by that article, the sums of the products that arise by multiplying each power by its respective distance from the axis of motion, being equal on the different sides of this axis, their efforts to move the plane must be equal and contrary, and destroy each other's effect. Therefore as the state of any system of bodies, as to motion or rest, depends on the motion or rest of the point called the centre of gravity, by what was shewn above in the last chapter; so it is another notable property of this point, that if the bodies be joined together, and to it, by inflexible lines void of gravity, and this point be sustained, the whole system shall be sustained and remain in *æquilibrio*.

8. When any powers B and D (*Fig. 25, 26.*) act upon a lever, endeavouring to turn it about the centre of motion c , or when they act upon a plane, endeavouring to turn it about the axis of motion IL , their effect is the same as if a power or weight equal to their sum was substituted in place of them at their common centre of gravity N . For, by § 14. chap. 2. $B \times BC + D \times DC = B + D \times NC$; or if Bb , Dd , Nn be perpendicular to IL in the points b , d , n , then, by the same article, $B \times Bb + D \times Dd = B + D \times Nn$. If G , the centre of gravity of all the powers, or weights, that act upon the lever, fall on one side of c the centre of motion; or the centre of gravity of all the powers that act upon the plane, is on one side of the axis IL ; then the preponderancy will be on that side, and will be the same as if, in place of all those powers, one power equal to their sum was substituted at their common centre of gravity. For it was shewn that $B \times BC + D \times DC - A \times AC = A + B + D \times GC$,

G C, when the power A acts on one side and the powers B and D on the other. Therefore, as when the centre of gravity of the powers rests upon the centre of motion, the whole is in *æquilibrio*, and the prop c sustains a force equal to their sum; so when the centre of gravity is not sustained by the prop, but falls on one side of it, the preponderancy is on that side, and is the same as if all the powers or weights were collected together at that centre. The analogy between these statical theorems, and those in the theory of motion relating to this centre, described in the last chapter, deserves our attention; and farther illustrate the simplicity of this doctrine and the harmony of all its parts.

9. Sir *Isaac Newton* demonstrates the fundamental proposition concerning the lever, from the resolution of motion. Let c (*Fig. 27.*) be the centre of motion in the lever KL; let A and B be any two powers, applied to it at K and L, acting in the directions KA and LB. From the centre of motion c, let cM and cN be perpendicular to those directions in M and N; suppose cM to be less than cN, and from the centre c, at the distance cN, describe the circle NH D, meeting KA in D. Let the power A be represented by DA, and let it be resolved into the power DG acting in the direction cD, and the power DF perpendicular to cD, by completing the parallelogram AFDG. The power DG, acting in the direction cD from the centre of the circle, or wheel, DHN towards its circumference, has no effect in turning it round the centre, from D towards H, and tends only to carry it off from that centre. It is the part DF only that endeavours to move the wheel from D towards H and N, and is totally employed in this effort. The power B may be conceived to be applied at N as well as at L, and to be wholly employed

in endeavouring to turn the wheel the contrary way, from N towards H and D. If therefore the power B be equal to that part of A which is represented by DF, these efforts, being equal and opposite, must destroy each other's effect; that is, when the power B is to the power A, as DF to DA, or, (because of the similarity of the triangles AFD, DMC) as CM to CD, or as CM, to CN, then the powers must be in *æquilibrio*; and those powers always sustain each other that are in the inverse proportion of the distances of their directions from the centre of motion; or, when the product of the one power multiplied by the distance of its direction from the centre, is equal to the product of the power on the other side multiplied by the like distance from it.

10. The demonstration commonly ascribed to *Archimedes* is founded upon this principle, that when any cylindric or prismatic body is applied upon a lever, it has the same effect as if its whole weight was united and applied at the middle point of its axis. Let AB (*Fig. 28.*) be a cylinder of an uniform texture, c its middle point; and it is manifest, that if the point c be supported, the equal halves of the cylinder, CA and CB, will ballance each other about the point c, and the body will remain in *æquilibrio*. Let the cylinder AB be distinguished into any unequal parts, AD and DB; bisect AD in E, and DB in F; then a power applied at E, equal to the weight of the part AD, with a contrary direction, will sustain it; and a power applied at F, equal to the weight of the part DB, with a contrary direction, will sustain that part; so that these two powers acting at E and F, respectively equal to the weights of AD and DB, have precisely the same effect as a prop at c, sustaining the whole cylinder AB, and may be considered as in *æquilibrio* with a power acting at c,
equal

equal to the whole weight of the cylinder. But the distance $CE = CA - AE = \frac{1}{2} AB - \frac{1}{2} AD = \frac{1}{2} DB$; and, in like manner, the distance $CF = CB - BF = \frac{1}{2} AB - \frac{1}{2} DE = \frac{1}{2} AD$; consequently, CE is to CF , as DB to AD ; that is, as the power applied at F to the power applied at E , these being in *æquilibrio* with the weight of the whole cylinder applied at c . From which it appears, that powers applied at E and F , which are to each other in the proportion of CF to CE , sustain one another about the centre c .

11. Suppose the lever AB (*Fig. 29.*) with the weights A and B , to turn round the centre c ; the bodies A and B will describe similar arcs Aa and Bb ; and Aa will be to Bb , as CA to CB , or as B to A ; consequently $A \times Aa = B \times Bb$; that is, the *momenta*, or quantities of motion, of A and B will be equal; and considering one of them as the power and the other as the weight, the power will be to the weight, as the velocity of the weight to the velocity of the power. Therefore in this, as in all mechanical engines, when a small power raises a great weight, the velocity of the power is much greater than the velocity of the weight; and what is gained in force is therefore said to be lost in time. In like manner, when a number of powers are supposed to act upon the lever, and it is turned round about their common centre of gravity c , the sums of the *momenta* on the different sides of c are equal.

12. The lever, or *veſtis*, is commonly distinguished into *three* kinds. In the *first*, the centre of motion is between the power and weight. In the *second*, the weight is on the same side of the centre of motion with the power, but applied between them. In the *third*, the power is applied between the weight and centre of motion. In this last, the

power must exceed the weight in proportion as its distance from the centre of motion is less than the distance of the centre from the weight. But as the first two serve for producing a slow motion by a swift one; so the last serves for producing a swift motion of the weight by a slow motion of the power. It is by this kind of levers that the muscular motions of animals are performed; the muscles being inserted much nearer to the centre of motion than the point where the centre of gravity of the weight to be raised is applied; so that the power of the muscle is many times greater than the weight which it is able to sustain. Tho' this may appear at first sight a disadvantage to animals, because it makes their strength less; it is, however, the effect of excellent contrivance: for if the power was, in this case, applied at a greater distance than the weight, the figure of animals would not only be awkward and ugly, but altogether unfit for motion; as *Borelli* has shewn in his treatise *de motu animalium*.

13. When the two arms of a lever are not in a right line, but contain any invariable angle at *c*, (*Fig. 30.*) the law of the *æquilibrium* is the same as in the former case; that is, if the power *p* be applied at *B* to the arm *c B*, and the weight *w* act, by means of a pulley *M*, in the direction *A M* perpendicular to the arm *c A*, the power and weight will sustain each other if *p* be to *w*, as *c A* to *c B*, or $p \times c B = w \times c A$. If several powers act upon the arm *c A*, find their centre of gravity *A*, on the arm *c A*, by § 13, chap 2. suppose all the powers to be united there; and if the power *p* be to their sum, as *c A* to *c B*, it will sustain them. The sum of the powers being supposed given, it is manifest that the farther their centre of gravity *A* is removed from the centre of motion *c*, the greater resistance they will oppose
against

against the power P , and it will require the greater force in the power to overcome them. From this *Galileo* justly concludes, that the bones of animals are the stronger for their being hollow, their weight being given; or, if the arm CBF represent their length, the circle CHD a section perpendicular to the length, P any power applied along their length, tending to break them; then the strength or force of all their longitudinal fibres, by which the adhesion of the parts is preserved, may be conceived to be united in A the centre of the circle CHD , which is the common centre of gravity of those forces, whether the section be a circle or *annulus*. But it is plain that when the area of the section, or the number of such fibres, is given, the distance CA is greater when the section is an *annulus*, than when it is a circle without any cavity; consequently the power with which the parts adhere, and which resists against P which endeavours to separate them, is greater in the same proportion. For the same reason, the stalks of corn, the feathers of fowls, and hollow spears, are less liable to accidents that tend to break them, than if they were of the same weight and length, but solid without any cavity. In this instance, therefore, art only imitates the wisdom of nature.

14. The same excellent author observes, that in similar bodies, engines, or animals, the greater are more liable to accidents than the lesser, and have a less relative strength; that is, the greater have not a strength in proportion to their magnitude. A greater column, for example, is in much more danger of being broke by a fall than a similar small one; a man is in greater danger from accidents of this kind than a child; an insect can bear a weight many times greater than itself, whereas a large animal, as a horse, can hardly bear a burthen equal to his own weight.

To

To account for this, it will be sufficient to shew, that, in similar bodies of the same texture, the force which tends to break them, or to make them liable to hurtful accidents, increases in the greater bodies in a higher proportion than the force which tends to preserve them entire, or secure against such accidents. Suppose the similar beams $A B D E$, $F G H K$, (*Fig. 31.*) of a cylindric or prismatic figure, to be fixed in the immoveable wall $I L$; and let us at present abstract from any other force that may tend to break them, besides their own weight. Bisect $A B$ in c , and $F G$ in m ; and their weights may be conceived to be accumulated at the points c and m , which are directly under their centres of gravity. For the greater facility of the computation, suppose $A B = 2 F G$, and consequently the weight of the beam $A B D E$ will be eight times greater than the weight of the similar beam $F G H K$; and the weight of the former being conceived to be accumulated in c , and that of the latter in m , and $A c$ being double the distance $F m$, it follows, that the force which tends to break the former at A , being eight times greater than that which tends to break the latter at F , and at the same time acting at a double distance, on both these accounts its effort must be sixteen times greater than that of the latter. Now, to compare the forces which tend to preserve those beams entire, and fixed in the wall, let $A R E$ be a section of the greater beam, and $F S K$ a section of the latter, perpendicular to their lengths at the points A and F ; bisect $A E$ in p , and $F K$ in q ; then the number of longitudinal fibres, whose adhesion tends to preserve the beams entire, or rather the quantity of this adhesion, in the greater beam, will be to the quantity of adhesion in the lesser beam, as the area of the section $A R E$ to the area of the section $F S K$, that is, in the present case (because of the similarity of the figures) as the square of

of AE to the square of FK , or as 4 to 1. But the adhesion of the parts that are in contact with each other in the section ARE may be conceived to be accumulated at p their centre of gravity; and the adhesion of the parts in contact with each other in the section FSK is to be conceived as accumulated in q , for the same reason. The adhesion, therefore, which tends to preserve the greater beam entire is quadruple of that which tends to preserve the lesser beam entire, and at the same time is to be conceived as acting at a double distance from the centre of motion, because $Ap = 2 Fq$; so that the effort which tends to preserve the greater beam from breaking, is eight times greater than that which tends to preserve the lesser beam entire. We have found, therefore, that the effort which tends to break the greater beam at A , is sixteen times greater than that which tends to break the lesser beam at F ; but that the effort, which, on the other hand, endeavours to preserve the adhesion of the greater beam entire, is only eight times greater than that which tends to preserve the adhesion in the lesser beam. In general, it will easily appear, in the same manner, that the efforts tending to destroy the adhesion of the beams, arising from their own gravity only, increase in the quadruplicate ratio of their lengths; but that the opposite efforts, tending to preserve their adhesion, increase only in the triplicate ratio of the same lengths. From which it follows, that the greater beams must be in greater danger of breaking than the lesser similar ones; and that, tho' a lesser beam may be firm and secure, yet a greater similar one may be made so long, as necessarily to break by its own weight. Hence *Galileo* justly concludes, that what appears very firm, and succeeds well, in models, may be very weak and infirm, or even fall to pieces

by

by its weight, when it comes to be executed in large dimensions according to the model.

15. From the same principles he argues, that there are necessarily limits in the works of nature and art, which they cannot surpass in magnitude. Were trees of a very enormous size, their branches would fall by their own weight. Large animals have not strength in proportion to their size; and if there were any land-animals much larger than those we know, they could hardly move, and would be perpetually subjected to most dangerous accidents. As to the animals of the sea, indeed, the case is different, as the gravity of the water sustains those animals in great measure, and in fact these are known to be sometimes vastly larger than the greatest land-animals. Nor does it avail against this doctrine to tell us, that bones have been found which were supposed to have belonged to giants of an immense size, such as the skeletons mentioned by *Strabo* and *Pliny*; the former of which was 60 cubits high, and the latter 46; for the naturalists have concluded, on just grounds, that in some cases those bones had belonged to elephants; and that the larger ones were bones of whales, which had been brought to the places where they were found, by the revolutions of nature that have happened in past times. Tho' it must be owned, that there appears no reason why there may not have been men that have exceeded, by some feet in height, the tallest we have seen. The reader will find a curious and useful dissertation on this subject, by the celebrated Sir *Hans Sloane*, in the *Philosophical Transactions*, or in the *Memoires de l'Academie Royale des Sciences*, 1727. If, in the other planets, the same law of cohesion and other attractions takes place as in the earth, it may be of use that the gravity near their surfaces should not be
vastly

vastly different from what it is near the surface of the earth; it was perhaps with some view to this, that Sir *Isaac Newton* insinuates, that it was not without design and contrivance that the gravities at the surfaces of the planets should differ so much less from each other, than, at first sight, might be expected from the attractions of bodies of so unequal magnitude.

16. It follows, from § 14th, that, in order to make bodies, engines, or animals, of equal relative strength, the greater ones must have grosser proportions. Thus in order that the greater cylinder *A B D E* may be as firm and secure against accidents as the lesser cylinder *F G H K*, the section *A R E* and its diameter *A E* must be increased till the effort arising from the adhesion of the parts bear as great a proportion to the effort that tends to overcome this adhesion, in the greater, as in the lesser cylinder. And this sentiment being suggested to us by perpetual experience, we naturally join the idea of greater strength and force with the grosser proportions, and the idea of agility with the more delicate ones. In architecture, where the appearance of solidity is no less regarded than real firmness and strength, this is particularly considered, in order to satisfy a judicious eye and taste; the various orders of the columns serving to suggest different degrees of strength. But by the same principle, if we should suppose animals vastly large, from the gross proportions, a heaviness and unwieldiness would necessarily arise, which would make them useless themselves, and disagreeable to the eye. In this, as indeed, in all other cases, whatever generally pleases tastes not vitiated by education, or by fabulous and marvellous relations, may be traced till it appear to

6 have

have a just foundation in nature; tho' the force of habits is so strong, and their effects upon our sentiments so quick and sudden, that it is often no easy matter to trace, by reflection, the grounds of what pleases us.

17. We have insisted at so great length on the lever, that we may be brief in treating of the other mechanical powers. The common *ballance* is a lever that has equal arms $A\ G$ and $G\ B$, (*Fig. 32.*) with the centre of motion c commonly placed directly over G . If the centre of motion was in G , equal weights suspended from A and B , would sustain each other, in any position of the lever $A\ B$; but when the centre of motion is above G , they sustain each other when the lever $A\ B$ is level only; and when the weight at A is but a little greater than the weight at B , the ends A and B descend and ascend by turns, till their common centre of gravity g settles in the vertical line $c\ G$; where they sustain each other, because their centre of gravity is sustained by c . The ballance is false when the arms $A\ G$ and $G\ B$ are unequal: and the exactness of this instrument chiefly depends upon making the friction, at the centre of motion c , as small as possible.

18. The *axis* and *wheel* has a near analogy to the lever; the power is applied at the circumference of the wheel, and the weight is raised by a rope that is gathered up (while the machine turns round) on the axis. The power may be conceived as applied at the extremity of the arm of a lever equal to the radius of the wheel, and the weight as applied at the extremity of a lever equal to the radius of the axis; only those arms do not meet at one centre of motion as in the lever, but in place of this centre, we have an axis of motion, *viz.* the axis of the whole

whole engine. But as this can produce no difference, it follows that the power and weight are in *æquilibrio* when they are to each other inversely as the distances of their directions from the axis of the engine; or when the power is to the weight, as the radius of the roller to the radius of the wheel, the power being supposed to act in a perpendicular to this radius; but if the power act obliquely to the radius, substitute a perpendicular from the axis on the direction of the power, in the place of the radius. Thus if $A B D E$ (*Fig. 33.*) represent the cylindric roller, $H P N$ the wheel, $L M$ the axis or right line upon which the whole engine turns, Q the point of the surface of the roller where the weight w is applied, P the point where the power is applied, $K Q$ the radius of the roller, $C P$ the radius of the wheel; then if the power P act with a direction perpendicular to $C P$, the power and weight will sustain each other when P is to w , as $K Q$ to $C P$ or $C H$: but if the power act in any other direction $P R$, let $C R$ be perpendicular from C , the centre of the wheel, on that direction; then P and w will sustain each other when P is to w , as $K Q$ to $C R$; because, in this case, a power P has the same effect as if it was applied at the point R of its direction, acting in a right line perpendicular to $C R$.

19. The *simple pulley* serves only to change the direction of the power, or motion, without any mechanical advantage, or any disadvantage but what arises from the friction. Let M (*Fig. 34.*) represent a simple pulley, $P N W$ the rope that goes over the pulley from the power P to the weight w : and it is manifest, that if P and w be equal, they will sustain each other as if suspended at equal distances, $M A$ and $M B$, from the centre of the lever $A B$. But, if besides the fixed pulley M , there be (*Fig. 35.*) another

another moveable pulley L , to which the weight w is fixed, and the rope that goes from the power P , over the fixed pulley M , and under the moveable pulley L , be fixed above at E , then it is manifest that the power P sustains only one half of w , because the rope KN sustains only one half of it, the other half being sustained by the rope KE .

There is an obvious analogy between this case of pulleys, and that wherein a power sustains a double weight at half its distance from the centre of motion, on the same side. For if AB be the diameter of the pulley L , at whose extremities the parallel ropes, AE and BN , touch it, the power P may be conceived to be applied at B , the weight w at L , and the centre of motion to be at A . If we suppose the power P and weight w to move, as P is equal to one half of w , so the velocity of w is one half of the velocity of P , or P multiplied by its velocity gives a product equal to w multiplied by its velocity; for, that the weight w may be elevated one inch, each of the parts of the rope EK and KN must be shortened by one inch; and the power P that draws the whole rope from E by K and N , must descend two inches. A similar reasoning may be applied to all the combinations of pulleys.

20. When a weight w (*Fig. 36.*) descends along an *inclined plane* AC , a part of its gravity is sustained by the reaction of the plane, and the remaining part produces its motion along the plane. Let AB be the height of the plane, BC the base, and the gravity of the weight w being represented by the vertical line wM , let this power be resolved into the power wN perpendicular to the plane, and wQ parallel to it. The former wN is destroyed by the reaction of the plane, and the latter wQ is that
which

which produces the motion of the body along the plane. Because the triangles $w Q M$ and $A B C$ are similar, $w Q$ is to $w M$, as $A B$ to $A C$; and the force with which a body descends along the plane is to its gravity, as the height of the plane to its length; consequently a force acting upon the body w , with the direction $Q w$ parallel to the plane $A C$, will sustain it, if it be to the whole weight of the body, as $A B$ to $A C$.

21. Let $A B C$ (*Fig. 37.*) represent a *wedge* driven into the cleft $E D F$, of which $D E$ and $D F$ are the sides; and if we suppose those sides $D E$ and $D F$ to re-act upon the wedge with directions perpendicular to $D E$ and $D F$, let the horizontal line $E F$ meet $D F$ in F ; then when the force impelling the wedge, supposed perpendicular to the horizon, is in *æquilibrio* with the resistances of the sides of the cleft $D E$ and $D F$, these three powers are in the same proportion as the three right lines $E F$, $D E$ and $D F$. For it follows from the composition of motion, that when three powers are in *æquilibrio* with each other, they are in the same proportion as the three sides of a triangle parallel to their respective directions, and, consequently, as the three sides of a triangle perpendicular to their directions; such a triangle being evidently similar to the former. But $E F$ is perpendicular to the direction in which the weight of the wedge, or the power that impells it, is supposed to act; and $D E$, $D F$ are perpendicular to the directions in which their resistances are supposed to act, consequently the power that impels the wedge and those resistances are in the same proportion as $E F$, $D E$ and $D F$. If other suppositions are made concerning the resistances of the sides of the cleft $D E$ and $D F$, the proportions of the powers may be determined, from the same principles.

22. When a point moves along the side of a cylinder, with an uniform motion, upon its curve surface, while this side is itself carried with an uniform motion about the axis of the cylinder, the line traced, by this compounded motion, upon the curve surface of the cylinder, is called a *spiral*. When this line is raised upon the external surface of the cylinder, it is called the *external screw*; but if it is carried on in the internal surface, it is called the *internal screw*. While one of these is converted about the other, one of them ought to be fixed; and they form a machine of great force for squeezing or moving bodies. If a power P (*Fig. 38.*) turn either of the screws with a direction parallel to the base, it will sustain the weight w which is to be raised, if it be to w in the same proportion as the distance between the two nearest spirals is to the circumference of the circle described by the power P ; because while the power makes a complete revolution, the screw advances by the distance of the two nearest spirals, and the velocity of the power is to the velocity of the weight, as the circumference described by P to that distance. The same will appear by considering the screw as an inclined plane involved about a cylinder. In this engine the friction is very great.

23. From these simple machines, compounded ones are formed by various combinations, and serve for different purposes; in which the same general laws take place, particularly that which was described in § 3, That the power and weight sustain each other when they are in the inverse proportion of the velocities which they would have in the directions wherein they act, if they were put in motion. By these the famous problem is resolved, of moving any given weight by any given power, provided the resist-

resistance arising from the friction can be overcome. It being of great importance to diminish this friction, several contrivances have been invented for that purpose. In wheel-carriages the friction is transferred from the circumference of the wheel (where it would act if the wheel did not turn round) to the circumference of the axis ; and, consequently, is diminished in the proportion of the radius of the axis to the radius of the wheel. In these, therefore, the friction is always diminished by diminishing the diameter of the axis, or by increasing the diameter of the wheel. The friction is likewise diminished by making the axis of an engine to rest upon the circumferences of wheels that turn round with it, instead of resting in fixed grooves that rub upon it ; for by this contrivance, the friction is transferred from the circumferences of those wheels to their pivots ; and the friction may be still diminished farther by making the axles of those wheels rest upon other friction-wheels that turn round with them. It is hardly possible to give general and exact rules concerning friction, since it depends upon the structure of bodies, the form of their prominent parts and cavities, and upon their rigidity, elasticity, their coherence, and other circumstances. Some authors have made the friction upon a horizontal plane equal to one third of the weight ; but others have found that it was only one fourth of it, and sometimes only $\frac{1}{6}$ or $\frac{1}{7}$ of it. Of late, authors have told us that the friction depends not on the surface of the body, but its weight only ; but neither is this found to be accurately true. In lesser velocities, the friction is nearly in the same ratio as the velocities ; but in greater velocities, the friction increases in a higher proportion, whether the bodies are dry or oiled.

24. The second general problem in mechanics, mentioned above, is to determine the proportion which the power and weight ought to bear to each other, that, when the power prevails, and the engine is in motion, the greatest effect possible may be produced by it in a given time. It is manifest that this is an enquiry of the greatest importance, though few have treated of it. When the power is only a little greater than that which is sufficient to sustain the weight, the motion is too slow; and tho' a greater weight is raised in this case, it is not sufficient to compensate the loss of time. When the weight is much less than that which the power is able to sustain, it is raised in less time; and this may happen not to be sufficient to compensate the loss arising from the smallness of the load. It ought, therefore, to be determined when the product of the weight multiplied by its velocity is the *greatest* possible; for this measures the effect of the engine in a given time, which is always the greater in proportion as the weight that is raised is greater, and as the velocity with which it is raised is greater. We shall, therefore, subjoin some instances of this kind that may be demonstrated from the common elementary geometry; wishing that farther improvements may be made in this most useful part of mechanics.

25. When the power prevails, and the engine begins to move, the motion of the weight is at first gradually accelerated. The action of the power being supposed invariable, its influence in accelerating the motion of the weight decreases while the velocity of the weight increases. Thus the action of a stream of water, or air, upon a wheel is to be estimated only from the excess of the velocity of the fluid above the velocity already acquired by the part of
the

the engine which it strikes, or from their relative velocity. On the other hand, the weight of the load that is to be elevated, and the friction, tend to retard the motion of the engine; and when these forces, *viz.* those that tend to accelerate it, and those that tend to retard it, become equal, the engine then proceeds with the uniform motion it has acquired.

Let $A B$ (*Fig. 39.*) represent the velocity of the stream, $A C$ the velocity of the part of the engine which it strikes, when the motion of the machine becomes uniform; and $C B$ will represent their relative velocity, upon which the effect of the engine depends. It is known that the action of a fluid, upon a given plane, is as the square of this relative velocity; consequently, the weight raised by the engine, when its motion becomes uniform, being equal to this action, it is likewise as the square of $C B$. Let this be multiplied by $A C$, the velocity of the part of the engine impelled by the fluid; and the effect of the engine in a given time will be proportional to $A C \times C B^2 =$ (supposing $C B$ to be bisected in D) $A C \times 2 C D \times 2 D B = 4 A C \times C D \times D B$; consequently the effect of the engine is greatest when the product of $A C$, $C D$, and $D B$ is greatest. But it is easy to see, that this product is greatest when the parts $A C$, $C D$ and $D B$ are equal; for, if you describe a semicircle upon $A D$, and the perpendicular $C E$ meet the circle in E , then $A C \times C D = C E^2$, and is greatest when C is the centre of the circle; so that in order that $A C \times C D \times D B$ may be the greatest possible, $A D$ must be bisected in C ; and $C B$ having been bisected in B , it follows that $A C$, $C D$, $D B$ must be equal; or that $A C$, the velocity of the part of the engine impelled by the stream, ought to be but one third of $A B$ the velocity of the stream. In this case, when (abstract-

ing from friction) the engine acts with the utmost advantage, the weight raised by it is to the weight that would just sustain the force of the stream, as the square of $c\ B$, the relative velocity of the engine and stream, to the square of $A\ B$, which would be the relative velocity if the engine was quiescent; that is, as 2×2 to 3×3 or 4 to 9 . Therefore, that the engine may have the greatest effect possible, it ought to be loaded with no more than $\frac{4}{9}$ of the weight which is just able to sustain the efforts of the stream. Of this the reader will find more in my *Treatise of Fluxions*, § 908.

26. For another example, suppose that a given weight p , (*Fig. 40.*) descending by its gravity in the vertical line, raises a greater weight w likewise given, by the rope $p\ M\ w$ (that passes over the fixed pulley M) along the inclined plane $B\ D$, the height of which $B\ A$ is given; and let it be required to find the position of this plane, along which w will be raised in the least time, from the horizontal line $A\ D$ to B . Let $B\ C$ be the plane upon which if w was placed, it would be exactly sustained by p , and, by § 20, of this chapter, p shall be to w , as $A\ B$ to $B\ C$; but w is to the force with which it tends to descend along the plane $B\ D$, as $B\ D$ to $A\ B$, by the same article; consequently the weight p is to that force, as $B\ D$ to $B\ C$. Therefore the excess of p above that force (which excess is the power that accelerates the motions of p and w) is to p , as $B\ D - B\ C$ to $B\ D$; or, taking $B\ H$ upon $B\ C$ equal to $B\ D$, as $C\ H$ to $B\ D$. But it is known that the spaces described by motions uniformly accelerated are in the compound ratio of the forces which produce them and the squares of the times; or, that the square of the time is directly as the space described in that time, and inversely as the force; consequently, the square of the time, in which $B\ D$ is described

described by w , will be directly as $B D$ and inversely as $\frac{C H}{B D}$, and will be least when $\frac{B D^2}{C H}$ is a *minimum*; that is, when $\frac{B C^2}{C H} + C H + 2 B C$, or (because $2 B C$ is invariable) when $\frac{B C^2}{C H} + C H$ is a *minimum*. Now as, when the sum of two quantities is given, their product is a *maximum* when they are equal to each other; so it is manifest, that, when their product is given, their sum must be a *minimum* when they are equal. Thus it is evident, that, as in the last section the rectangle or product of the equal parts $A C$ and $C D$ was $C E^2$; so the rectangle or product of any two unequal parts, into which $A D$ may be divided, is less than $C E^2$, and $A D$ is the least sum of any two quantities the product of which is equal to $C E^2$. But the product of $\frac{B C^2}{C H}$ and $C H$ is $B C^2$, and consequently given; therefore the sum of $\frac{B C^2}{C H}$ and $C H$ is least when these parts are equal, that is, when $C H$ is equal to $B C$, or $B D$ equal to $2 B C$. It appears, therefore, that when the power P and weight w are given, and w is to be raised by an inclined plane, from the level of a given point A to the given point B , in the least time possible, we are first to find the plane $B C$ upon which w would be sustained by P , and to take the plane $B D$ double in length of the plane $B C$; or, we are to make use of the plane $B D$ upon which a weight that is double of w could be sustained by the power P .

27. Let a fluid, moving with the velocity and direction $A C$ (*Fig. 41.*) strike the plane $C E$, and suppose that this plane moves parallel to itself in the direction $C B$; perpendicular to $C A$, or that it cannot move in any other direction; then let it be required to find the most advantageous position of the plane

c E, that it may receive the greatest impulse from the action of the fluid. Let A P be perpendicular to c E in P, draw A K parallel to c B, and let P K be perpendicular upon it in K; and A K will measure the force with which any particle of the fluid impells the plane E c, in the direction c B. For the force of any such particle being represented by A c, let this force be resolved into A Q parallel to E c, and A P perpendicular to it; and it is manifest that the latter A P only has any effect upon the plane c E. Let this force A P be resolved into the force A L perpendicular to c B, and the force A K parallel to it; then it is manifest, that the former, A L, has no effect in promoting the motion of the plane in the direction c B; so that the latter A K, only, measures the effort by which the particle promotes the motion of the plane c E, in the direction c B. Let E M and E N be perpendicular to c A and c B, in M and N; and the number of particles, moving with directions parallel to A c, incident upon the plane c E, will be as E M. Therefore the effort of the fluid upon c E, being as the force of each particle and the number of particles together, it will be as $A K \times E M$; or, because A K is to A P ($= E M$), as E N to c E, as $\frac{E M^2 \times E M^2 \times E N}{c E}$; so that c E being given, the problem is reduced to this, to find when $E M^2 \times E N$ is the greatest possible, or a *maximum*. But because the sum of $E M^2$ and of $E N^2$ ($= c M^2$) is given, being always equal to $c E^2$, it follows that $E N^2 \times E M^4$ is greatest when $E N^2 = \frac{1}{3} c E^2$; in the same manner as it was demonstrated in § 25. that when the sum of A c and c B was given, $A c \times c B^2$ was greatest when $A c = \frac{1}{3} A B$. But when $E N^2 \times E M^4$ is greatest, its square-root $E N \times E M^2$ is of necessity at the same time greatest. Therefore the action of the fluid upon the plane c E in the direction c B is greatest

est when $EN^2 = \frac{1}{3} CE^2$, and consequently $EM^2 = \frac{2}{3} CE^2$; that is, when EM the sine of the angle ACE in which the stream strikes the plane is to the radius, as $\sqrt{2}$ to $\sqrt{3}$; in which case it easily appears, from the trigonometrical tables, that this angle is of $54^\circ. 44'$.

28. Several useful problems in mechanics may be resolved by what was shewn in the last article. If we represent the velocity of the wind by AC , a section of the sail of a wind-mill perpendicular to its length by CE , as it follows from the nature of the engine, that its axis ought to be turned directly towards the wind, and the sail can only move in a direction perpendicular to the axis, it appears, that, when the motion begins, the wind will have the greatest effect to produce this motion, when the angle ACE in which the wind strikes the sail is of $54^\circ. 44'$. In the same manner, if CB represent the direction of the motion of a ship, or the position of her keel, abstracting from her lee-way, and AC be the direction of the wind, perpendicular to her way, then the most advantageous position of the sail CE , to promote her motion in the direction CB , is when the angle ACE , in which the wind strikes the sail, is of $54^\circ. 44'$. The best position of the rudder, where it may have the greatest effect in turning round the ship, is determined in like manner. And how this same angle enters into the determination of the figure of the rhombus's that form the bases of the cells in which the bees deposit their honey, in the most frugal manner, I have shewn in a letter to the learned and worthy *Martin Folkes*, Esq; president of the Royal Society. *Philosophical Transactions*, N^o. 471.

29. But it is to be carefully observed, that when the sine of the angle $A C E$ is to the radius as $\sqrt{2}$ to $\sqrt{3}$, or (which is the same thing) when its tangent is to the radius, as the diagonal of a square to its side, this is the most advantageous angle only at the beginning of the motion of the engine; so that the sails of a common wind-mill ought to be so situated, that the wind may indeed strike them in a greater angle than that of $54^{\circ}.44'$. For we have demonstrated elsewhere, that when any part of the engine has acquired the velocity c , the effort of the wind upon that part will be greatest, when the tangent of the angle in which the wind strikes it is to the radius, not as the $\sqrt{2}$ to 1, but as $\sqrt{2 + \frac{9cc}{4aa} + \frac{3c}{2a}}$ to 1, the velocity of the wind being represented by a . If for example $c = \frac{1}{3}a$ then the tangent of the angle $A C E$ ought to be double of the radius, that is, the angle $A C E$ ought to be of $63^{\circ}.26'$. If $c = a$ then $A C E$ ought to be of $74^{\circ}.19'$. This observation is of the more importance, because in this engine, the velocity of the parts of the sail remote from the axis, bear a considerable proportion to the velocity of the wind, and perhaps sometimes are equal to it; and because a learned author, Mr. *Daniel Bernoulli*, has drawn an opposite conclusion from his computations in his *hydrodynamics*, by mistaking a *minimum* for a *maximum*; where he infers, that the angle in which the wind strikes the sail ought to decrease as the distance from the axis of motion increases, that if $c = a$ the wind ought to strike the sail in an angle of 45° , and that, if the sail be in one plane, it ought to be inclined to the wind, at a *medium*, in an angle of about 50° . How he fell into these mistakes, we have explained elsewhere*. In like manner, tho'

* *Treatise of Fluxions*, § 914.

the angle $A C E$ of $54^{\circ} 44'$. be the most advantageous at the beginning of the motion, when a ship sails with a side-wind, yet it ought to be enlarged afterwards as the motion increases. In general, let $A a$, parallel to $c B$, be to $A c$, as the velocity which the engine has already acquired in the direction $c B$, to that of the stream; upon $A c$ produced take $A D$ to $A c$ as 4 to 3, draw $D G$ parallel to $c B$, and let a circle described from the centre c with the radius $c a$ meet $D G$ in g ; and the plane $c E$ shall be in the most advantageous situation for promoting the motion of the engine, when it bisects the angle $a c g$. It is generally supposed, that a direct wind always promotes the motion of a ship, the sail being perpendicular to the wind, more than any side-wind; and this has been affirmed in several late ingenious treatises; but to prevent mistakes, we are obliged to observe, that the contrary has been demonstrated in our treatise of *fluxions*, § 919; where other instances of this second general problem in mechanics are given, to which we refer.

30. The mechanical powers, according to their different structure, serve for different purposes; and it is the business of the skilful mechanic to chuse them, or combine them, in the manner that may be best adapted to produce the effect required, by the power which he is possessed of, and at the least expence. The lever can be employed to raise weights a little way only, unless the engine itself be moved, as, for example, to raise stones out of their beds in quarries. But the axis and wheel may serve for raising weights from the greatest depths. The pulleys being easily portable aboard ships, are therefore much employed in them. The wedge is excellent for separating the parts of bodies; and the screw, for compressing or squeezing them together; and its
great

great friction is even sometimes of use, to preserve the effect already produced by it. The strength of the engine, and of its parts, must be proportioned to the effects, which are to be produced by it. As we found, that, when the centre of motion is placed between the power and weight, it must sustain the sum of their efforts; a small ballance ought not to be employed for weighing great weights; for these disorder its structure, and render it unfit for serving that purpose with accuracy. Neither are great engines proper for producing small effects: the detail of which things must be left to the skilful and experienced mechanic.

31. But, besides the raising of weights and overcoming resistances, in mechanics we have often other objects in view. To make a regular movement, that may serve to measure the time as exactly as possible is one of the most valuable problems in this science; and has been most successfully effected, hitherto, by adapting pendulums to clocks; tho' many ingenious contrivances have been invented to correct the irregularities of those movements that go by springs. Some have endeavoured to find a perpetual movement, but without success: and there is ground to think, from the principles of mechanics, that such a movement is impossible. In many cases, when bodies act upon each other, there is a gain of absolute motion; but this gain is always equal in opposite directions, and the quantity of direct motion is never increased. To make a perpetual movement, it appears necessary that a certain system of bodies, of a determined number and quantity, should move in a certain space for ever, and in a certain way and manner; and for this, there must be a series of actions returning in a circle, to make the movement continual; so that any action by which the absolute

fronting pag. 188.
Fig. 25.



Fig. 28.

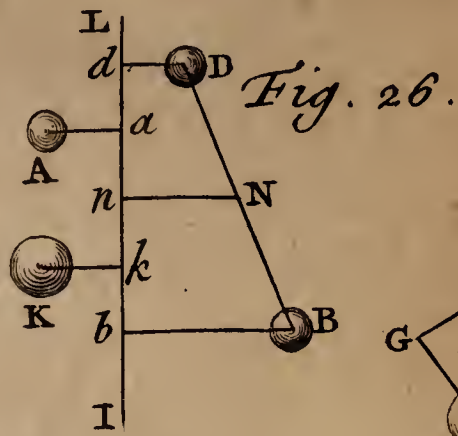
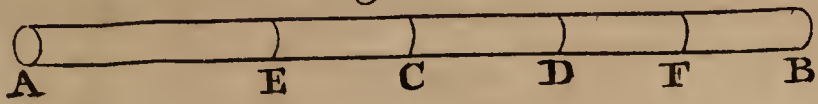


Fig. 26.

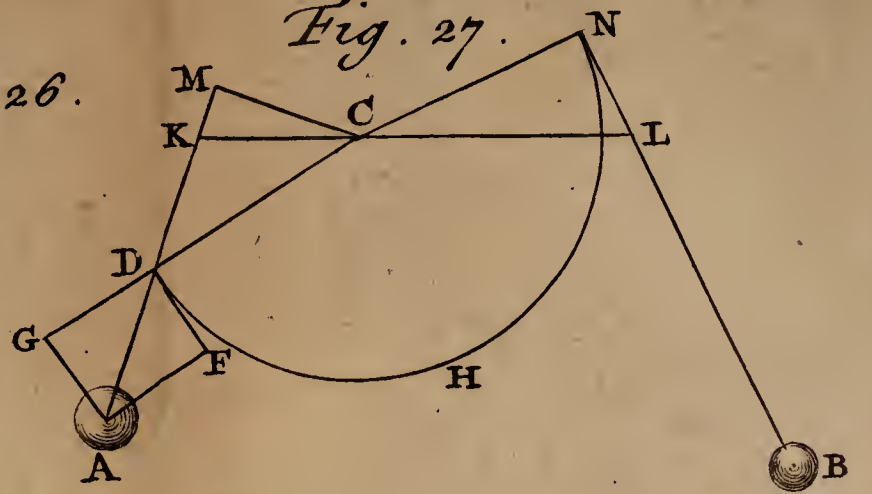


Fig. 27.

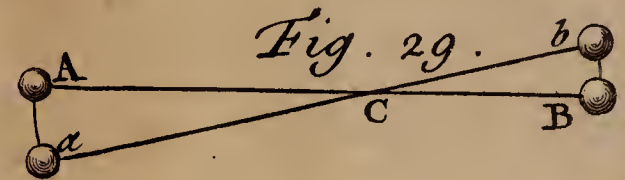


Fig. 29.

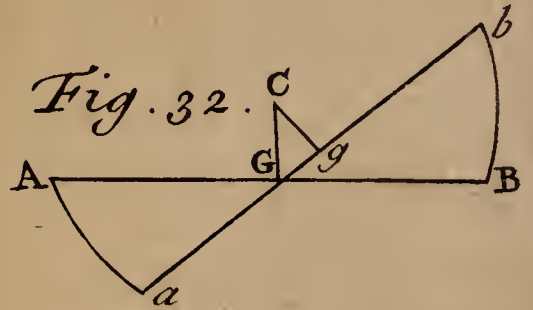


Fig. 32.

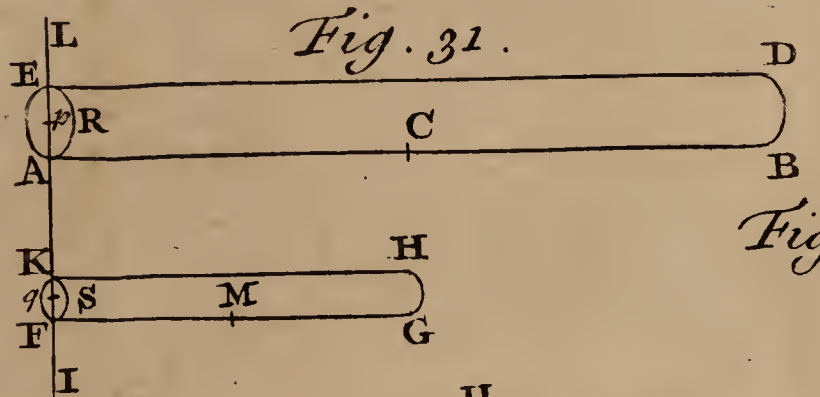


Fig. 31.

Fig. 30.

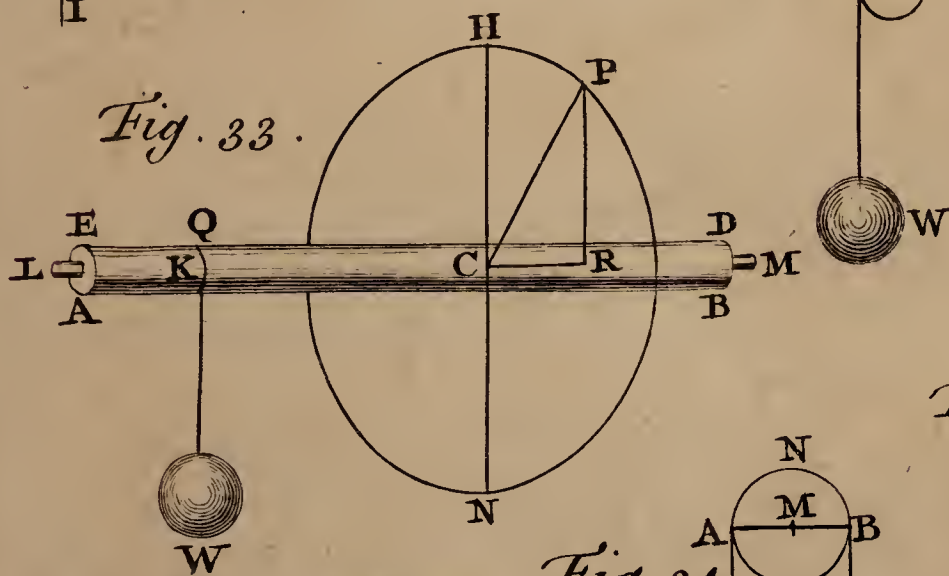
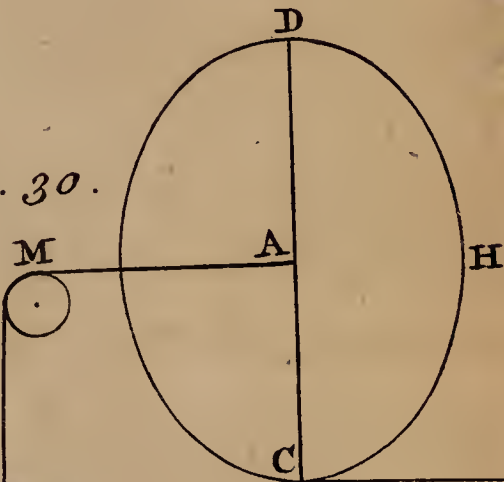


Fig. 33.

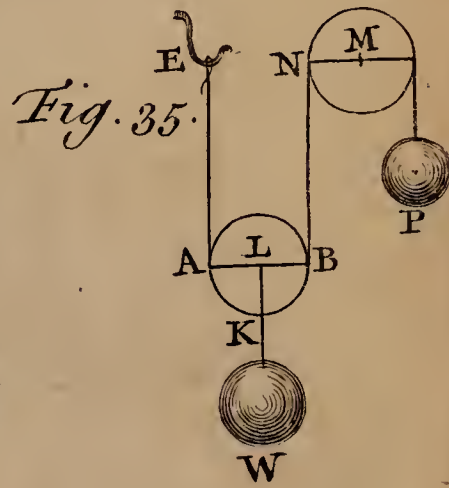


Fig. 35.

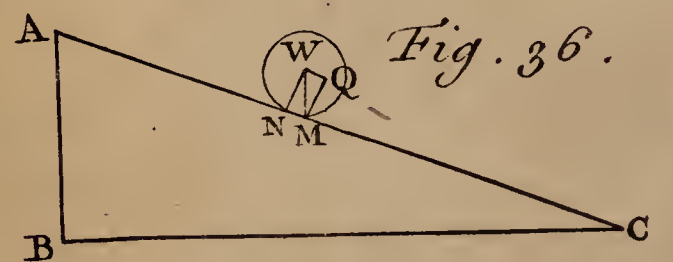


Fig. 36.

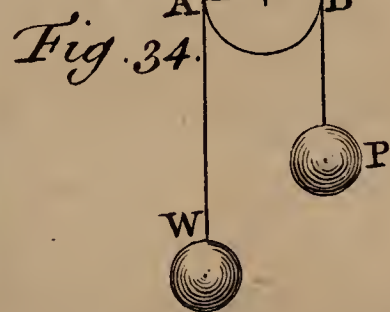


Fig. 34.

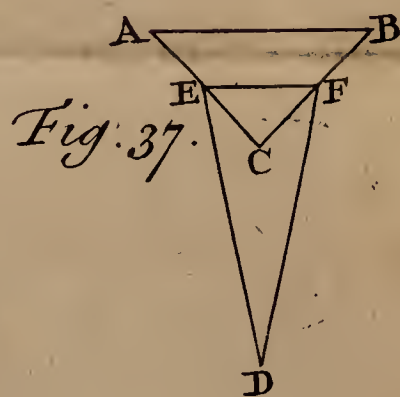


Fig. 37.

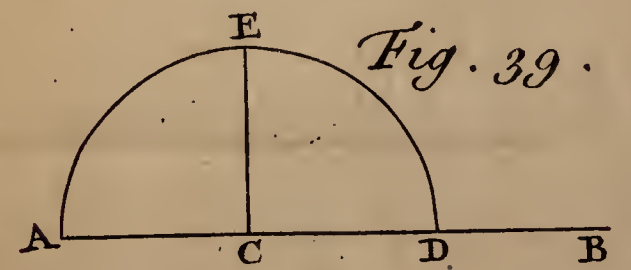


Fig. 39.

Fig. 38.

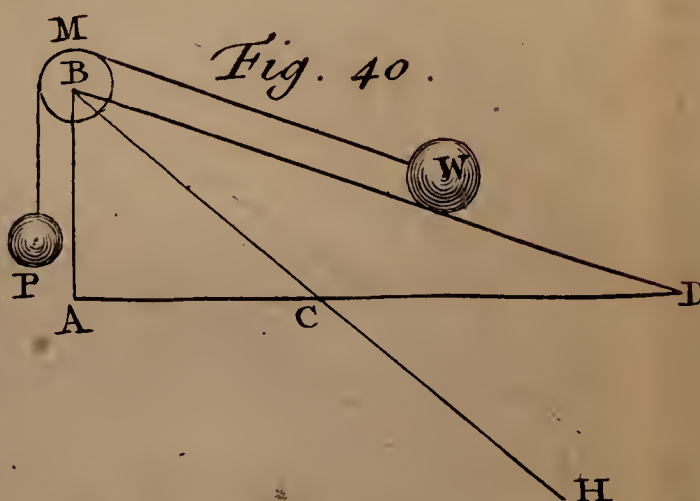
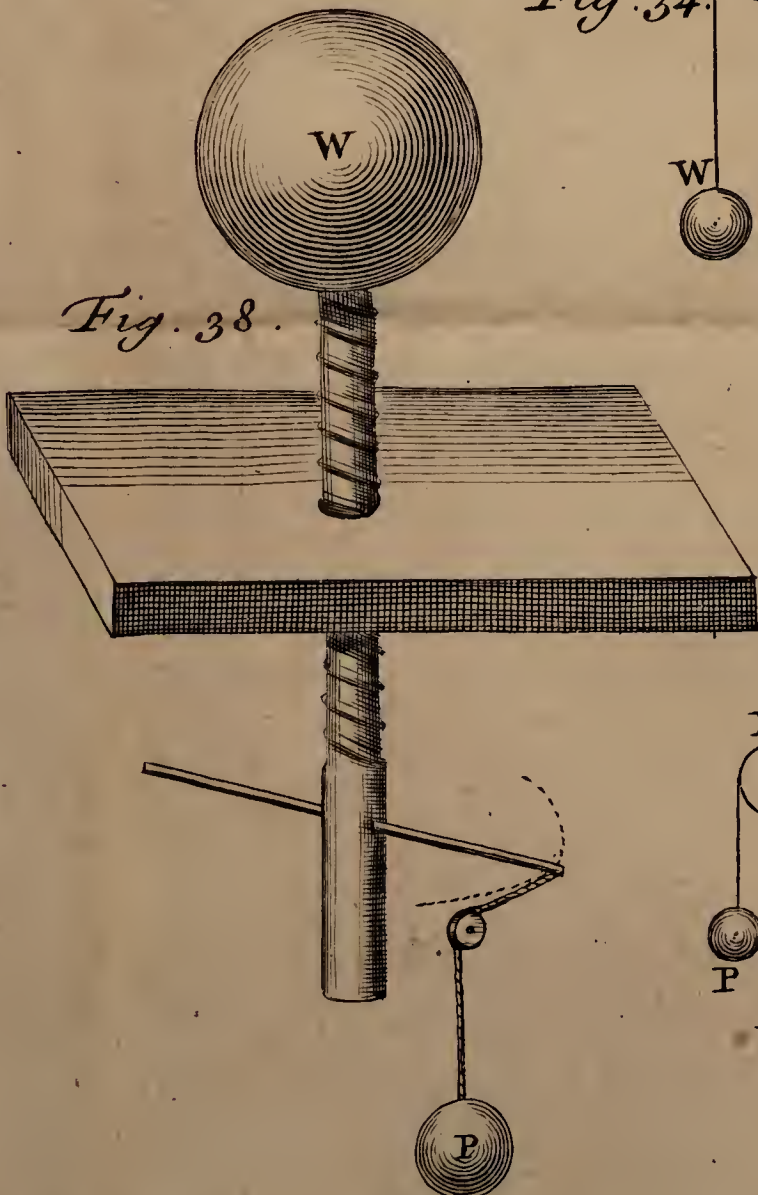


Fig. 40.

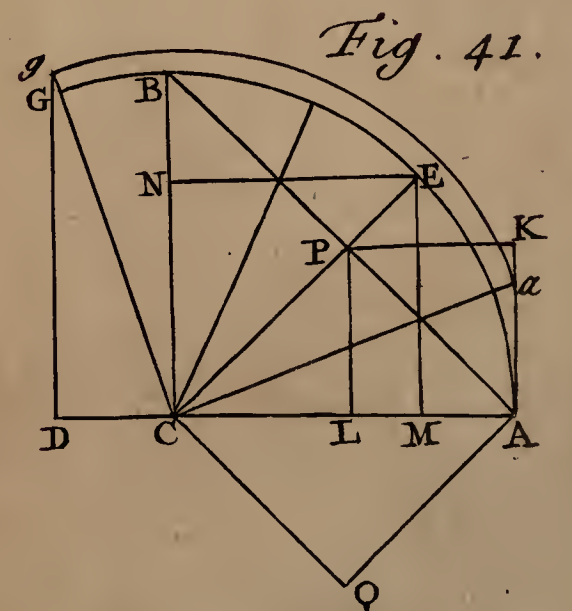


Fig. 41.

absolute quantity of force is increased, of which there are several sorts, must have its corresponding counter-action, by which that gain of force is destroyed, and the quantity of force restored to its first state. Thus, by these actions, there will never be any gain of direct force, to overcome the friction and the resistance of the *medium*. But every motion will be abated, by these resistances, of its just quantity; and the motions of all must, at length, languish and cease.

32. To illustrate this, it is allowed, that, by the resolution of force, there is a gain or increase of the absolute quantity of force; as the two forces A B and A D (*Fig. 2.*) taken together, exceed the force A C which is resolved into them. But you cannot proceed resolving motion *in infinitum*, by any machine whatsoever; but those you have resolved must be again compounded, in order to make a continual movement, and the gain obtained by the resolution will be lost again by the composition. In like manner, if you suppose A and B (*Fig. 42.*) to be perfectly elastic, and that the lesser body A strikes B quiescent, there will be an increase of the absolute quantity of force, because A will be reflected; but if you suppose them both to turn round any centre c, after the stroke, so as to meet again in *a* and *b*, this increase of force will be lost, and their motion will be reduced to its first quantity. Such a gain, therefore, of force as must be afterwards lost in the actions of the bodies can never produce a perpetual movement. There are various ways, besides these, by which absolute force may be gained; but since there is always an equal gain in opposite directions, and no increase obtained in the same direction; in the circle of actions necessary to make a perpetual movement, this gain must be presently lost, and will

will not serve for the necessary expence of force employed in overcoming friction and the resistance of the *medium*.

33. We are to observe, therefore, that tho' it could be shewn that in an infinite number of bodies, or in an infinite machine, there could be a gain of force for ever, and a motion continued to infinity, it does not therefore follow that a perpetual movement can be made. That which was proposed by Mr. *Leibnitz*, in *August* 1690, in the *Leipsick* acts, as a consequence of the common estimation of the forces of bodies in motion, is of this kind; and, for this and other reasons, ought to be rejected. It is, however, necessary to add, that tho' on many accounts, it appear preferable to measure the forces as well as motions of bodies by their velocities, and not by the squares of their velocities; yet, in order to produce a greater velocity in a body, the power or cause that is to generate it must be greater in a higher proportion than that velocity; because the action of the power upon the body depends upon their relative motion only; so that the whole action of the power is not employed in producing motion in the body, but a considerable part of it in sustaining the power, so as to enable it to act upon the body, and keep up with it. Thus the whole action of the wind is not employed in accelerating the motion of the ship, but only the excess of its velocity above that of the sail on which it acts, both being reduced to the same direction. When motion is produced in a body by springs, it is the last spring only which acts upon the body by contact, and the rest serve only to sustain it in its action; and hence a greater number of springs is requisite to produce a greater velocity in a given body, than in proportion to that velocity. A double power, like that of gravity, will produce a double motion

motion in the same time; and a double motion in an elastic body may produce a double motion in another of the same kind. But two equal successive impulses, acting on the same body, will not produce a motion in it double of what would be generated by the first impulse; because the second impulse has necessarily a less effect upon the body, which is already in motion, than the first impulse which acted upon it while at rest. In like manner, if there is a third and fourth impulse, the third will have less effect than the second, and the fourth less than the third. From this it appears what answer we are to make to a specious argument that is adduced to shew the possibility of a perpetual motion. Let the height $A B$ (*Fig. 43.*) be divided into four equal parts $A C$, $C D$, $D E$, $E B$: suppose the body A to acquire by the descent $A C$, a velocity as 1 , and this motion by any contrivance to be transmitted to an equal body B ; then let the body A , by an equal descent $C D$, acquire another motion as 1 , to be transmitted likewise to the same body B , which in this manner is supposed to acquire a motion as 2 , that is sufficient to carry it upwards from B to A ; and because there yet remain the motions which A acquires by the descents $D E$ and $E B$, that may be sufficient to keep an engine in motion, while B and A ascend and descend by turns, it is hence concluded that a sufficient gain of force may be obtained in this manner, so as to produce a perpetual movement. But it appears from what has been shewn, that a motion as 2 cannot be produced in B , by the two successive impulses transmitted from A , each of which is as 1 .

Some authors have proposed projects for producing a perpetual movement, with a design to refute them; but, by mistaking the proper answer, have rather confirmed the unskilful in their groundless

less expectations. An instance of this we have in Dr. Wilkins's *Mathematical Magick*, book 2. chap. 13. A load-stone at A (*Fig. 44.*) is supposed to have a sufficient force to bring up a heavy body along the plane F A, from F to B; whence the body is supposed to descend by its gravity, along the curve B E F, till it return to its first place F; and thus to rise, along the plane F A, and descend, along the curve B E F, continually. But supposing B Z E to be the surface upon which, if a body was placed, the attraction of the load-stone and the gravity of the body would ballance each other, this surface shall meet B E F at some point E between A and F, and the body must stop in descending along A E F at the point E.

C H A P IV.

Of the collision of bodies.

1. **T**H O' the laws of motion and principles of mechanics are sufficiently explained and established in the preceding chapters, it will be of use, before we proceed to apply them to subjects of a higher nature, to consider the most simple and obvious motions and phænomena that are derived from them; by which they may be farther tried and examined, and our methods of reasoning from them justified: and these are the motions which are produced by bodies impinging upon one another, which fall frequently under our observation, and can be repeated by us in experiments. It is always from the most simple kind of phænomena that we can trace with the greatest certainty the analysis of the laws of nature; from which we afterwards may proceed to such as are more complicated and abstruse: but it would be contrary to the rules of good method to begin

begin with the latter. It would be very preposterous, for example, in defining or ascertaining the true notion of the *inertia* of body, to begin with chymical experiments concerning fermentation, the solutions of bodies by menstrooms, the phænomena of generation and corruption, or others of that complicated kind. If we should begin with fixing our attention on these, we should be apt to ascribe to body an activity which is really repugnant to its nature. It is from observations and experiments concerning the sensible and gross bodies, that we must acquire our knowledge of the first principles of this science. The doctrine of the collision of bodies was very plain and clear, and deduced in a satisfactory manner from the laws of motion, before some late authors endeavoured to cloud it, by introducing abstruse notions into it, in favour of their new doctrine concerning the estimation of the forces of bodies in motion. But we shall have no regard to these; and shall endeavour to deduce it, in a plain and satisfactory manner, from the principles established and illustrated in the second chapter.

2. Bodies have been commonly distinguished into three sorts. Those are called perfectly *hard* whose parts yield not at all in their collisions, but are absolutely inflexible; and such the last elements of bodies, or atoms, are supposed to be. Those are called *soft* whose parts yield in their collisions, but restore not themselves again towards their first positions. Those are said to be *elastic* which yield in their collisions, but restore themselves so as to recover their first situation; and they are said to be perfectly elastic, when they restore themselves with the same force with which they are compressed. The actions of perfectly hard or inflexible bodies on one another are consummated in a moment: and, as
O there

there is no spring, nor any force, to separate them, they must go on together after their collision as if they formed one body. But when an elastic body is acted on by any force or power, its parts yield at first, and afterwards restore themselves by degrees to their first situations. There is a time required for this, which may be distinguished into two portions; the first is the time during which the parts yield and become more and more compressed; the other is the time during which they restore themselves to their first situations. When two spherical elastic bodies meet, at first they touch one another in a point, but their contact gradually increases, as the parts that touch and press on one another yield, till their greatest compression: and afterwards these parts recover by the same steps, tho' in a contrary order, their first situations. The actions of elastic bodies may be explained by imagining springs KL placed betwixt hard bodies A and B (*Fig. 14.*); for the springs must have the same effect in this case, as the elasticity of the parts of the bodies in the other case. If A move towards B and compress the springs, and, by their mediation, act on B , the springs will become more and more compressed, till the two bodies have equal velocities in the same direction; and then, no force acting on the springs, they will have liberty to begin to expand themselves; which they will do by the same degrees as they were compressed, in a contrary order: and this is the second period of the action of the bodies on one another. In the first period of the action of elastic bodies, or of bodies acting by the intervention of springs, the same effects are produced as if the bodies were perfectly hard. At the end of this period the respective velocity of the bodies is destroyed, and in the instant when it ceases the second begins, the velocities of the bodies in the same direction being now equal. In this second period of
the

the action of the bodies, if the elasticity is perfect, the springs expanding themselves by the same force with which they were compressed, the bodies must be separated with a respective velocity equal to that they had before their collision; and whatever motion was added to, or subducted from, either body, in the first period, as much will be added to, or subducted from it, in the same direction, in the second; so that there will be twice as much force lost, or twice as much gained, by either, as if the bodies had been perfectly hard.

3. The effects produced in the first period of the action of bodies that have an imperfect elasticity are the same as when the bodies are perfectly elastic; but, because their parts recover their first situations with less force than that whereby they were displaced from them, there is less force lost or gained in the second than in the first period. There is, however, a constant proportion observed between what is lost or gained in these two periods, in the same sort of bodies; so that there is a constant proportion between their respective velocities before and after their collision. In glass, for example, this proportion is observed to be that of 16 to 15.

4. In soft bodies, whose parts yield so as not to restore themselves at all to their first situations, the actions must be the same as in the first period of perfectly elastic bodies, and the same as in perfectly hard bodies. By their collision their respective velocity is destroyed, the *inertia*, or resistance of the parts, having the same effect in this case, as their spring in the other. After the collision they go on together as one mass, there being no spring to separate them. Because the parts yield, in their collisions, certain philosophers have imagined that some force must be

lost in producing this effect : but there is no motion communicated to any one part that it can lose without communicating it to others ; a body moving in a fluid loses no force but what it communicates to the parts of the fluid ; and a body acting upon a soft body can lose no force but what must be communicated to the parts of that body, which therefore must be accumulated to the force of the whole. The parts are indeed moved out of their first places, but this can produce no loss of force ; for it is manifest, that if A move and strike B, (*Fig. 45.*) and make it go into the place *b*, and there strike c, so that it remain itself in the place *b*, all the force which A had at first must be still found in A or c, and there can be none lost or consumed in carrying B from its first place B, to its last place *b*, since A lost none but what it gave to B, and B could lose none, but what is communicated to c. There can be no force lost in this case more than if B had struck c in its first place B, nor would there be more force lost in B moved twice or thrice as far before it struck c. In like manner, when a body acts upon a soft body and moves its parts out of their places, the force which the first body loses is employed in moving those parts indeed, by which they acquire whatever is lost by it, and lose none of what they thus acquire, but by communicating to other particles ; nor is it of moment how far they are moved from their places, but what force is communicated to them, which it is not possible to conceive they can lose by merely moving out of their places, but by acting on other particles.

5. This will still be found true, tho' you suppose the particles of the soft body to cohere with some certain degree of force. That case may be explained by supposing particles, B, c and D, (*Fig. 46.*) cohering by a string of a certain degree of strength,
and

and that A impelling c changes the situation of the particles with respect to one another. In this case, A will lose no force which will not be all communicated to c, but some part, by mediation of the string, must be imprinted on B and D, and all that A loses and is not given to c, must be communicated to B and D, if we suppose the string infinitely fine, or abstract from its *inertia*, and reckon all the force in the same direction. It is true the string will be stretched by the force which is at first imprinted on c, but as c can lose none but what B and D receive, there can be no force lost from that cause; and, if the string should break, the only consequence can be, that there will be no more force communicated from c to B and D after that happens. From the equality of action and reaction it follows, that the string acts equally on c and B, and on c and D; so that it adds as much force to B and D as it takes from c; and, as this is always true, it must hold in the instant when the string breaks, as well as before: the cohesion of the particles, therefore, can be the occasion of no loss of force, taking in all that are affected in the collision, and there appears no ground for supposing that any force is consumed, in making the parts of soft bodies yield, but what is accumulated to the whole mass of body, while its parts continue all together.

6. These things being premised, first let the bodies A and B (*Fig. 47.*) be supposed void of elasticity, let c be their centre of gravity, and let A D and B D represent their velocities before the stroke. Then supposing the stroke to be direct, after it they will proceed together as forming one mass, and their centre of gravity being carried along with them, their common velocity will be the same as the velocity of that centre, which (by § 15. chap. 2.) is the same

after the stroke as before it. But while the bodies described $A D$ and $B D$ before the stroke, their centre of gravity moves from c to D , the place where they meet, or the one overtakes the other; therefore the common velocity of A and B after the stroke is measured by $c D$, their velocities before the stroke being represented by $A D$ and $B D$ respectively. The right line $c D$ shews the direction as well as the velocity of their motions after the stroke; for it is always in the direction from c to D . If D fall upon c , then $c D$ vanishes, and their motions are destroyed by the stroke. This proposition serves for determining the cases when the bodies are either perfectly hard, or perfectly soft.

7. But if the bodies are perfectly elastic, take $c E$ equal to $c D$ in an opposite direction; and the velocities of A and B after the stroke, with their directions, will be represented by $E A$ and $E B$ respectively. For the change produced in their motions by the stroke, being, in this case, double of what it was in the former, by § 2; and the difference of $A D$ and $c D$ (the change produced in the velocity of A in the former case) being equal to the difference of $c D$, or $c E$, and $E A$, it follows that the velocity of A after the stroke is measured by $E A$; and the difference of $E B$ and $c D$, or $c E$, *viz.* $c B$, being equal to the difference of $c D$ and $B D$, it follows, that $E B$ is the velocity of B after the stroke. If B have no motion before the stroke, let $A B$ represent the velocity of A , take $c E$ equal and opposite to $c B$, and $E A$, $E B$, will represent the velocities of A and B after the stroke: in which case, the velocity of A before the stroke is to the velocity of B after it, as $A B$ to $E B$, or $2 c B$; that is, as one half $A B$ to $c B$, and therefore (by the property of the centre of gravity) as half the sum of the bodies A and B to A .

From

From this theorem, all the cases relating to the motion of bodies that have a perfect elasticity may be immediately deduced. For example, if the bodies *A* and *B* be equal, then $cA = cB$, and since $cE = cD$, it follows that $EA = BD$, and $EB = AD$; that is, the bodies exchange their velocities by the stroke.

8. But if the elasticity of the bodies is imperfect, take cE (*Fig. 48. n. 1.*) equal and opposite to cD , but ca is less than cA , and cb less than cB , in the same proportion as their elasticity is less than a perfect elasticity; and the right lines Ea and Eb will represent their velocities after the stroke, by § 3; because if we distinguish the time in which the bodies act upon each other into two periods, as in that article, the effect produced in the second period will be less than the effect produced in the first period, in that ratio. In this case their respective velocity after the stroke is represented by ab , and is to their respective velocity before the stroke, as ab to AB . In glass, *Sir Isaac Newton* found this ratio to be that of 15 to 16, as was observed above; consequently in determining the effect of their collisions, we are to take $ca = \frac{15}{16} cA$, and $cb = \frac{15}{16} cB$.

9. If motion be communicated, in this manner, from a body *A* to a series of bodies in a geometrical progression, then the velocity successively communicated to those bodies will be likewise in a geometrical progression; and if *A* and *B* be the two first bodies, the common ratio of the velocities will be that of half the sum of *A* and *B* to *A*; that is, if the bodies *A*, *B*, be represented by the right lines oa and ob , (*Fig. 48. n. 2.*) and ab be bisected in e , the common ratio of any two subsequent velocities in the progression will be that of oe to oa ; and if n

O 4

repre-

represent the number of bodies without including the first a , the velocity of the last will be to the velocity of the first, as the power of $o a$ whose exponent is n to the same power of $o e$.

10. Any three bodies being represented by $o a$, $o b$, and $o d$, take $o f$ to $o d$, as $o a$ is to $o b$; then supposing the motion to begin from the first $o a$ (which was supposed to strike $o b$ quiescent, and $o b$ afterwards to strike $o d$ quiescent) the velocity communicated in this manner, to the third shall be to the velocity of the first, as $o a$ is to one fourth part of the sum of $o a$, $o b$, $o f$, and $o d$. For the velocity of the first $o a$ is to the velocity of the second $o b$, as the sum of $o a$ and $o b$ to $2 o a$; the velocity of $o b$ is to that of $o d$, as the sum of $o b$ and $o d$ to $2 o b$; consequently the velocity of the first $o a$ is to the velocity of the third $o d$, in the compound ratio of $o a + o b$ to $2 o a$ and of $o b + o d$ to $2 o b$, that is, (since $o a$, $o b$, $o f$, $o d$, are proportional, so that $o a$ is to $o b$, as $o a + o f$ to $o b + o d$, and $o a + o b$ to $o b$, as the sum of $o a$, $o b$, $o f$, and $o d$ to $o b + o d$) as the sum of $o a$, $o b$, $o f$, and $o d$ is to $4 o a$. Hence the velocity of $o a$ being given, the velocity communicated to $o d$ is inversely as the sum of $o a$, $o b$, $o f$, and $o d$, and is greatest when this sum is least; that is, if $o a$ and $o d$ be given, when $o b$ and $o f$ coincide with each other and with $o k$ the mean proportional between $o a$ and $o d$. Therefore the velocity communicated to $o d$ is greatest when $o b$, the body interposed between $o a$ and $o d$, is a mean proportional between them. This is one of Mr. *Huygens's* theorems; from which it follows, that the more such geometrical mean proportionals are interposed between $o a$ and $o d$, the greater is the velocity communicated to $o d$. There is, however, a limit which the velocity communicated to $o d$ never amounts to, (the bodies $o a$, $o d$,
and

and the velocity of $o a$ before the stroke, being given) to which it approaches continually, while the number of such bodies interposed between $o a$ and $o d$ is always increased. And this limit is a velocity which is to the velocity of the first $o a$ before the stroke, in the subduplicate ratio of $o a$ to $o d$; as we have demonstrated in our *fluxions*, § 514.

II. The same principles will serve for determining the effects of the collisions, when a body strikes any number of bodies at once, in any directions whatever. Let the bodies first be perfectly hard and void of elasticity, and the body c (*Fig. 49.*) moving in the direction $c d$ with a velocity represented by $c d$, strike at once the bodies $A, B, E, \&c.$ that are supposed at rest before the stroke, in the directions $c f, c h, c k, \&c.$ in the same plane with $c d$, and let $d a, d b, d e$, be perpendicular to $c f, c h, c k$, in a, b , and e , respectively. Determine the point p where the common centre of gravity of the bodies $c, A, B, E, \&c.$ would be found, if their centres were placed at the points $c, a, b, e, \&c.$ respectively, (by § 13. chap. 2.); join $d p$, and $c l$ parallel to $d p$ shall be the direction of the body c after the stroke. Let $p r$, perpendicular to $d p$, meet $c d$ in r , and $d l$, perpendicular to $c d$, meet $c l$ in l ; then if $c l$ be divided in g , so that $c g$ be to $c l$ in the ratio compounded of that of $c d$ to $c r$, and that of the body c to the sum of all the bodies, the velocity of c after the stroke will be represented by $c g$; that is, the velocity of c after the stroke will be to its velocity before it, as $c g$ is to $c d$. Let $g f, g b$, and $g k$ be respectively perpendicular to $c f, c h$, and $c k$, in f, b , and k ; and the velocities of A, B , and E , after the stroke, will be represented by $c f, c b$, and $c k$.

But if we now suppose the bodies to be perfectly elastic, or the relative velocities, before and after the stroke, to be always equal when measured on the same right line; produce DG till Dg be equal to $2\,DG$, join Cg , and the body C will describe Cg after the stroke, in the same time that it would have described a right line equal to CD , before the stroke. And, in like manner, the motions are determined when the elasticity is imperfect, if the relative velocity after the stroke is always in a given ratio to the relative velocity before it in the same right line. Mr. *Bernoulli* has resolved only a very limited case of this problem, in his *Essay* on motion, *Paris* 1726; for he supposes the bodies to be perfectly elastic, and that, for each body on one side of the line of direction CD , there is always an equal body on the other side, that is impell'd in a right line forming an equal angle with CD ; so that the body C moves with the same direction after the stroke as before. The solution of this particular case, (which he represents as a matter of uncommon difficulty, and magnifies as the fruit of the new doctrine concerning the forces of bodies) he derives from this principle, "that the sum of the bodies multiplied by the squares of their velocities is the same before and after the stroke;" which principle, however, had never been demonstrated by him; for it cannot be considered as an immediate consequence of the equality of action and re-action, as he too hastily concluded, by what was shewn above. But the solution of these and other problems of this kind is derived, in a natural, easy, and general manner, from the laws concerning the sum of the motions of a system of bodies estimated in a given direction, and concerning the motion of their centre of gravity, which is never affected by their collisions.

12. The same things being supposed as in § 7. because $cE = cD$, (*Fig. 47.*) it follows that $AD^2 - AE^2 = 4CE \times CA$; and that $EB^2 - BD^2 = 4CE \times CB$. But $A \times 4CE \times CA = B \times 4CE \times CB$, by the property of the centre of gravity c : therefore $A \times AD^2 - A \times AE^2 = B \times EB^2 - B \times BD^2$, or $A \times AD^2 + B \times BD^2 = A \times AE^2 + B \times EB^2$; that is when the bodies are perfectly elastic, the sum arising when each is multiplied by the square of its velocity, is the same after the stroke as before it. The same things being now supposed as in the last article, let DQ, gq, fm, bn, kr , be perpendiculars to cG , in Q, q, m, n , and r ; then the rectangles contained by cm and cG , cn and cG , cr and cG , will be respectively equal to the squares of cf , cb , and ck . If the bodies c, A, B, E , be supposed to have no elasticity, their velocities after the stroke will be represented by cG, cf, cb , and ck , the velocity of c before the stroke being represented by cD , because, in this case, no relative velocity is generated by the stroke in their respective directions; and the sum of $A \times cm, B \times cn, E \times cr$ is equal to $c \times GQ$, because the sum of the motions which would be communicated to A, B , and E , in the direction cG , is equal to the motion which c would lose in the same direction, by § 4. chap. 2. Therefore the sum of $A \times cf^2, B \times cb^2, E \times ck^2$ is equal to $c \times cG \times GQ$; and to these if we add $c \times cG^2$, the sum of all the bodies multiplied by the squares of their velocities in this case would be $c \times cG \times cQ$. But when the bodies are supposed to be perfectly elastic, the velocities of A, B , and E , are to be represented by $2cf, 2cb$, and $2ck$, respectively; the sum of $A \times 4cf^2, B \times 4cb^2$ and $E \times 4ck^2$, is equal to $c \times 4cG \times GQ$ or (*Elem. 8. 2.*) $c \times cQ^2 - c \times cq^2$; to which if we add $c \times cg^2$ (or $c \times cq^2 + c \times DQ^2$) the whole sum of the products, when each body is multiplied

multiplied by the square of its velocity, is equal to $c \times c D^2$; and consequently the same after the stroke as it was before the stroke. When therefore the bodies are void of elasticity, this sum is less after the stroke than before it, in the ratio of $c G \times c Q$ to $c D^2$, or of $c G$ to $c L$, L being the point where $L D$ perpendicular to $c D$ meets $c G$. And when the bodies A, B, E , move, before the stroke, in directions different from those in which c acts upon them, the proposition will appear by resolving their motions into such as are in those directions (which alone are affected by the stroke,) and such as are in perpendiculars to those directions, from *Elem.* 47. 1. This proposition likewise holds when bodies of a perfect elasticity strike any immoveable obstacle as well as when they strike one another, or when they are constrained, by any power or resistance, to move in directions different from those in which they impell one another. But it is manifest, that it is not to be held a general principle or law of motion, since it can take place in the collisions of one sort of bodies only. The solutions of some problems which have been deduced from it may be obtained, in a general and direct manner, from plain principles that are universally allowed, by determining first the motions of hard bodies, which are supposed to have no elasticity, and thence deducing the solutions of other cases, when the relative velocities before and after the stroke are equal, or in any given ratio.

13. From what was shewn in the last article, we are led to the principle, which, by Mr. *Huygens*, was called the *conservatio vis ascendantis*. It is well known, and was proved in § 11. chap. 1. that the heights to which bodies will rise against the direct resistance of an uniform gravity are as the squares of the velocities with which they set out. In the last article

article we found that the sum of the products, when the bodies are multiplied by the squares of their velocities, is the same after as before the stroke; provided the bodies be perfectly elastic. If, therefore, we suppose the motion of the bodies to be turned upwards in vertical lines, the sum of the products when each body is multiplied by the height to which it would arise is the same after as before the stroke. But by the property of the centre of gravity, in § 15, chap. 2. the sum of the products of the bodies multiplied by those heights is equal to the product of the sum of the bodies multiplied by the height to which their centre of gravity would arise. Therefore when the motions of bodies are supposed to be converted upwards in vertical lines, before or after their collisions, their common centre of gravity will always arise to the same height; and that is what is meant by Mr. *Huygens* when he tells us the *vis ascendens* of any system of bodies is not affected by their collisions or mutual actions, provided they be perfectly elastic; for if they are soft bodies, or have an imperfect elasticity (which indeed is the case of all bodies we have access to examine,) then it is obvious that by their collisions their motions are often diminished, and sometimes totally destroyed; so that the centre of gravity will necessarily arise to a less height after their collision than before it, if the motions of the bodies be supposed to be converted upwards in vertical lines.

14. When bodies are moved by their gravity, and at the same time act upon each other, it will still be found, that the sum of the products that arise when each body is multiplied by the square of the velocity acquired by it, is equal to the difference of the sum of the products of those that descend multiplied by the

the

the squares of the velocities that would have been acquired by the same descents, if the bodies had fallen freely without acting upon each other, and of the sum of the products of the bodies that ascend multiplied by the squares of their respective velocities that would be acquired by falling freely along the respective altitudes to which they have arisen; provided that the elasticity of the bodies be perfect; or if it be imperfect, that there be no collision, or sudden communication of motion from one body to another. For if the relative velocities in their respective directions be less immediately after that action than before it; in those cases, the sum of the products of the bodies multiplied by the squares of their velocities will be less than it would have been if the bodies had descended freely from the same respective altitudes; and if the bodies be supposed to ascend with their respective velocities at any time, and their motions be retarded by their gravity only, the common centre of gravity will not ascend to the same level from which it descended; as we have shewn at length in our *Treatise of Fluxions*, from § 521 to 533.

15. The true general principle on this subject, is, that when any number of bodies, moved by their gravity, are connected together in any manner so as to act upon each other while they move, the ascent of their common centre of gravity, in their vibrations or revolutions, will be always found to be either equal to its descent, or less than it, but never to exceed it. And, from this principle, the impossibility of a perpetual motion is justly derived. For it appears, that, in such vibrations and revolutions, the successive ascents of the centre of gravity must continually diminish, in consequence of the attri-
tion

tion of the parts of bodies, and the resistance of the medium ; since the ascent of the centre of gravity being never greater than the descent (tho' often less than it,) there can be no gain of force to overcome those resistances. All motion, therefore, must be abated and gradually languish in our mechanical engines, unless they be supplied by new and repeated influences of the power.

16. It is very well known, that, when allowance is made for the defect of elasticity in bodies, for attrition, and the resistance of the medium, these conclusions are perfectly agreeable to experience ; and therefore serve to confirm the general laws of motion with their corollaries, and our methods of reasoning from them.

C H A P. V.

*Of the motion of projectiles in vacuo ; of the cycloid, and the motion of a pendulum in it *.*

L E M M A I.

Suppose the motion of a body to be uniformly accelerated ; let the time be represented by the right line AM , (*Plate IV. Fig. I.*) and any part of it by AK , draw MN , KL perpendiculars to AM in M and K , and AN intersecting them in N and L : then the velocities acquired in the times AM , AK , reckoned from the beginning of the motion, will be as the

* To render the second book more complete, we have added this *supplement*, from two pieces which the author used to give his scholars. The substance of them is taken from the learned Mr. Cotes's *Traëts*, printed at the end of his *Harmonia Mensurarum*.

perpendiculars $M N$, $K L$, but the spaces described in these times will be as the areas $A M N$, $A K L$.

This proposition has been demonstrated elsewhere; but we shall here add the proof that is commonly given of it, by the method of *indivisibles*.

Since the motion of the body is supposed to be uniformly accelerated, that is, to receive equal increments of velocity in equal times, the velocities acquired will be always proportional to the times: so that if $M N$ represent the velocity acquired in the time $A M$, it follows, because $A M : A K :: M N : K L$, that $K L$ will represent the velocity acquired in the time $A K$. After the same manner, the velocities acquired in the times $A B$, $A C$, $A D$, &c. will be represented by the perpendiculars $B E$, $C F$, $D G$, &c. respectively.

The space described by any uniform motion is as the rectangle contained by the right lines that represent the velocity and the time: therefore the spaces described in the times $A B$, $B C$, $C D$, $D H$, &c. with the velocities $B E$, $C F$, $D G$, $H I$, &c. are as the rectangles $A E$, $B F$, $C G$, $D I$, &c. and the spaces described in the whole time $A K$ as the sum of these rectangles. That the motion may be uniformly and continually accelerated, suppose the number of the parts $A B$, $B C$, $C D$, &c. into which the line $A K$ is divided, to be increased *in infinitum*, and the sum of the rectangles $A E$, $B F$, $C G$, &c. will become equal to the triangle $A K L$. Therefore, in a motion uniformly accelerated, the spaces described in any times $A K$, $A M$, from the beginning of the motion, are as the areas $A K L$, $A M N$.

Corol. 1. The space described by a motion uniformly accelerated, in any time, is half the space that would be described, in the same time, by an uniform motion with the velocity acquired at the end of that time.

The space described by a motion uniformly accelerated, in the time $A K$, is represented by the triangle $A K L$; the space that would be described by an uniform motion, in the same time, with the velocity $K L$, is represented by the rectangle contained by $A K$ and $K L$, but the triangle $A K L$ is half of that rectangle; and the proposition is manifest.

Corol. 2. The spaces described by a motion uniformly accelerated, are as the squares of the times from the beginning of the motion; for those spaces are as the similar triangles $A K L$, $A M N$, whose homologous sides $A K$, $A M$, represent the times. For the same reason, the spaces are also as the squares of ($K L$, $M N$,) the velocities acquired at the end of those spaces.

Corol. 3. If the accelerating force is supposed to be greater or lesser in any given ratio, the velocities generated by it, in a given time, will be increased or diminished in the same ratio. And in any times, the velocity generated by this force, will be to that generated by the former, in the compounded ratio of the forces and of the times.

Corol. 4. The fall of heavy bodies, either perpendicular or along inclined planes, being a motion uniformly accelerated, the preceding *Lemma* and its collaries may be applied to them.

LEMMA II. Fig. II.

If two heavy bodies fall from rest at c to the horizontal line AB , one in the vertical CB , and the other along the inclined plane CA ; the time of descent from c to B , will be to the time of descent from c to A , as CB to CA ; and the velocities acquired at B and A will be equal.

For let the force of gravity by which the body descends in the vertical CB , be represented by CB , and resolved into the forces BD perpendicular to CA , and CD ; the other body is urged along the inclined plane by CD only. Therefore the accelerating forces by which the bodies descend in the vertical CB and along the inclined plane CA , are represented by CB and CD . The spaces described in equal times, by the uniform continued action of any forces, are in the same ratio as those forces: therefore the bodies will fall from c to B , and from c to D , in equal times. But the time of descent from c to D is to the time of descent from c to A (by *Corol. 2. and 4. Lem. 1.*) in the subduplicate ratio of CD to CA , that is, (because CD, CB, CA , are in continued proportion) in the ratio of CD to CB , or of CB to CA .

Again, the velocities generated in the falls are in the compound ratio of the generating forces, and of the times of their generation (*Corol. 3. Lem. 1.*) that is, in the present case, in the compound ratio of CA to CB , and of CB to CA ; which compound ratio is that of equality.

L E M M A III.

Upon the same horizontal plane, let there be raised another plane ca , whose elevation is cB ; from c draw cI parallel to ca meeting BA in I , and from B the line Bd perpendicular to cI . Then cB representing, as before, the constant force of gravity, cD , cd will represent the accelerating forces along the planes cA and (cI or) ca ; and their ratio being compounded of those of cD to cB , and of cB to cd , that is, of cB to cA , and (cI to cB or) ca to cB ; it follows that those accelerating forces are directly as the elevations of the planes, cB , ca , and inversely as their lengths cA , ca .

Corol. 1. Compound now these three ratios; that of cA to cB , of \sqrt{cB} to \sqrt{ca} , and of ca to cB , their sum gives the ratio of the times of falling thro' cA and ca , being the direct ratio of the lengths cA , ca , and the inverse subduplicate of the elevations cB , or ca .

Corol. 2. The velocities acquired being as the accelerating forces and the times in which they act; compound the ratio of these found in the preceding *Lemma* and *Corollary*, and there will result that of the velocities, viz. the direct subduplicate of the elevations cB , ca .

Corol. 3. Hence likewise it is inferred, that if (*Fig. III.*) a body fall from rest at c , to A in the horizontal line AB , along any number of planes cD , DE , EA , inclined to each other any how, as at D and E , the velocity at A will be the same as if the body had fallen in the vertical cB ; abstracting however

from the loss of velocity that happens by its impulses at D and E , upon the contiguous planes.

That multiplying the number of planes from c to A , till the path of the body becomes curvilinear, the velocity at A will be accurately the same as in the perpendicular fall $c B$.

And lastly, that if a series of planes, $c d$, $d e$, &c. similar and similarly situated to the former, or two similar and similarly situated arcs of a curve, be the path of the body; the velocities will be as the lengths of the paths; and the times in the subduplicate ratio of those lengths, of the heights $c B$, $c b$, or of any two homologous lines belonging to the figures.

Corol. 4. Let $A D$ (*Fig. IV.*) be the diameter of a circle touching the horizontal line in A ; $C A$, $c a$, any two chords drawn to A . Then, if bodies descend by the force of gravity along these chords, the times of descent will be equal; and the velocities will be proportional to the chords $C A$, $c A$.

For, joining $D c$, $D c$, and making $c E$, $c e$, perpendiculars to the diameter; because the triangles $D C A$, $E C A$ are similar, as also $D c A$, $e c A$; it is easily shewn that $C A$ is to $c A$ in the subduplicate ratio of the elevations $A E$, $A e$: and this compounded with the same ratio inverted, gives the ratio of equality; which, by *Corol. 1.* is that of the times.

And, by *Corol. 2.* the velocities are in the subduplicate ratio of $A E$ to $A e$, or that of $C A$ to $c A$.

I. Of the motion of projectiles.

PROPOSITION I. Fig. V.

The line described by a heavy body, thrown in any direction not perpendicular to the horizon, is a parabola.

Suppose a body projected in the direction AD , with the velocity it would have acquired by falling from B to A , the body, by that force alone acting upon it, would uniformly describe the right line AD ; and any part of the line of direction, as AH , represents the time in which it would be described.

Suppose that the force of gravity, acting alone, would have, in the same time, carried the body from A to P ; complete the parallelogram $APMH$, and, at the end of the time represented by AH , the body will actually be found in M . Since, by the first *Corollary* of the first *Lemma*, the time in which the body falls from B to A is the same in which it would describe $2AB$ by an uniform motion, with a velocity equal to that acquired at A , therefore that time will be represented by $2AB$. But the time in which the body would fall from A to P being represented by AH , it follows, from the second *Corollary* of the same *Lemma*, that $AP : AB :: AH^2 : 4AB^2$, and $4AB \times AP = AH^2 = PM^2$: from which it appears that the point M is a point in the *Parabola* whose diameter is AP and vertex A , having the parameter of that diameter equal to $4AB$.

Corol. I. It is evident that the line AH is a tangent to the *Parabola* in A , because it is parallel to the ordinate PM .

Corol. 2. Since $4 AB$ is the parameter of the diameter AP , it follows that the parameters belonging to the vertex A of the diameter AP are always in the duplicate ratio of the velocities of the projection, the space AB being always as the square of the velocity acquired by falling from B to A . It follows also that the parameter of AP is the same when the velocity of the projection is the same, whatever the direction AH of the projectile be.

Corol. 3. If from A as centre you describe the semicircle BQL , its circumference shall be the *locus* of all the foci of the parabolas that can be described by a projectile thrown from A , with the velocity it could acquire falling from B to A : for, by a known property of the parabola, the distance of the focus from A is always equal to $\frac{1}{4}$ of the parameter of the diameter that passes thro' A : that is, to $\frac{1}{4}$ of $4AB$ or to AB itself; all the foci must therefore be found in the semicircle BQL .

Corol. 4. Hence it is easy to determine the parabola described when the direction of the projectile is given; for you need only draw AF so as to make the angle FAD equal to the given one DAB , which the direction AD makes with the perpendicular AB , and the point F where AF cuts the semicircle BQL shall be the focus required; and if you draw thro' F the line FN parallel to AB cutting the *directrix* BE in N , it shall be the axis, and I , the middle point betwixt F and N , shall be the vertex of the parabola, $4FI$ being the parameter of the axis.

Corol. 5. If you draw a line thro' the vertex I parallel to the *directrix*, meeting AB in C it must be bisected by the line of direction in D ; and if you draw

draw a line from the focus F , to D , it will be perpendicular to the tangent, and will pass thro' B if produced, as appears from the properties of the parabola: and therefore a semicircle described upon AB as diameter will always pass thro' the point D , where the line of direction cuts CI the tangent to the vertex of the parabola.

Definition. If you draw a line thro' the point A , parallel to the horizon, cutting the axis in O and the parabola in K , then AK is called the *amplitude* of the parabola.

PROPOSITION II.

The amplitude of any parabola is always equal to four times the sine of double the angle which the line of direction makes with the vertical, taking the half of AB for radius.

For $AK = 2AO = 2CI = 4CD$; but AK is the amplitude of the parabola, and CD is the sine of the angle DGB , which is the double of BAD , if you take GB ($= \frac{1}{2} AB$) for radius.

Therefore the amplitude is equal to 4 times the sine of double the angle BAD , which the vertical makes with the line of direction.

Corol. 1. The velocity of projection being given, the amplitudes are to one another as the sines of double the angles of inclination.

Corol. 2. If the angle BAD does not exceed 45° , then it is plain that the more acute that angle is, the amplitude AK must be the less; since the sine of

double that angle must become less, and the amplitude is equal to four times the sine.

When the angle BAD vanishes, then the parabola AIK coincides with the straight line AB ; and the projectile, instead of describing a curve, will only rise to B and fall again to A .

On the other hand, the more the angle BAD approaches to 45° , the line CD , which is the sine of double that angle, becomes the greater: and therefore the amplitude AK , which is quadruple of that sine, must also become the greater.

Corol. 3. When the angle BAD becomes 45° , the points F and O shall fall on the point Q , where the semicircle BQL cuts the horizontal line AK ; the sine CD of double BAD becomes now the sine of 90° , and therefore is equal to the radius GA .

But since the radius is the greatest sine, it is plain that now the amplitude AK is the greatest that can be described by any projectile thrown from A with a velocity which it would have acquired by falling from B to A : and this greatest amplitude is always double of BA ; for AK in this case is equal to $4AG = 2AB$. Hence it appears, that if you throw a body in a direction that makes an angle of 45° with the horizon, it will be carried farther on the horizontal line, than if you threw it with the same force in any other direction.

Corol. 4. When the angle BAD is greater than 45° , then according as it approaches to a right angle, the parabola becomes more and more open, but the amplitudes AK decrease as the angle BAD increases;
for

for $A K = 4 C D$, and $C D$ must, in this case, decrease according as $B A D$ increases.

If of two directions $A D$ and $A d$, the elevation of the one exceeds that of 45° as much as the elevation of the other wants of it, their amplitudes will be equal; for the sines of double these angles must be equal, because they are supplements to two right angles, to one another: but the amplitudes of the parabola are always quadruple of these sines, and therefore they must also be equal to one another. That the doubles of these angles are supplements to one another appears thus: let their difference from 45° be called A , and the greater shall be $45^\circ + A$, the lesser $45^\circ - A$, their doubles shall be $90^\circ + 2 A$ and $90^\circ - 2 A$, which are supplements to each other because together they make up 180° .

Corol. 5. When the angle $B A D$ becomes a right angle, then $A B$ becomes the axis, and A the vertex of the parabola, $C D$ vanishes, and $A K$ becomes $= 0$.

Corol. 6. When the angle $B A D$ becomes greater than a right one, then the curve described shall be only a portion of the parabola that we have considered in the preceding corollaries, lying on the other side of A .

Corol. 7. If there is given the *impetus* or velocity wherewith the projectile is thrown, and the angle of elevation, or its complement $B A D$, you may find the amplitude $A K$, and the altitude of the parabola described by this projection. For seeing the amplitude of 45° is $2 A B$ (which is the line that always expresses the velocity, since by falling thro' it the velocity is acquired) you may say as the radius (or
fine

fine of 90°) is to the fine of double the angle BAD , so is $2AB$ to AK the amplitude sought, (by *Cor. 1.*): the amplitude being found, you may find the altitude by saying, as the radius is to the tangent of the angle of elevation, so is $CD (= \frac{1}{4} AK)$ to AC the altitude sought.

Corol. 8. If you have given the amplitude AK , and the angle of elevation DAB , you may find the impetus necessary to describe a parabola that shall have that amplitude, by this proportion; as the fine of double the angle of elevation, is to the radius, so is one half of the given amplitude to AB , the space thro' which a body must fall to acquire the necessary impetus.

Corol. 9. If the impetus and amplitude be given, the direction may be found by this rule. First find AB , by falling thro' which the given impetus may be acquired; then say, as the double of this line to the given amplitude, so is the radius to the fine of double the angle of elevation, and this angle or its complement will satisfy the problem.

PROPOSITION III. *Fig. VI.*

A projectile thrown in the direction AE , with the velocity it would acquire by falling from B to A , will strike any line AN in K , so that AK shall be equal to $4CD$: supposing AG perpendicular to the line AN , the angle $GBA = GAB$, and that the circle described from G as centre, with the radius GA , cuts the direction AE in D , and that DC is parallel to AN , meeting AB in C .

For it is plain that the angle $ADC (= DAK) = DBA$, by *Eucl. 32. 3.* and that consequently the triangles

triangles $A D C$, $A D B$ are similar, having the angle at A common, and the angle $A D C = A B D$; therefore $A C : A D :: A D : A B ::$ (because of the similar triangles $A C D$, $P A K$) $A P : P K ::$ (by the property of the parabola) $P K : 4 A B$, therefore $A D = \frac{1}{4} P K$, and consequently $C D = \frac{1}{4} A K$, or $A K = 4 C D$.

Corol. 1. Draw thro' D a parallel to $A B$ meeting the circle in d , and draw $A d$; then will the projectile thrown in the direction $A d$ strike the line $A N$ in the same point K ; for $C D = c d$.

Corol. 2. Let $H L$, parallel to $A B$, touch the circle in H , then shall $A H$ be the direction which will carry the projectile farthest on the line $A N$; because when D comes to H , then $C D$ is the greatest it can possibly be, and consequently $A K (= 4 C D)$ is then the greatest distance the projectile can be carried to, on the line $A N$, by the velocity acquired by falling from B to A . But it is plain that the angle $H A N = H B A = H A B$, therefore the direction $A H$ bisects the angle $B A N$ which the line $A N$ makes with the vertical $A B$.

Corol. 3. The lines $A D$, $A d$, make equal angles with $A H$, also the angle $D A N = d A B$; and when these angles are equal the distance $A K$ is the same.

Corol. 4. When $A K$ is given and the direction is required, take $A R = \frac{1}{4}$ of $A K$, and thro' R draw $R D$ parallel to $A B$, meeting the circle in D and d ; then draw $A D$, $A d$; and these will be the directions*.

* See more on this subject in Mr. Gray's Treatise of Gunnery, London 1731.

II. Of the cycloid; and the motion of a pendulum in it.

Definitions. If the circle $c d h$ (*Fig. VII.*) roll on the given streight line $A B$, so that all the parts of the circumference be applied to it one after another, the point c that touched the line $A B$ in A , by a motion thus compounded of a circular and rectilineal motion, will describe the curve line $A c e B$ which is called the *cycloid*. The streight line $A B$ is called the *base*; the line $E F$ perpendicular to $A B$, bisecting it in F , the *axis*; and the point E the *vertex* of the cycloid. The circle by whose revolution the curve line is described, is called the generating circle. The line $c k$ parallel to the base $A B$, meeting the circle in c and the axis in k , is called an *ordinate* to the axis; and a line meeting the curve in one point, that produced does not fall within the curve, is called a *tangent* to the curve in that point.

P R O P O S I T I O N I.

On the axis $E F$ describe the generating circle $E G F$, meeting the ordinate $c k$ in G ; and the ordinate will be equal to the sum of the arc $E G$ and its right sine $G K$; I say, $c k = E G + G K$.

It is plain, from the definition, that the line $A B$ is equal to the whole circumference of the generating circle, and therefore $A F$ must be equal to the semicircumference $E G F$. It is also obvious, from the description of the curve, that the arc $c d$ is equal to the line $A d$, and consequently the arc $c h$ equal to $D F$ or $I K$ or $c G$; but the arc $c h$ is equal to the arc $E G$; therefore $c G$ is equal to the arc $E G$, and the

the ordinate $c\kappa$ ($= c\text{G} + g\kappa$) must be equal to the sum of the arc E G and the right line $g\kappa$.

PROPOSITION II.

The line $c\text{H}$, parallel to the chord E G , is a tangent to the cycloid in c .

Draw an ordinate $c\text{k}$ very near $c\kappa$, meeting the curve in c , the circle in g , and the axis in k : let $c\text{u}$ and $g\text{n}$, parallel to the axis, meet the ordinate $c\text{k}$ in u and n ; and from o the centre of the circle E G F , draw the radius $o\text{G}$. Since $c\text{k} = \text{E g} + g\text{k}$, therefore $c\text{u} = g\text{g} + g\text{n}$; and if you suppose the ordinate $c\text{k}$ to approach to the ordinate $c\kappa$, and at length to coincide with it, as $g\text{g}$ and $g\text{n}$ vanish, the triangles $g\text{g n}$ and $g o \kappa$ become similar, whence $g\text{g} : g\text{n} :: o\text{G} : o\kappa$, and $g\text{g} + g\text{n} : g\text{n} :: o\text{G} + o\kappa (= \text{F K}) : o\kappa$; but $g\text{n} : g\text{n} :: g\kappa : o\kappa$, therefore $g\text{g} + g\text{n} : g\text{n} :: \text{F K} : g\kappa :: g\kappa : \text{E K}$; and consequently $c\text{u} : c\text{u} :: g\kappa : \text{E K}$; and if you draw the chord $c\text{c}$, the triangles $c\text{u c}$, E K G will be similar; so that the chord $c\text{c}$, as the points c and c coincide, becomes parallel to E G : therefore the tangent of the cycloid at c is parallel to E G .

PROPOSITION III.

The arc of the cycloid E L is double of the chord E M of the corresponding arc of the generating circle E M F .

Let κL and $k\text{s}$ be two very near ordinates of the cycloid, meeting the generating circle in M and Q ; produce the chord E M till it meet the ordinate $k\text{s}$ in P ; let Q o be the perpendicular from Q on M P ; then draw the lines E N and M N , touching the circle in E and M .

Because the triangles ENM , PQM are similar, and $EN = NM$, therefore PQ is equal to QM ; and the triangle PQM being isosceles, the perpendicular QO bisects the base PM ; so that MP is double of MO : but, by the last proposition, LS is parallel, and consequently equal, to MP , and LS is equal to $2MO$. The line LS is the increment of the curve EL , generated in the same time that the chord EM increases by MO , since EQ is equal to EO , when the points Q and M come together: Therefore the curve increases with double the velocity that the chord increases; and since they begin, at E , to increase together, the arc of the cycloid EL will be always double of the chord EM .

Corol. The semi-cycloid ELB is equal to twice the diameter of the generating circle, EF ; and the whole cycloid $ACEB$ is quadruple of the diameter EF .

PROPOSITION IV.

Let ER be parallel to the base AB , and CR parallel to the axis of the cycloid; and the space ECR , bounded by the arc of the cycloid EC and the lines ER and RC , shall be equal to the circular area EGK .

Draw cr parallel to CR ; and since $cu : cu :: GK : EK$; therefore $EK \times cu = GK \times cu$, and consequently $Rr \times CR = GK \times Kk$: therefore the little space $CRrc = GKkg$. So that the areas ECR , EGK increase by equal increments; and since they begin to flow together, therefore they must be equal.

Corol.

Corol. 1. Let $A T$, perpendicular to the base $A B$, meet $E R$ in T , and the space $E T A C E$ will be equal to the semicircle $E G F$.

Corol. 2. Since $A F$ is equal to the semicircumference $E G F$, the rectangle $E F A T$, being the rectangle of the diameter and semicircumference, will be equal to four times the semicircle $E G F$: and therefore the area $E C A F E$ will be equal to three times the area of the generating semicircle $E G F$.

Corol. 3. If you draw the line $E A$, the area intercepted betwixt the cycloid $E C A$ and the straight line $E A$ will be equal to the semicircle $E G F$; for the area $E C A F E$ is equal to three times $E G F$, and the triangle $E A F = A F \times \frac{1}{2} E F$ the rectangle of the semicircle and radius, and consequently equal to $2 E G F$; therefore their difference, the area $E C A E$, is equal to $E G F$.

P R O P O S I T I O N V.

Take $E b = O K$, draw $b z$ parallel to the base meeting the generating circle in x , and the cycloid in z , and join $c z$, $F x$: then shall the area $c z E c$ be equal to the sum of the triangles $G F K$ and $b F x$.

Draw $z d$ parallel to the axis $E F$, meeting $E T$ produced in d , and the trapezium $R c z d$ will be equal to $\frac{1}{2} \overline{C R} + \frac{1}{2} \overline{z d} \times R d =$ (because $z d = E b = O K$) $\frac{1}{2} O E \times R d$. But $R d = R E + E d = C K + b z = E G + G K + E x + b x$; therefore the trapezium $R c z d$ is equal to the sum of the rectangles of half the radius and the arcs $E G$, $E x$, added to their sines $G K$, and $b x$. But the area $E G F$, *i. e.* the triangle $E G F$ and the segment

ment cut off by the chord EG , is equal to the rectangle contained by half the radius and the sum of the arc EG and its right sine GK ; and the area EXF consisting of the sector EOX and the triangle XOF is equal to the rectangle of half the radius and the sum of the arc EX and its right sine bx ; therefore the trapezium $RCZd$ is equal to the sum of the areas EGF and EXF . By the last proposition, the area ECR is equal to EGK , and $EZd = EbX$; from the trapezium $RCZd$ subtract the areas ECR , EZd , and from the areas EGF , EXF , subtract the areas EGK , EbX , and there will remain the area $CZEc$ equal to the sum of the triangles, GFK , bFX .

Corol. 1. Hence, an infinite number of segments of the cycloid may be assigned that are perfectly quadrable. For example, if the ordinate cK be supposed to cut the axis in the middle of the radius OE , then K and b coincide; and the area ECK becomes in that case equal to the triangle GKF , and Ebz becomes equal to bFX ; and these triangles themselves become equal.

Corol. 2. Suppose now that K comes to the centre O , and c comes to i ; then because OK vanishes, therefore Eb vanishes, and the space $CZEc$ becomes in this case $ECiE$, which is equal to $\frac{1}{2} OE^2$; for the triangle bFX in this case vanishes.

But to return from this digression;

PROPOSITION VI. *Fig. VIII.*

Let ATC be a semi-cycloid having its base EC parallel to the horizon, and its vertex A downwards: suppose a string, with a pendulum, of the length of the semi-

semi-cycloid, suspended at c, and applied to the semi-cycloid c T A; the body p, by its gravity, will gradually separate the string from the semi-cycloid c T A, and will describe an equal semi-cycloid A P V, having its vertex in v, and its axis perpendicular to the horizon.

On the axis A E describe the generating semicircle A G E, draw A B cutting the vertical line c v in D, and on D v, taken equal to A E, describe the semicircle D H V. Then, since the semi-cycloid c T A is equal to $2 A E$ or c v, (by *Cor. Prop. III.*) therefore the body p will come to v, when the string c T P comes to a vertical situation. Thro' T and P draw T G and P H parallel to A D, meeting the semicircles in G and H; and since the streight part of the string T P is equal to the curve T A to which it was applied, therefore $T P = 2 A G = 2 T K$, and consequently T K and K P are equal, and the points G and H must be equally distant from the line A D: and therefore the arc A G will be equal to D H, and consequently the angle $G A D = A D H$; and the chords G A, D H, are parallel. But T P, being a tangent to the cycloid in T, is parallel to G A; therefore D K P H is a parallelogram, and D K is equal to P H. But the arc A G is equal to G T, by *Prop. I.* and therefore the arc A G $= A K$; and since A D = A G E, it follows that D K or P H $= G E$ or H V: and if P H be produced till it meet the axis in R, then shall the ordinate P R be equal to the sum of the arc H V and its right sine H R, and therefore the point P, by *Prop. I.* must be in a semi-cycloid, whose generating circle is D H V, its axis D v, and vertex v.

Corol. If another semi-cycloid equal to c T A, as c t B, be placed in a contrary situation, it is plain, that, by means of these semi-cycloids, a pendulum

Q

may

may be made to describe the cycloid $A v B$ in its oscillations.

P R O P O S I T I O N VII.

Let $v L$, perpendicular to $D v$, be equal to any arc of the cycloid $v M L$; describe with the radius $v L$ the semicircle $L z l$; and supposing the pendulum to begin an oscillation from L , the velocity acquired at M , in the cycloid, will be as $M x$ the ordinate of the circle at the corresponding point M in the straight line $v L$: and the force by which the motion of the pendulum is accelerated in M , is as the arc of the cycloid $v M$ that remains to be described.

Let $L R$, $M S$ be perpendiculars to the axis $D v$, meeting the generating circle in O and Q , and draw the chords $v O$, $v Q$: then by *Cor. 3. Lemma 3.* the velocity of the pendulum at M , will be the same as would have been acquired by a body directly falling from R to s , and the velocity acquired at v will be the same as would have been acquired by a body directly falling from R to v ; but these velocities are to one another as $\sqrt{R S}$ to $\sqrt{R v}$, by *Cor. 2. Lemma 1.* and since $R v : S v :: v O^2 : v Q^2$, and $R v : R v - S v (= R S) :: v O^2 : v O^2 - v Q^2 :: v L^2 : v L^2 - v M^2$ (because $v L = 2 v O$ and $v M = 2 v Q$), it follows that the velocity of the pendulum acquired in M is to the velocity acquired in v , as $\sqrt{v L^2 - v M^2}$ to $\sqrt{v L^2}$, or as $M x$ to $v z$.

The force of gravity that is supposed invariable, acting in the direction of the diameter $D v$, may be represented by $D v$; and may be resolved into the two forces $D Q$ and $v Q$, whereof the first $D Q$, parallel

parallel to $t M$ the string, serves only to stretch the string, and does not at all contribute to accelerate the motion of the pendulum; it is only the force represented by the chord $v Q$ that accelerates the motion of it along the curve $M m$, and is all employed to produce that effect, the direction $v Q$ being parallel to the tangent of the cycloid at M , by *Prop. II.* But $v M = 2 v Q$, by *Prop. III*; therefore the force that accelerates the pendulum at M , is as the arc of the curve $v M$.

Corol. It is obvious from the demonstration, that the part of the gravity which the string sustains in any point M , is to the whole weight of the pendulum, as the chord $D Q$ to the diameter.

PROPOSITION VIII.

Suppose that the circle $L z l$ is described by the body x with an uniform motion, by the velocity acquired by the pendulum in v ; and any arc of the cycloid, as $M N$, will be described by the pendulum, in the same time as the arc of the circle $x y$ by that uniform motion: taking $v N$, on the straight line $v L$, equal to $v N$ in the cycloid, and drawing $N Y$ parallel to $v z$, meeting the circle in y .

Let $x m$ be an ordinate very near to $x M$, and draw $x r$ parallel to the diameter $L l$, meeting $x m$ in r ; then, since the triangles $x r x$ and $v x M$ are similar, it follows that $x x : M m (= x r) :: v x : M x$, that is, as the velocity of the body x to that of the body M : and consequently the spaces $x x$ and $M m$ will be described in the same time by these bodies, the times being always equal when the spaces are taken in the same ratio as the velocities. After the same manner, the other corresponding parts of the lines $M N$ and

$Q z$

$x y$

$x y$ will be described in the same time; and therefore the whole space $m n$ will be described in the same time as the arc $x y$.

Cor. Therefore the pendulum will oscillate from L to v , in the same time as the body x will describe the quadrant $L z$.

P R O P O S I T I O N IX.

The time of a complete oscillation in the cycloid is to the time in which a body would fall thro' the axis of the cycloid $D v$, as the circumference of a circle to its diameter.

The time in which the semi-circumference $L z l$ is described by the body x , is to the time in which the radius $L v$ could be described with the same velocity; as the circumference of a circle, to its diameter. But the same time, in which the semi-circumference $L z l$ is described by the body x , is equal to the time of the complete oscillation $L v p$ in the cycloid, by the *Corollary* of the last proposition. The time in which a body falls from o to v , along the chord $o v$, is equal to the time in which $L v (= 2 o v)$ could be described by the velocity acquired at the point v , by *Cor. 1. Lem. 1.* and *Cor. 3. Lem. 3.* and the time of the fall thro' the chord $o v$ is equal to the time of the fall thro' the diameter $D v$, by *Cor. 4. Lem. 3.* consequently the time in which $L v$ could be described by a velocity equal to that of the body x , is equal to the time of a fall thro' the diameter $D v$. It follows therefore that the time of the entire oscillation $L v p$, is to the time of a fall thro' the diameter $D v$; as the circumference of a circle, to its diameter.

Corol.

Corol. 1. Hence the oscillations in the cycloid are all performed in equal times ; for they are all in the same ratio to the time in which a body falls thro' the diameter $D V$. If therefore a pendulum oscillates in a cycloid, the time of the oscillation in any arc is equal to the time of the oscillation in the greatest arc $B V A$, and the time in the least arc is equal to the time in the greatest.

Corol. 2. The cycloid may be considered as coinciding, in v , with any small arc of a circle described from the centre c ; passing thro' v ; and the time in a small arc of such a circle will be equal to the time in the cycloid ; and hence is understood why the times in very little arcs are equal, because these little arcs may be considered as portions of the cycloid as well as of the circle.

Corol. 3. The time of a complete oscillation in any little arc of a circle, is to the time in which a body would fall thro' half the radius ; as the circumference of a circle to its diameter : and since the latter time is half the time in which a body would fall thro' the whole diameter, or any chord, it follows that the time of an oscillation in any little arc, is to the time in which a body would fall thro' its chord ; as the semicircle, to the diameter.

Suppose $n v$ a small arc of the circle described from the centre c ; then the time in the arc $n v$ is so far from being equal to the time in the chord $n v$, even when they are supposed to be evanescent, that the last ratio of these times is that of the circumference of a circle to four times the diameter : and hence an error in several mechanical writers is to be corrected, who, from the equality of the evanescent arcs and

their chords, too rashly conclude the time of a fall of a body in any of these arcs equal to the time of the fall of a body in their chords.

Corol. 4. The times of the oscillations in cycloids, or in small arcs of circles, are in a subduplicate ratio of the lengths of the pendulums. For the time of the oscillation in the arc LVP is in a given ratio to the time of the fall thro' DV , which time is in the subduplicate ratio of the space DV , or of its double CV the length of the pendulum.

Corol. 5. But if the bodies that oscillate be acted on by unequal accelerating forces, then the oscillations will be performed in times that are to one another in the ratio compounded of the direct subduplicate ratio of the lengths of the pendulums, and inverse subduplicate ratio of the accelerating forces: because the time of the fall thro' DV is in the subduplicate ratio of the space DV directly and of the force of gravity inversely; and the time of the oscillations is in a given ratio to that time. Hence it appears, that if oscillations of unequal pendulums are performed in the same time, the accelerating gravities of these pendulums must be as their lengths; and thus we conclude that the force of gravity decreases as you go towards the equator; since we find that the lengths of pendulums that vibrate seconds are always less at a less distance from the equator.

Corol. 6. From this proposition we learn how to know exactly what space a falling body describes in any given time: for finding, by experiment, what pendulum oscillates in that time, the half of the length of the pendulum will be to the space required, in the duplicate ratio of the diameter to the circumference; because spaces described by a falling body,

from the beginning of its motion, are as the squares of the times in which they are described; and the ratio of the times, in which these spaces are described, is that of the diameter to the circumference: and thus Mr. *Huygens* demonstrates that falling bodies, by their gravity only, describe 15 *Parisian* feet and 1 inch in a second of time.

Schol. That it may be understood how the time in a small arc is not the same with that in its chord, tho' the evanescent arc is equal to its chord, we may here demonstrate, that if vk and nk be two planes touching the arc nv in v and n . Tho' the evanescent chord nv be equal to the sum of these tangents vk and nk , yet the time in the chord is to the time in these tangents as 4 to 3.

By *Cor.* 1. *Lem.* 3. the time in nk is to the time in nv as nk to nv , or as 1 to 2; but kv being horizontal, the motion in kv must be uniform, and it will be described by that uniform motion in half the time the body falls from n to k : therefore if the time in which kv is described uniformly be called τ , the time in which nk is described will be 2τ , and the time in which the chord nv will be described will be 4τ : and consequently the time in which a body would fall along the two tangents, is to the time in which it would describe the chord, as 3 to 4.

B O O K III.

Gravity demonstrated by analysis.

C H A P. I.

Of the theory of gravity, as far as it appears to have been known before Sir Isaac Newton.

I. **F**ROM experiments and observations alone, we are enabled to collect the history of nature, or describe her phænomena. By the principles of geometry and mechanics, we are enabled to carry on the *analysis* from the phænomena to the powers or causes that produce them; and, by proceeding with caution, we may be satisfied that our foundations are well laid, and that the superstructure raised upon them is secure. The first views which philosophers had of nature were no better than those of the vulgar, being the immediate suggestions of sense. But by comparing these together, examining the nature of the senses themselves, correcting and assisting them; and by a just application of geometrical and mechanical principles, the scheme of nature soon appears very different to a philosopher from that which is presented to a vulgar eye. At first sight, the surface of the earth appears of an unbounded extent, and of a most irregular form; while all the rest of the universe, the clouds, meteors, moon, sun, and stars of all sorts, appear in one concave surface bent towards the earth. This was the opinion concerning the system that most com-

commonly prevailed at first, while their imagination, influenced by such prejudices, made men fancy that they saw and heard things impossible. Thus the *Roman* poet represents their army when in *Portugal* (the western boundary of the great continent) as hearing the sun enter with a hissing noise into the ocean,

Audiit herculeo stridentem gurgite solem.

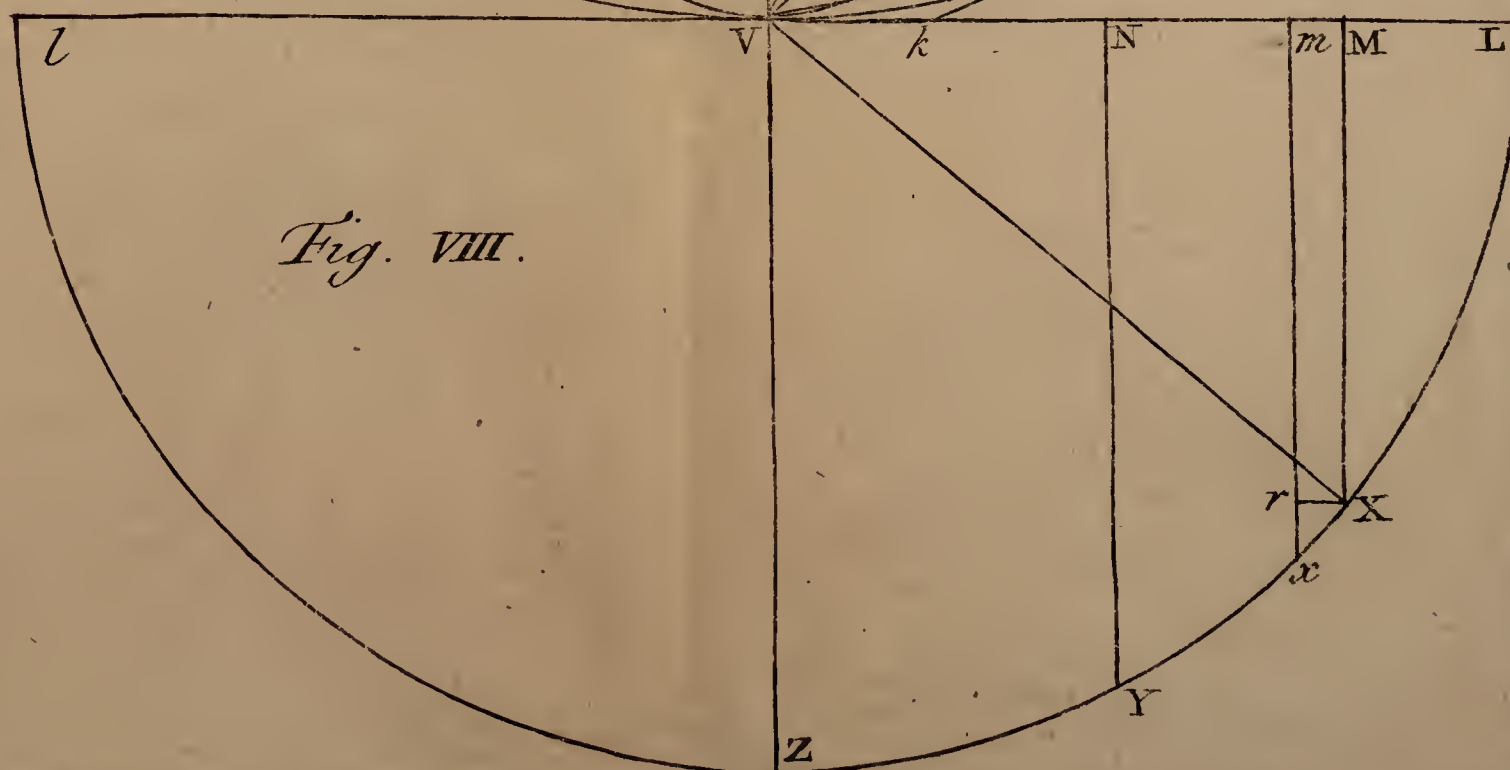
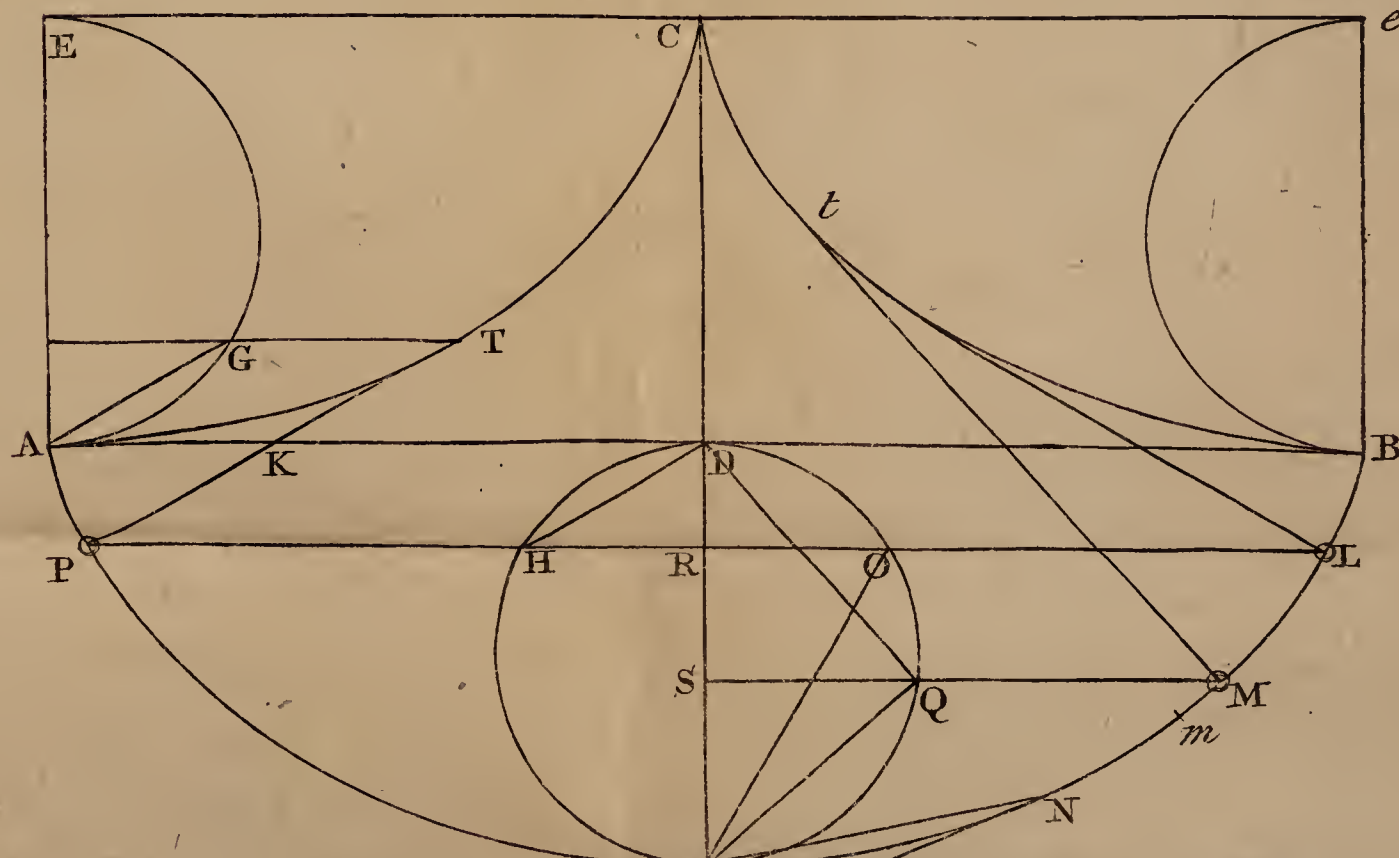
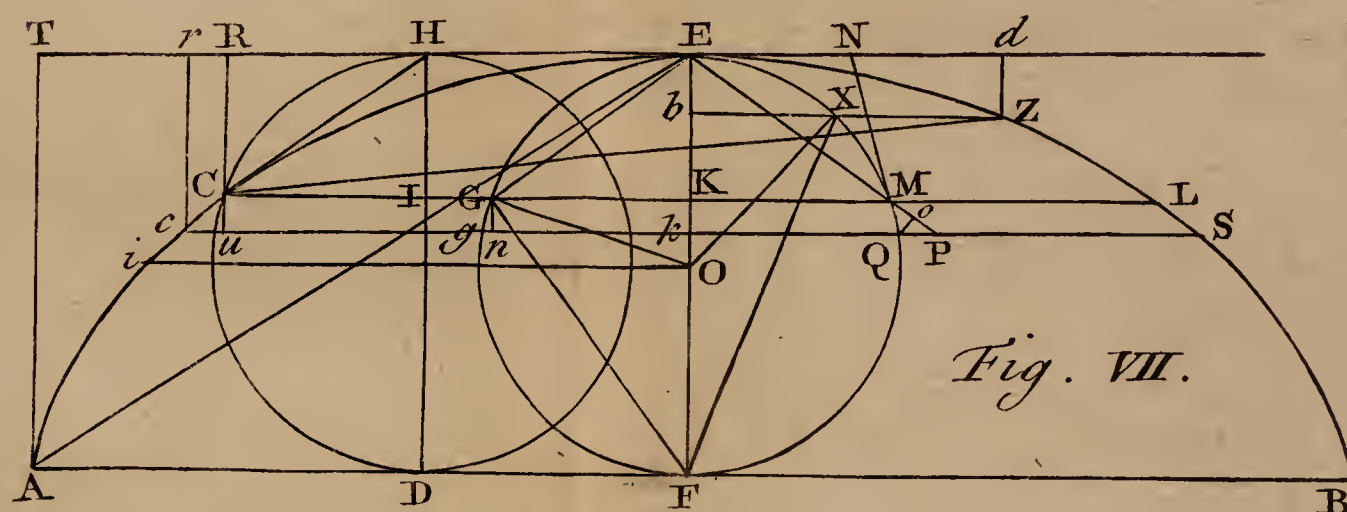
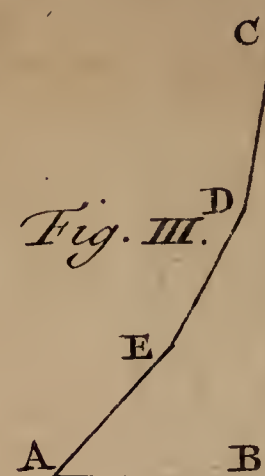
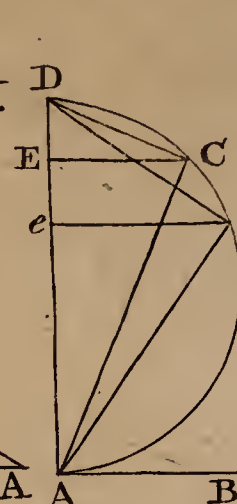
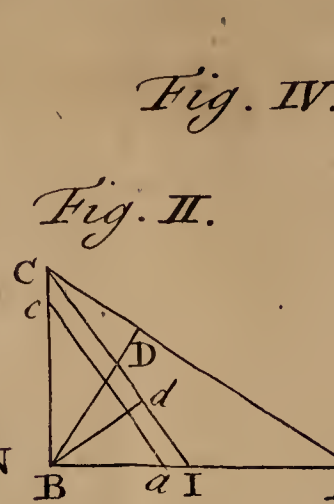
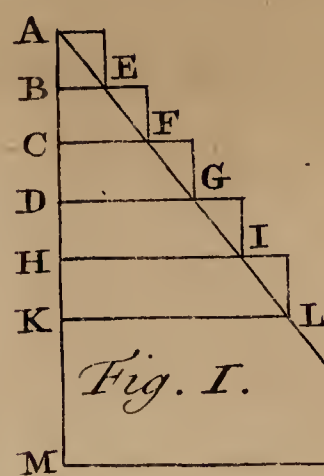
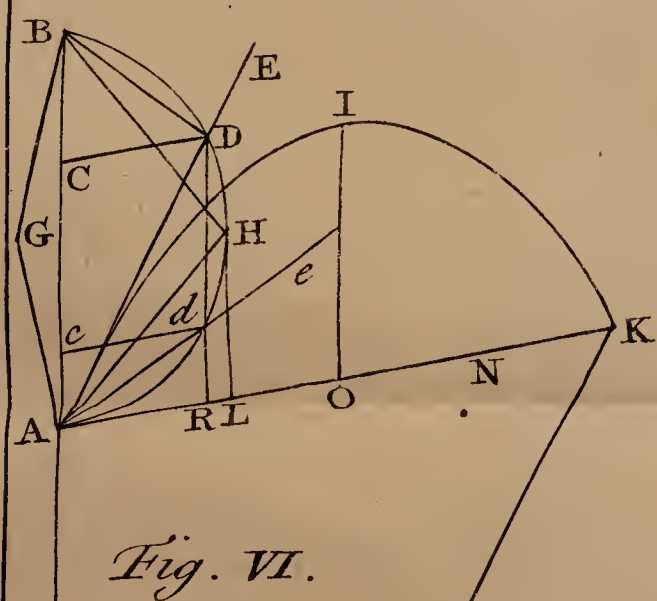
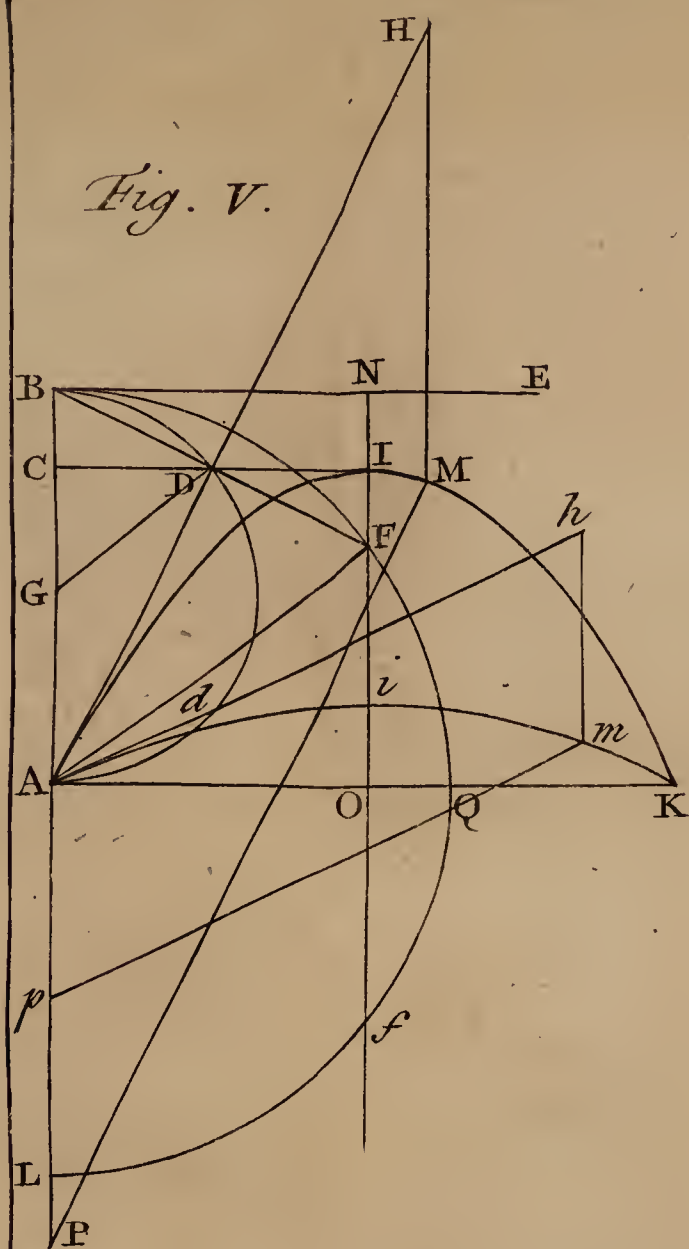
LUCAN.

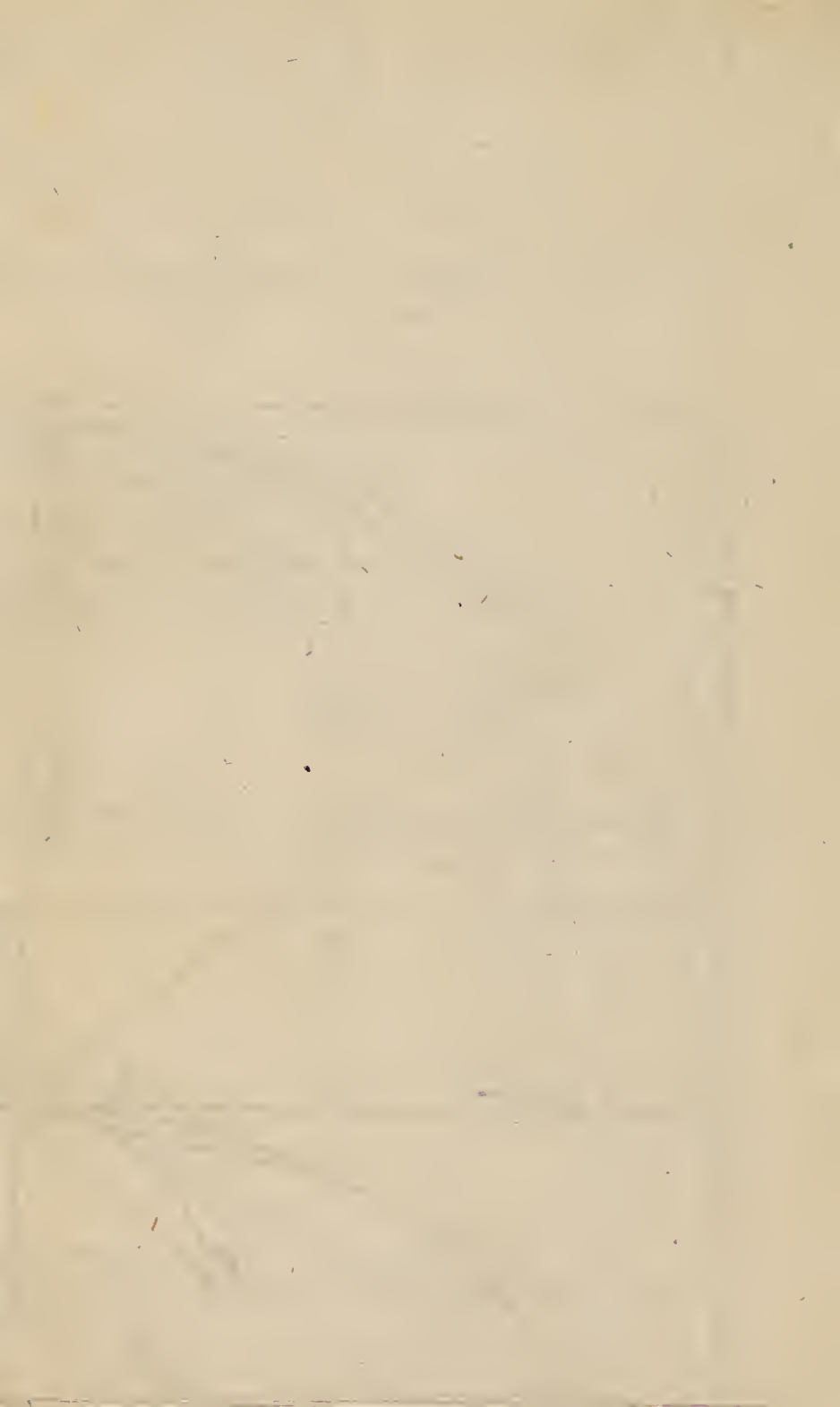
while other travellers have talked of a vast cavity in the most remote parts of the east, from whence the sun was heard to issue every morning with an unsufferable noise. But philosophers soon discovered that the earth was not of an unbounded extent, but of a globular form; and that the meteors, planets, and stars, were not confined to one concave surface, but dispersed in *space* at very different distances; that their real magnitudes and motions are very different from their apparent ones, and are not to be deduced from the appearances in any one place, but from views taken from divers points of sight, compared together by geometrical principles.

2. As our *analysis* of the system must be founded upon the real figures, magnitudes and motions, of the bodies of which it is composed; so we shall have an excellent instance of the method of proceeding by *analysis* and *synthesis* if we describe in what manner we are enabled, from the apparent phænomena, to deduce an account of the real; without the knowledge of which our enquiries into the powers or causes that operate in nature must be doubtful or erroneous. The knowledge of the disposition and motions of the celestial bodies must precede a just enquiry into their causes. The former is more simple, the latter more arduous; and the former will pre-

prepare the way for the latter, and serve to make the reader acquainted with this method (the only one by which certainty can be acquired in this science) in easy cases; before he proceed to those of a more complicated nature. We shall therefore begin with the plainest and most simple instance of this kind, by shewing briefly how, from the phænomena, the true figure, magnitude, and motions of the earth are derived; and how, these being established, innumerable phænomena are deduced by *synthesis*.

3. It is to *sight* that our knowledge of the distant parts of the system is owing, those objects that are very near us falling under the observation of the other senses only: but this sense, however admirable, has its imperfections. Vision depends upon the picture of external objects formed on the *retina*, together with a judgment of the understanding, acquired by habit and experience; which is so immediately connected with the sense, that it is impossible, by an act of reflection, to trace it, or, when it is erroneous, suddenly to correct it. If vision depended upon the picture only, then equal pictures upon the *retina* would suggest ideas of equal magnitudes of the objects; and if the smallest fly was so near that it could cover a distant mountain from it, the fly ought to appear to us to be equal to the mountain. But we have, by habit, acquired a faculty of compounding the opinion, or prejudice, formed concerning the distance with the apparent magnitude or bulk of the image formed on the *retina*; and this with an inconceivable quickness of thought, so that the idea or image we form to ourselves of its magnitude is the result of both; an allowance being made for the greater distance, agreeable to the notion we have conceived of it. Hence it is easy to see how many fallacies in vision must arise: for as
we





we may be often mistaken in our notion of distance, so every such mistake must produce a corresponding error in our idea of the magnitude of the object. Besides, in many cases, this notion of distance arises without reflection, from the force of habit; and we find the effect of it takes place even after the understanding is better informed, and the judgment corrected. Thus the moon continues to appear bigger to us at the horizon than at the meridian, even after it has been demonstrated to us that her distance is then greater, so that she ought really to appear less. Because (according to *Kepler's* observation) the heavens appear to us, not in an hemispherical dome, but as a segment of a sphere less than the hemisphere, we have been accustomed to ascribe a greater real magnitude to objects seen at a great distance along the horizon, than to those of an equal apparent magnitude (or that have equal images on the *retina*) seen at a considerable elevation about it; and hence he ingeniously accounts for the moon's appearing bigger to us at the horizon than at the meridian. But after we are better informed, and know that the apparent magnitude of the moon is less at the horizon in the same proportion as the distance is greater, we continue to make an allowance not on this account only, but a much greater than this requires, from the great influence of habit and custom*; the effect of which on the mind and its operations is a subject that well deserves the particular attention of philosophers,

* Perhaps the concave surface of the heavens appears to us as a portion less than a hemisphere, because we have been always accustomed to see greater distances along the horizon than in the vertical line towards the zenith. But whatever the reason of this appearance (supposing it true) may be, it would seem that an habitual way of thinking to the contrary ought to have some effect; and some observe that the moon never appears to them so large at the horizon, as it did formerly when they were young and unacquainted with her motions.

but is improper to be insisted on in this place, lest we should seem to mix, without necessity, what is obscure and uncertain with what is clear and satisfactory. For the *analysis* we are to describe, depends not on any disputed principles, but on those of practical geometry applied to the heavens.

4. Experience has taught us several ways of forming a judgment concerning the distances of objects, when they are not very remote from us; as by the different disposition of our eyes when we look at a near object with both; it being manifest that when the object is near, the eyes must be turned more towards each other, in order that they may be directed towards the same point of it, than when it is at a greater distance. We soon learn from experience, likewise, that when the object is very near, the image is obscure and confused, and we are obliged to strain the eye to render it tolerably distinct. The image is also found to be more luminous and bright when the object is near than when it is remote. But the most usual way of estimating the distance is from the intervening objects; or, when the object itself is of a kind with which we are well acquainted, by the bulk its image bears in the picture upon the *retina*. By these, and perhaps other methods, we are enabled to form some judgment of the distance of near objects*. But when they are very remote, and no objects

* A learned author, of a distinguished character, begins an ingenious treatise upon this subject, by observing, “ it is, I think, agreed by all, that distance, of itself and immediately, cannot be seen. For distance being a line directed endwise to the eye, it projects only one point in the fund of the eye, which point remains invariably the same, whether the distance be longer or shorter.” The distance here spoken of, is distance from the eye; and what is said of it is not to be applied to distance in general. The apparent distance of two stars

objects intervene, as is the case of the celestial bodies, these methods fail us, the sense is at a loss in comparing their distances together, and is unable to determine which are greater or less, without the aid of geometry, or some equivalent art. In such cases, therefore, the objects are all referred by the sense to one concave surface. Thus the clouds, meteors, planets and stars of all kinds, appear to the sense in one concave surface of heaven, though there be the greatest variety in their real distances. It is in these cases that practical geometry brings us its necessary and sure aid. By it we soon find that the clouds are not only nearer us than the celestial bodies, which they often cover from us, but that their distance is only of a few miles; a small change of the place producing a great change in their position with respect to us, while those that are seen by us at one place are different in position from those that are seen at the same time in places remote from it. We soon perceive that the moon is at a vastly greater distance; because she is seen over one half of the earth at once, and nearly in the same direction, or in the same situation among the fixed stars. We easily learn that the moon is at a less distance from us than the sun, because by coming between us and the sun she produces the solar eclipses; and that *Venus* and

is capable of the same varieties as any other quantity or magnitude. Visible magnitudes consist of parts into which they may be resolved as well as tangible magnitudes, and the proportions of the former may be assigned as well as of the latter; so that this author goes too far, when he tells us that visible magnitudes are to be no more accounted the object of geometry than words; and when he concludes of distance in general, what had only been shewn of distance directed “end-wise to the eye;” and pretends “to demonstrate that the ideas of space, outness, and “things placed at a distance, are not, strictly speaking, the object of sight; and are not otherwise perceived by the eye than “by the ear.”

Mercury are nearer to us in their inferior conjunctions than the sun, because they are then seen as dark spots upon his disk. If our instruments were absolutely perfect, and our observations could be made with the utmost accuracy, then each celestial body might have its distance precisely ascertained, and the whole disposition of the system might be exactly known. But this subject being of the utmost importance in our present *analysis*, it deserves some farther illustration.

5. Let *A* and *c* (*Plate III. Fig. 50.*) represent two spectators, or two different stations of the same spectator, *D* the object or phænomenon whose distance is required. This object appears to the spectator at *A* in the right line *A D F*, and to the spectator at *c* in the right line *c D E*; the angle contained by which, *A D c*, shews how much the position of the object *D* varies with respect to the two spectators. When this angle is great, the distance *A D* bears not a great proportion to *A c*; but when this angle is very small, as when the object is removed from *D* to *H*, then its distance from *A* must be much greater than *A c* the distance of the two spectators or stations; because *A c* is always to *A D*, as the sine of the angle *A D c* to the sine of *A c D*, by common trigonometry. Thus when *A c* consists of some miles, and *D* represents a cloud, the angle *A D c* is found to be considerable; and thence we learn that its distance is not very great. If *E D c* represent the right line in which the sun shines, then *c* will represent the shadow of the cloud upon the plane *A c*; and the proportion of *A D* to *A c* may be determined by observations taken from one station *A*. But tho' the right line *A c* consist of hundreds of miles, if *H* represent the moon, it is found that the angle *A H c* is exceeding small; and thence we conclude, that the distance of the moon

moon is not to be expressed but by a great number of miles.

6. Let c (*Fig. 51.*) represent the centre of the earth, A a place upon its surface, $c A e$ the vertical line of this place, d any object or phænomenon in the zenith; $A D F$ a tangent to the surface of the earth at A , the sensible horizon at that place. Then the object d being supposed to project upon the fixed star e , when in the vertical line, to a spectator at A as well as at c , it will be otherwise when the object d comes to the horizon at D . For tho' the centre c , the object D and the star E (abstracting from their proper motions) be still in a strait line, yet D and E are no longer in a right line with A the place of the spectator: but while D appears to be set at F , the star appears still elevated above the horizon by the arc $E F$, which measures the angle $E D F$, or $A D C$; the sine of which is to the radius, as $c A$ the semidiameter of the earth is to $c D$ the distance of the object from the centre of the earth. This angle $A D C$ is what is called the *horizontal parallax* of the object or phænomenon, and shews under what angle the semidiameter of the earth $c A$ would appear if viewed at the distance of the object $c D$. And to find this horizontal parallax of any object, is no more than to determine how great (or under how many *minutes* and *seconds*) the semidiameter of the earth would appear viewed at that object. Suppose any number of objects in the right line $A F$, as D, G, H ; and spectators at each of these viewing the semidiameter of the earth $c A$; it will appear to them under the respective angles $c D A, c G A, c H A$, which are the respective parallaxes of those objects, and which gradually decrease as their distances increase. We discover therefore the distances of those objects by determining what appearance, as to bulk or ap-
parent

parent magnitude, the earth's semidiameter makes at those objects: and it is obvious that this method is well founded, it being manifest, that the distances at which the earth appears great to a spectator must be less, and that those distances at which the earth appears small to him must be greater. Thus to a spectator carried to a few hundred miles distance only, the earth would appear very large; to a spectator at the moon, the semidiameter of it would appear under an angle less than a degree; to a spectator at *Venus*, of much about the same bigness as *Venus* appears to us; and to a spectator as remote as *Jupiter* or *Saturn* it would hardly be visible at all, unless his sense was more acute than ours, or assisted by art. And as, when the proportion of the distance of the spectator from the centre of the earth to its semidiameter is known, it is easily ascertained how great an appearance the earth will make to that spectator; so conversely, when this appearance is determined, it is easy to assign the spectator's distance from it.

7. In this manner, mensuration is carried from the earth to the heavens; and the distances of the celestial bodies compared with semidiameters of the earth, and with one another. For the further illustration of what is of such importance in astronomy, a science that affords us so noble and extensive views of nature, let us imagine a spectator at *A* viewing the immense expanse around him, while a right line *D L*, perpendicular to *A D* and equal to the semidiameter of the earth, moves off on the right line *A F* from the least to the greatest distances; then the parallax belonging to any distance is nothing else than the angle which the semidiameter of the earth at that distance subtends to the spectator at *A*. Thus the parallaxes belonging to the several distances *A D*, *A G*,
A H,

$\angle H$, $\angle c$. are the respective angles DAL , GAM , HAN , $\angle c$; which measure the apparent magnitude of the semidiameter of the earth viewed, at those distances, by a spectator at A . While we suppose this semidiameter to be carried off *in infinitum*, these apparent magnitudes gradually decrease, nearly in the same proportion as the distance increases. The parallaxes decrease in the same manner; and a scale of the one affords us a scale of the other. It is obvious, that, from the moment any object departs from the vertical line, it appears to a spectator at A depressed towards the horizon, and is the more depressed in proportion as it is nearer to him. The true place of the object D is at E , where it would be seen from the centre c ; but its apparent place to a spectator at A is at F , and its depression or parallax is measured by the arc EF , or by the angle EDF equal to ADC . Now, in order to find this depression, it is sufficient to make use of the fixed star E , which has no sensible parallax, and was supposed to be in conjunction with the object in the vertical line Ade ; for the depression of the object D below the star E , viewed from A , gives the parallax. By processes of this kind, it is found, from astronomical observations, that the mean distance of the moon from the centre of the earth, is about $60\frac{1}{2}$ semidiameters of the earth.

8. The figure of a body is more easily known when we are able to view it from great distances than from very small ones; because when it is at a great distance, the eye takes in a considerable portion of it in one view, from which the figure of the whole is more easily collected: whereas when it is viewed at a small distance, small irregularities on its surface have too great an effect upon the sense, and are apt to mislead us in our judgment concerning the whole.

It is very easy to see, for example, that the sun and moon are globular, because in all positions they constantly appear to us as bounded by a circle, a property which belongs to the sphere or globe alone. But the figure of the earth is not so easily discovered by us, because the largest views we are able to take of it, from the tops of the highest mountains, bear a small proportion to the whole surface; and the curvature or sphericity is hardly sensible in those prospects of it. However, we have undoubted proofs that the earth is globular, though not exactly spherical. We are assured that the meridian sections of the earth, or sections thro' its poles, are circular, because, as we go southwards the northern stars are depressed, and the southern stars elevated, nearly in a regular course; so that a degree of depression of the former, or elevation of the latter, always corresponds to 60 *Italian* or geographical miles on the meridian; whence we conclude, that a meridian section of the earth is a circle, a degree of which is 60 such miles, and the whole circumference is 60×360 , or 21600, of the same miles. At the equator, both the poles are in the horizon; as we remove northwards, the northern pole rises till we come to the pole of the earth, where the celestial pole is in the zenith; and, in general, the elevation of the pole increases gradually and regularly with the distance from the equator. The equator and its parallels appear to be circular from the regular daily progress of light, from east to west, along their surface. The sun arrives at the meridian of places that are more easterly, sooner than to the meridian of those that are towards the west, in proportion to the distance of the meridians measured upon the equator. The spherical figure of the earth appears likewise from *levelling*, where it is found necessary to make an allowance for the difference between the apparent and

and the true level; the former being a plane that touches the earth's surface, the latter the globular surface itself, which falls below the tangent plane.

9. But we have the plainest and most simple proof of the globular figure of the earth, from that of its shadow projected on the moon in a lunar eclipse. For this shadow being always bounded by an arc of a circle, it follows that the earth which projects it is of a spherical figure. If there was any remarkable angle, or very considerable irregular protuberance, on the earth, it would, on some occasion or other, appear by the shadow. The mountains, indeed, are irregularities on the surface of the earth; but they bear so small a proportion to its vast bulk, that they make no appearance upon its shadow. There is likewise a gradual rising from the sea-shore towards the inland parts of the great continents; as in *Europe* from the shores of the ocean, the *Mediterranean*, and the *Euxine* sea, towards *Switzerland*; but this gradual rising is small, and has little effect on the figure of the earth. If it was considerable, it would carry the inland parts too high in the atmosphere; but it is sufficient for giving a course to the rivers, and preserving the beautiful circulation of water, so necessary to the good condition of this globe; and the extent of the continents has been probably contrived with a view to this great purpose. Upon the whole, the earth is evidently globular tho' not an exact sphere, and if seen at a distance would appear to us as the sun or moon; that is, always terminated by a circular figure, unless this distance was so great as to make it appear like *Venus* or *Mars*; when, in consequence of the contraction of the apparent diameter, the whole surface would appear to be crowded in one point, and the *Alps*, *Pyrenees*, and even the distant *Cordelleras*, would reflect undistin-

R 2

guished

guished rays. At such distances its figure could not be discerned by sense, unless it was assisted by a telescope, or some equivalent instrument.

10. The ocean, which covers a great part of the surface of the earth, is more accurately globular than the solid parts; and it is manifest that this arises from the gravitation of its parts towards the earth, acting in right lines perpendicular to its surface. For if its direction formed an acute angle with the surface, the fluid water would necessarily move towards that side, and could not be in *æquilibrio* till the direction of gravity became perpendicular to the surface every where, so as to give no inclination to the fluid to move towards either side. The perpendiculars to a spherical surface meet all in the centre of the sphere. Therefore, since the earth is nearly a sphere, the direction of the gravity is nearly towards its centre; not as if there was really any virtue or charm in the point called the centre, by which it attracted bodies, but because this is the result of the gravitation of bodies towards all the parts of which the earth consists; as will appear more fully afterwards. The direction of gravity is not any one fixed or determined one, as the vulgar are apt to imagine; nor is there any occasion for pillars or instruments of any kind to support the earth; that direction being always downwards which is towards the centre, or (to speak more accurately) which is perpendicular to the fluid surface or level, on the concave side; and that direction being upwards which lies in a perpendicular to the surface on the convex side. Was the earth all fluid, all the surface would be on one level, and no one part would have a pre-eminence above the rest in this respect; and bodies would be sustained by the earth equally round all its surface with equal firmness and security. Thus there is no difficulty
in

in conceiving that there are *Antipodes* ; and it appears equally absurd that bodies should fall off from any other part of the earth, as that they should rise here into the air.

II. This principle of gravity extends to all bodies around the earth. For the gravity of the air being established beyond all dispute, by the celebrated experiments of *Galileo* and *Torricelli*, and many others of the same kind, it easily appears that all terrestrial bodies whatsoever are heavy, or gravitate towards the earth ; and that the apparent levity of some of them proceeds only from the greater gravity of the ambient air, which makes them rise upwards, for the same reason that cork rises in water, and lead in quick-silver ; or from their being carried off by some medium entangled in its parts. The gravity of terrestrial bodies must the rather be allowed to be universal, because, by the most accurate experiments, it is always found to observe the same proportion as their quantities of matter ; and not to depend on the figure or bulk of bodies, or the contexture of their parts, but always to measure their quantity of matter, and to be measured by it only, abstracting from the influence of the medium in which they swim. For gravity always generates the same velocity, in bodies of all sorts, in the same time ; and therefore must act equally on equal portions of matter, and on a greater portion with a force proportionally greater. The direction of this power is nearly towards the centre of the earth ; for, at present, we abstract from the variation of its figure from that of a perfect sphere, arising from its motion on its axis. The force of this power is such, that it carries all bodies downwards about $15\frac{1}{2}$ feet, of *Paris* measure, in a second of time. This is the result of accurate experiments ; every body would fall just so

much if it descended freely in the plumb-line, or perpendicular to the horizon, and met with no resistance from the air or ambient medium. When a body is projected in a right line that is not perpendicular to the horizon, it moves in a curve, but so as to fall always below the point in the line of projection which is directly over it, as much as it would have fallen by descending freely in the perpendicular in the same time; provided we suppose gravity to act in parallel lines, as was usual before Sir *Isaac Newton* found it necessary to consider this subject more accurately, and which may be admitted, without any sensible error, in such motions as our engines are able to produce.

12. The globular figure of the earth, with the direction and force of gravity, being discovered by this *analysis*, a great variety of phænomena may be thence deduced by the *Synthetic* method. The whole doctrine of the sphere may be explained from the figure of the earth, either in the *Pythagorean* or *Ptolemaic* system. As the sun appears to go round the whole circle of 360 degrees in 24 hours, so in one hour he appears to describe 15 degrees, and one degree in 4 minutes of time, on the equator or its parallels. Hence the distance of meridians at two places, measured upon the equator, or their difference of longitude, being known, it is easy to compute how much the hours at one place precede the same hours at the other, by allowing four minutes of time for each degree of that distance; and conversely, the difference of time being given, the difference of longitude is computed by allowing one degree for each four minutes of time, and proportionally in greater or lesser differences. And it is obvious that the hours of the day, which are successive in any one place, are co-existent when you take in the whole globe;

globe ; so that no hour of the day can be assigned, but a meridian can be likewise assigned where it is that hour at this present time. The *sensible horizon* of any place is a plane perpendicular to the plumb-line at that place, and tangent to the earth's surface there. The *rational horizon* is a plane through the earth's centre parallel to this, whose poles are the *zenith* and *nadir*, in the same manner as the north and south poles of the world are the poles of the equator. The particular phænomena of places depend upon the position of their horizon with respect to the circles of the apparent diurnal motion of the sun and stars. The horizon of a place at the equator passes through the poles, and divides equally the equator and its parallels. Hence the days and nights are always equal in such places, and each of the stars performs one half of its revolution above their horizon, and the other half under it. The circles of diurnal motion are all perpendicular to their horizon, and therefore they are said to be in a *right sphere*. When the sun moves in the equator, he rises directly from their horizon to their zenith, and then descends directly to their horizon again ; in other cases, after rising perpendicularly, he slopes away in his parallel towards the north or south side of their zenith, according to the season of the year ; which must be a considerable relief to them, as the heat must thereby be abated. At the poles, their horizon coincides with the equator ; so that the northern celestial hemisphere must be always in view of the northern pole, being above their horizon, while no part of the southern hemisphere is visible to them, being always beneath it. The circles of the diurnal motion being parallel to the equator, and consequently to their horizon, the sun and stars appear to them to move in parallels to their horizon ; the fixed stars never rise nor set, and the sun rises at

the vernal equinox and sets at the autumnal; so that they have day for one half year and night for the other. They are said to be under a *parallel sphere*. In intermediate places, the circles of the diurnal motion are oblique to their horizon; one pole is always elevated above it by an arc equal to the latitude of the place, and the other pole is depressed under it by an equal arc. All the stars whose distance from the elevated pole exceeds not the latitude of the place are constantly above their horizon; and those within the same distance of the other pole are depressed under it, and are never visible to them. The equator and horizon being great circles divide each other equally, whence the days and nights are equal everywhere when the sun describes the celestial equator. But when the sun is on the same side with the elevated pole, a greater portion of his parallel is above the horizon than under it, and therefore the days are longer than the nights: and when the sun is on the other side of the equator, a greater portion of his diurnal parallel is below the horizon than above it; and consequently the nights are longer than the days. These are said to be under an *oblique sphere*. In all those different places, the time in which they have day (that is, when the centre of the sun is above the horizon) is equal to the time in which they have night, or when the centre of the sun is beneath their horizon, taking the whole year together; abstracting from the effects of refraction and the elliptic figure of the earth's orbit, which are not considered in the doctrine of the sphere. But these equal times are distributed with a good deal of variety. At the equator they have 12 hours day and 12 hours night, perpetually succeeding each other. At the poles they have their day all at once and their night at once, each of half a year. In intermediate places,
the

the length of their days at one season is compensated by the length of the nights at another. Within the polar circles, they have the sun continually for some days, or weeks, circulating above their horizon; but, in the opposite season of the year, he continues as long beneath their horizon; and thus the equality of the times of day and night is preserved, when we abstract from the sun's having a sensible diameter, from the effects of refraction and twilight, and the elliptic figure of the earth's orbit; but, in consequence of these, the time in which they have day considerably exceeds what is commonly called night, particularly in the northern hemisphere. The *amplitude* of the sun, or his range upon the horizon, has likewise great varieties, which are easily deduced from the same principles. It is least at the equator, amounting there to $23^{\circ} 29'$ on each side, towards the north and south of the east and west points. In the latitude of 56° it amounts to above 45° , on each side of the same points; and the arc between the most northern and southern points where he rises, and sets, is above a quadrant. At the polar circles, his range on the horizon is the whole semicircle from north to south. A circle perpendicular to the meridian and horizon is called the *prime vertical*, and, being a great circle, it cuts the equator equally, and all places that are under it bear due east or west from us; whence many of the geographical paradoxes are explained. The art of *dialling* is deduced from the same principles. The most simple kind of dial is an equinoctial one, where the shadow is received upon a plane parallel to the circles of the sun's diurnal motion, and is projected by a *stylus*, or right line, perpendicular to those planes. Because the sun moves over equal arcs on its parallel in equal times, the motion of the shadow in this dial must likewise be uniform, so that the intervals between the hours must

must be equal ; which is therefore made by dividing a circle into 24 equal parts. The construction of other dials is easily deduced from this : but our design obliges us to mention these things very briefly. We have a remarkable instance of the beauty of truth when we observe what a variety of phænomena arise from so few simple principles as the spherical figure of the earth, its diurnal motion, and the obliquity of its axis, as we take a survey of the earth from the torrid to the frigid zone, or from the equator to the poles, and attend to the phænomena of heat and cold, as well as of those of day and night, and of the apparent motions of the stars. A diversity of phænomena so very great, arising from two principles of so simple a nature, affords a curious speculation to the understanding, as well as a pleasing entertainment to the imagination, and serves to suggest the admirable fertility of which nature is capable in its productions ; insomuch that upon one globe we have some image or representation, in the climates from the equator to the poles, of that great variety that we may suppose to take place in the solar system, from *Mercury*, the nearest and hottest, to *Saturn*, the remotest and coldest of all the planets.

13. Tho' the doctrine of the sphere may be explained from the *Ptolemaic*, as well as from the *Pythagorean* or *Copernican* system, by supposing the *primum mobile* to penetrate the whole universe (the earth and its appendices only excepted) and to carry every thing round the earth's axis every day ; yet this hypothesis, to every thinking person who has not devoted his judgment entirely to the prejudices of sense or dictates of superstition, appears so very absurd, that it is now almost universally exploded. The motions of the *comets*, performed with so much free-

freedom in the celestial spaces, shew us that the solid orbs are imaginary, and that there can be no such universal mover that carries all the universe along with it: nor is there any axis upon which this immense machine can be supposed to turn. The prodigious velocity, which, according to this doctrine, must be ascribed to the remote fixed stars, cannot but shock those that have any just notion of the vast extent of the universe. The ascribing so extraordinary a pre-eminence to the earth, to which it appears to have no title, argues a partiality unworthy of philosophers; especially since we see that most of the other bodies of the system, even the sun himself, turn round upon their axes, which would induce us, if we were upon the surface of any of them, to ascribe the same pre-eminence to that one, and to place it in the centre of the whole. But besides these and other considerations, the retardation of pendulums carried to the equator, with the increase of the degrees of the meridian from thence to the poles, are observations that demonstrate a centrifugal force, greatest at the equator, and gradually diminishing towards either pole, where it vanishes. Now this centrifugal force is an evident proof of the diurnal rotation of the earth upon its axis; therefore, in treating of the celestial motions, we shall entirely abstract from the apparent diurnal motions of the planets, as pertaining to the earth only: and thus our *analysis* of the causes that produce the celestial motions is founded on the real state of things, and not on fallacious appearances.

14. The doctrine of the sphere is easily deduced from these true motions. One half of the earth is illuminated by the sun at all times, and the other half always deprived of his light. The boundary of light and darkness is a great circle of the earth.

It is day at any place while it revolves in the illuminated part, but night while it moves in the part that is hid from the sun's rays. The diurnal motion is from west to east, and the sun rises to any place when it arrives at the boundary of light and darkness on the west-side, and sets when it arrives at the same boundary on the east. The point where a right line joining the centres of the sun and earth cuts the surface of the earth, is that which has the sun in the vertex or zenith, and is the pole or middle point of the illuminated disk. The circle described by the earth's annual motion, or the sun's apparent motion, is the *ecliptic*; and, because the axis of the earth is oblique to the plane of this circle, it cuts the equator (in an angle of $23^{\circ} 29'$), and the two points of intersection are called the equinoctial points; in which the sun appears when the axis of the earth is perpendicular to the right line drawn from its centre to the centre of the sun. Those are called the solstitial points which are at 90° distance from the former, and where the sun appears when he declines most towards the poles. The equator being a great circle, so as to be equally divided by the boundary of light and darkness, the day therefore at the equator is always equal to the night. It is obvious that when the sun appears on the north side of the equator, the northern pole must be in the illumined hemisphere; so that it must be day there from the vernal to the autumnal equinox, but that they must be deprived of the sun's light from the autumnal to the vernal equinox; and that it is the contrary at the south pole. In any place that is on the same side of the equator with that which has the sun in the zenith, a greater part of the parallel to the equator described by that place must be in the illuminated hemisphere than in the other; so that the day must be longer than the night: but it is the contrary when the place

is

is on the opposite side of the equator, and then the night must be longer than the day. In the same manner all the other phænomena of the doctrine of the sphere may be deduced from the true motions in the system.

15. We have given a summary account of what was known concerning the gravity of terrestrial bodies, before Sir *Isaac Newton*. As the figure of the earth is owing to this principle; so, as *Copernicus* very justly observed *, it is highly reasonable to suppose that by a like principle, diffused from the sun and planets, their figures are preserved in their various motions. Various attempts and schemes have been proposed, for explaining the nature of this power and its cause; but all have proved unsuccessful. *Des Cartes* deduced it from the centrifugal force of his subtle matter revolving on the axis of the earth; but this account has been already refuted †. Others considered it as a sort of magnetism; but the powers of gravity and magnetism differ widely in most essential circumstances. Others derived it from the pressure of the atmosphere; altho' the air is so far from producing gravity, that it constantly subducts from the weight of bodies. But all we want to conclude here, is, that this power extends universally to all sorts of sensible bodies, at or near the earth's surface; and that it has these two remarkable properties; first, that it is proportional to the quantity of matter in bodies; secondly, that it acts incessantly or continually, and with the same force upon a body that is already in motion as upon a body that is at rest. The last property appears from hence, that it produces equal accelerations in

* See Book I. Chap. 3. § 2.

† See Book I. Chap. 4. § 4.

falling bodies in equal times. Both these properties distinguish it from such causes as are wholly mechanical; which either act in proportion to the surface or to the bulk of bodies, and produce a less acceleration in a body that is already in motion, in the direction in which the cause acts, than upon a body at rest, in the same time. We here observe these things concerning gravity, not with a view to determine any thing concerning its cause, but only to pave the way for what follows concerning the universality of this principle.

C H A P II.

The moon is a heavy body, and gravitates towards the earth in the same manner as terrestrial bodies.

1. **S**IR *Isaac Newton* considering that the power of gravity acts equally on all matter on the surface of the earth or near it, that it is not sensibly less on the tops of the highest mountains, that it affects the air and reaches upward to the utmost limits of the atmosphere, and that it cannot be owing to the influence of any sensible terrestrial matter; he could not believe that it broke off abruptly, but was induced, on these grounds, to think it might be a more general principle, and extend to the heavens; so as to affect the moon at least, which is by much the nearest to us of all the bodies in the system. The absurdity of those who had taught that the heavenly bodies were made of some inexplicable substance, essentially different from that of our earth, had sufficiently appeared from modern discoveries: the philosophers no longer made that distinction, which had been founded on superstition and vulgar prejudices only. The earth was allowed to be of the
number

number of the planets, and the planets were considered as like our earth. To complete this resemblance, our author has shewn that they consist of the same heavy gravitating substance of which the earth is formed.

2. The effects of the power of gravity upon terrestrial bodies may be reduced to three classes : *First*, in consequence of it, a body at rest, supported by the ground, or suspended by a string or line of any kind, or that is any way kept from falling, endeavours, however, always to move ; and in such cases, its gravity is measured by the pressure of the quiescent body upon the obstacle that hinders its motion. *Secondly*, when a body descends in the vertical or plumb-line, its motion is continually accelerated, in consequence of the power of gravity's acting incessantly upon it ; or if it be projected upwards in the same right line, its motion is continually retarded, in consequence of the same power's acting incessantly upon it with a contrary direction : and, in such cases, the force of gravity is measured by the acceleration or retardation of the motion produced in a given time, by the power continued uniformly for that time : but if the body descend or ascend along an inclined plane, or move in a resisting medium, then, in measuring this power, due regard must be had to the principles of mechanics described in the preceding book. *Thirdly*, When a body is projected in any direction different from the vertical line, the direction of its motion is continually varied, and a curve line is described, in consequence of the incessant action of the power of gravity, which in such cases is measured by the flexure or curvature of the line described by it ; for the power is always the greater, *cæteris paribus*, the more it bends the way or course of the body from the tangent or direction
in

in which it was projected. Effects of the power of gravity, of each kind, fall under our constant observation, near the surface of the earth; for the same power which renders bodies heavy while they are at rest accelerates them when they descend perpendicularly, and bends their motion into a curve line when they are projected in any other direction than that of their gravity. But we have access to judge of the powers that act on the celestial bodies by the effects of the last kind only: we see bodies near the earth falling towards it; but this is a proof of the moon's gravity that cannot be had, till the present state of things comes to its dissolution. When a body is projected in the air, we do not see it fall in the perpendicular towards the earth, but we see it falling every moment from the tangent to the curve, that is, from the direction in which it would have moved if its gravity had not acted for that moment. And this proof we have of the moon's gravity: for tho' we do not see her falling directly towards the earth in a right line, yet we observe her descending every moment towards the earth from the right line which was the direction of her motion at the beginning of that moment; and this is no less evidently a proof of her being acted upon by gravity, or some power like to it, than her rectilineal descent would be, was she allowed to fall freely towards the earth.

3. If we had engines of a sufficient force, bodies might be projected from them so as not only to be carried a vast way without falling to the earth, but so as to move over a quarter of a great circle of it, or (abstracting from the effects of the air's resistance) so as to move round the whole earth without touching it, and, after returning to their first place, commence a new revolution with the same force they first received from the engine, and after that a third,
and

and thus revolve as a moon or satellite round the earth for ever. If this could be effected near the earth's surface it might be done higher in the air or even as high as the moon, could the engine, or an equivalent power, be carried up and made to act there. By increasing the force of the power, a body proportionally larger might be thus projected: and, by a power sufficiently great, a heavy body not inferior to the moon might be put in motion at first; which, being perpetually restrained by its gravity from going off in a right line, might revolve for ever about the earth. Thus Sir *Isaac Newton* saw that the curvilinear motion of the moon in her orbit, and of any projectile at the surface of the earth, were phænomena of the same kind, and might be explained from the same principle extended from the earth so as to reach the moon; and that the moon was only a greater projectile that received its motion, in the beginning of things, from the Almighty Author of the universe.

4. But to make this perfectly evident, it was necessary to shew that the powers which act on the moon, and on projectiles near the earth, and bend their motions into a curve line, were directed to the same centre, and agreed in the quantity of their force as well as in their direction. All we know of force relates to its direction or quantity, and a constant coincidence and agreement in these two respects is sufficient ground to conclude them to be the same, or similar, phænomena derived from the same, or from like causes. It was shewn in the last chapter, that the gravity of heavy bodies is directed towards the centre of the earth; and it appears from the observations of astronomers, that the power which acts on the moon, incessantly bending her motion into a curve, is directed towards the same centre: for they

find that the moon does not describe an exact circle about the earth; but an *ellipse* or oval; and that she approaches to the earth, and then recedes from it, in every revolution, but so as to have her motion accelerated while she approaches to the centre of the earth, and retarded as she recedes from it; which is an indication that she is acted on by a power directed, accurately or nearly, towards the centre.

5. That this may appear more fully, let us suppose that a body is projected in any right line, and, if no new force act upon it, then must it proceed in that line, describing equal spaces in equal times, by the first law of motion; and if you imagine a ray drawn always from the body to some fixed point, that is not in the line of its motion, while the body moves over equal spaces in equal times, that ray will describe equal triangular spaces * in equal times;

* All the reasoning here supposes only one proposition very generally known, that "triangles on the same base, or on equal bases, that have the same height, are equal to each other;" from which it easily follows, 1. That while a body by an uniform motion describes the line AF , (*Fig. 52.*) and moves over the equal parts AB , BC , in equal times, the triangles described by a ray drawn always from the body to the given point s , *viz.* ASB , $BS C$, must be equal, because their bases AB , BC are equal, and they have their common vertex in s . 2. Suppose a force to act on the body in B , directed toward s , that would carry it to E , if it acted alone upon the body, in the same time in which the body by its uniform motion would describe BC , and the body will now describe BD the diagonal of the parallelogram $BEDC$ in the same time, and the ray drawn from the body to s will describe the triangle $BS D$ equal to $BS C$, because they are on the same base BS and between the parallels BS , CD ; that is, the space described now by the ray is equal to the space that would have been described by it if no new force had acted on the body B : from which it appears, that the space described by the ray is not increased or diminished by any action of the body directed towards s , and therefore the ray drawn from the body to s will still continue to describe equal spaces in equal times, if no new force act upon it but what is directed towards s .

because

because these triangles, described by the ray in equal times, will have equal bases on the line of projection, and one common vertex in that fixed point. Suppose next that a force, directed to the same fixed point, acts upon the body, and it will now be carried out of the first line of its motion into a new direction, but the area or space described by the ray, drawn always from the body to that fixed point, will be equal to the space that would have been described by the ray in the same time if no such force had acted upon the body; for these spaces are triangles standing on the same base (*viz.* the first distance of the body from that fixed point) and between the same parallel lines. The power, therefore, directed towards the given point has no effect on the magnitude of the area or space described by the ray that is supposed to be drawn always from the body to that point; it may accelerate or retard the motion of the body, but affects not the area. Therefore the ray must still continue to describe the same spaces in equal times about the given point, as it would have done if no new force had acted on the body, but it had been permitted to proceed uniformly in the line of projection.

6. As one impulse towards the given point has no effect on the area, or space, described by the ray tending always from the body to that point, so any number of successive impulses directed to the same point can have no effect on that area, so as to accelerate or retard its description; and, if you suppose the power directed to that point to act continually, it will bend the way of the body's motion into a curve; and may accelerate or retard its velocity, but can never affect the area described in a given time by the ray supposed to be drawn always from the body to the given point; which therefore will

be always of an invariable quantity, equal to that which would have been described in the same time, if the body had proceeded uniformly in a right line, from the beginning of the motion.

7. The converse of this theorem shews, that the equable increase of the areas described by a ray, drawn always from a body to a given point, is an indication that the direction of the power that acts upon the body, and bends its way into a curve, is directed to that point. It is easy to see, that if that power was directed to either side of the point *, it would increase or diminish the area described by the ray drawn from the body to the point; so that if equal areas continue to be described about it in equal times, we may be assured that the power is directed to that point. If a body describe a circle with an equable motion, so as to move over equal arcs in equal times, the areas described in equal times by a ray drawn from the body to the centre of the circle will be equal, and it is plain that the force which bends the body into the curve must tend to that centre; for if it was directed to any other point, the body would be accelerated in its motion as it approached to that point, and retarded as it removed to a greater distance from it. We have explained this proposition at some length, because it is of the greatest consequence in this philosophy. From it we learn, that the force which retains the moon in her orbit is directed to the centre of the earth, because she describes by a ray drawn to the centre of the earth, equal spaces in equal times, being accelerated

* If a new force acted upon the body at B , that was directed to either side of s , the body, instead of being found in the line $c\ D$, would, in the same time, either pass that line or fall short of it, and the area described by the ray drawn from the body s would either be greater or less than $B\ s\ c$.

in her motion as she approaches to the earth, and retarded as she recedes from it. We shall, afterwards, see that a small inequality in these spaces only serves to confirm our author's philosophy.

8. There is, therefore, a power which acts on the moon, like to gravity, directed to the centre of the earth; and as this power makes her fall from the direction of her motion every moment towards the earth; so, if her projectile motion was destroyed, the same power would make her fall to the earth, in a direct line: and because this power acts incessantly, bending, every moment, her way into a curve, it therefore would make her descend to the earth with an accelerated motion, like that of heavy bodies in their fall. It remains only to shew, that the power which acts on the moon agrees with gravity in the quantity of its force, as well as in all other respects. But, before we compare them in this particular, we are to observe, that the power which acts upon the moon, is not the same at all distances from the earth, but is always greater when she is nearer to the earth. To be satisfied of this, it is only necessary to see that to bend the motion of a body into a curve, when it moves with a greater velocity, requires the action of a greater power than when it describes the same curve with a less velocity. This is obvious enough, but may appear more fully thus: imagine a tangent (*Fig. 53.*) drawn at the beginning of a small arc described by the body, and as this is the line which the body would have followed if no new power had acted upon it, the effect of that power is estimated by the depression of the other extremity of the arc under that tangent: now it is plain, that in arcs of the same curvature or flexure, the greater the arc is, the farther must one extremity of it fall below the tangent drawn at the other extremity;

tremity; and consequently when a body describes a greater arc, it must be acted on by a greater power than when it describes a lesser arc in the same time. Now as the moon approaches to the earth, her motion is accelerated, is swiftest at her least distance, and slowest at her greatest distance, and the arcs which she describes at her greatest and least distance have the same curvature, therefore the force which acts upon her at her least distance, when her motion is swifter, must be the greater force.

9. It will not be difficult to see according to what law this power varies, at her greatest and least distances from the earth. That it may appear more easily, let us assume a simple case, and suppose that her least distance is the half of her greatest distance. If this was true, the moon would move with a double velocity in her least distance, that the area described there by a ray from her to the earth might be equal to the area described by such a ray in the same time, at her greatest distance; so that she would describe at her least distance an arc, in one minute, equal to the arc she would describe in two minutes at her greatest distance; and would fall as much below the tangent at the beginning of the arc, in one minute in the lower part of her orbit, or the *perigæum*, as in two minutes in the higher part of it, or her *apogæum*. If therefore her projectile motion was destroyed at her least distance, she would fall towards the earth as much in one minute, as in two minutes if her projectile motion was destroyed at her greatest distance. But the spaces described by a heavy body in its descent are as the squares of the times, by *Book II. Chap. 1. § 11*; and such a body descends thro' a quadruple space in a double time; so that the moon descending freely at her greatest distance, would necessarily fall four times as far in two minutes

minutes as in one minute. Therefore she would fall thro' four times as much space, in one minute, at her least distance, as at her greatest distance in the same time. But the forces with which heavy bodies descend, are in the same proportion as the spaces described, in consequence of those forces, in equal small parts of time; consequently the power that acts at the least distance is quadruple of that which acts at the greater distance, when the latter is supposed to be double of the former; or the forces are as 4 to 1, when the distances are as 1 to 2. We find, therefore, that the force which acts upon the moon, and bends her course into a curvilinear orbit, increases as the distance from the centre of the earth decreases, so as to be quadruple at half the distance. In the same manner it is shewn, that if her least distance was the third part only of her greatest distance, her velocity would be triple at the least distance, to preserve the equability of the areas described by a ray drawn from her to the centre of the earth; and that she would be acted upon by a power which would have the same effect there in one minute, as in three minutes at her greatest distance; so that if she was allowed to descend freely from each distance, she would fall nine times as far from the least distance as from the greatest, in the same time; consequently, the power itself which causes her descent, would be nine times greater at the third part of the distance; or the distances being as 1 to 3, the force of gravity at those distances would be as 9 to 1, that is, inversely as the squares of the distances. In the same manner, it appears that when the greatest and least distances are supposed to be in any proportion of a greater to a lesser number, the velocities of the revolving planet are in the inverse ratio of the same numbers; and that the powers,

S 4

which

which bend its motion into a curve, are in the inverse ratio of the squares of those numbers.

10. In general, let T (*Fig. 53.*) represent the centre of the earth, $A L P$ the moon's elliptical orbit, A the *apogæum*, P the *perigæum*, $A H$ and $P K$ the tangents at those points, $A M$ and $P N$ any small arcs described by the moon in equal times, at those distances; $M H$, $N K$, the subtenses of the angles of contact, terminated by the tangents in H and K : then $M H$ and $N K$ will be equal to the spaces which would be described by the moon, if allowed to fall freely from the respective places A and P , in equal times; and will be in the same proportion to each other, as the powers which act upon the moon, and inflect her course, at those places. Let $A m$ be taken equal to $P N$, and $m b$, parallel to $A P$, meet the tangent at A in b ; then, because the curvature of the ellipse is the same at A as at P , $m b$ is equal to $K N$; and, if the moon was to fall freely, from the places P and A , towards the earth, her gravity would have a greater effect at P than at A , in equal times, in proportion as $m b$ is greater than $M H$. But $m b$ is the space which the moon would describe freely by her gravity at A , in the time in which $A b$ would be described by her projectile motion at A ; and $M H$ is the space thro' which she would descend freely by her gravity at A , in the time in which $A H$ would be described by her projectile motion; and those spaces being as the squares of the times, it follows that $m b$ is to $M H$, as the square of $A b$ to the square of $A H$, or (because of the equality of the areas $T A H$, $T P K$) as the square of $T P$ to the square of $T A$. Therefore the gravity at P is to the gravity at A , as the square of $T A$ to the square of $T P$; that is, the gravity of the moon towards the earth increases in the same proportion as the square of the distance from the

the centre of the earth decreases. Sir *Isaac Newton* shews the universality of this law, in all her distances, from the direction of the power that acts upon her, and from the nature of the *ellipsis*, the line which she describes in her revolution; and it follows from the properties of this curve, that, if you take small arcs described by the moon in equal times, the space by which the extremity of any arc descends towards the earth below its tangent at the other extremity, is always greater in proportion as the square of the distance from the focus is less: from which it follows that the power which is proportional to this space observes the same proportion.

II. The moon's orbit, according to the observations of astronomers, differs not much from a circle of a radius equal to sixty times the semi-diameter of the earth; and the circumference of her orbit, is, therefore, about sixty times the circumference of a great circle of the earth; which, by the *French* mathematicians, was found to be 123249600 *Parisian* feet. The circumference of the moon's orbit is easily computed from this; and, since she finishes her revolution in 27 days, 7 hours and 43 minutes, it is easy to calculate what arc she describes in one minute. Now, to compute by what space one end of this arc falls below a tangent drawn at the other end, we learn from geometry that this space is nearly a third proportional to the diameter of her orbit and the arc she describes in a minute; and by an easy calculation this space is found to be $15\frac{1}{2}$ *Parisian* feet. This space is described in consequence of her gravity towards the earth, which, therefore, is a power, that, at the distance of sixty semi-diameters of the earth, is able to make her descend in one minute

nute through $15\frac{1}{2}$ *Parisian* feet. This power increases as she approaches to the earth: in order to see what its force would be at the surface of the earth, let us suppose her to descend so low in her orbit, at her least distance, to pass by the surface of the earth. She would then come sixty times nearer to the centre of the earth, and move with a velocity sixty times greater, that the areas, described by a line drawn from her to that centre in equal times, might still continue equal. The moon therefore passing by the surface of the earth, at her lowest distance, would describe an arc in one second of time (which is the sixtieth part of a minute) equal to that which she describes in a minute at her present mean distance, and would fall as much below the tangent at the beginning of that arc in a second, as she falls from the tangent at her mean distance in a minute; that is, she would fall near the surface of the earth $15\frac{1}{2}$ *Parisian* feet in one second of time. Now this is exactly the same space through which all heavy bodies are found by experience to descend by their gravity, near the surface of the earth, as we observed above. The moon, therefore, would descend at the surface of the earth with the same velocity, and every way in the same manner, as heavy bodies fall towards the earth; and the power which acts upon the moon, agreeing in direction and force with the gravity of heavy bodies, and acting incessantly every moment, as their gravity does, they must be of the same kind, and proceed from the same cause.

12. The computation may be made also after this manner: the mean distance of the moon from the earth being sixty times the distance of heavy bodies at the surface from its centre, and her gravity increasing in proportion as the square of her distance from

from the centre of the earth decreases, her gravity would be 60×60 times greater near the surface of the earth than at her present mean distance, and therefore would carry her through $60 \times 60 \times 15\frac{1}{2}$ *Parisian* feet in a minute near the surface: but the same power would carry her through 60×60 times less space in a second than in a minute, by what has been often observed of the descent of heavy bodies; and, therefore, the moon in a second of time would fall by her gravity near the surface of the earth $15\frac{1}{2}$ *Parisian* feet; which therefore is the same with the gravity of terrestrial bodies.

13. Thus Sir *Isaac Newton* shewed that the power of gravity is extended to the moon; that she is heavy, as all bodies belonging to the earth are found by perpetual experience to be; and that the moon is retained in her orbit from the same cause, in consequence of which a stone, bullet, or any other projectile, describes a curve in the air. If the moon, or any part of her, was brought down to the earth, and projected in the same line and with the same velocity as a terrestrial body, it would move in the same curve; and if any body was carried from our earth to the distance of the moon, and was projected in the same direction and with the same velocity with which the moon is moved, it would proceed in the same orbit which the moon describes, with the same velocity. Thus the moon is a projectile, and the motion of every projectile gives an image of the motion of a satellite or moon. These phænomena are so coincident, that it is manifest they must flow from the same cause.

C H A P. III.

Of the solar system ; and the parallaxes of the planets and fixed stars.

I. **H**A V I N G shewed that gravity is extended from the surface of the earth to the moon, and to all distances upwards, decreasing in a regular course as the squares of those distances increase, our author did not stop here : as any considerable discovery in nature generally opens a new scene, so valuable a one as this could not be barren in Sir *Isaac Newton's* hands. The gravity of the moon suggested to him the universal gravitation of matter ; and so successful an account of her motion led him to explain all the curvilinear motions in the solar system, from the same principle. The earth cannot be considered as the centre of the motions of any body in the system but of the moon only, with which she forms one of those lesser systems of which the vast solar system consists. The inferior planets, *Mercury* and *Venus*, do not so much as include the earth within their orbits, but manifestly revolve round the sun ; for sometimes they are farther distant from us than the sun, and at other times pass between him and us, but never are seen opposite to the sun, or appear removed from him beyond a certain arc, which is called their *greatest elongation*. The higher planets, *Mars*, *Jupiter* and *Saturn*, move in orbits which include the earth indeed ; but it appears from their motions, which viewed from the earth are subject to many irregularities, that the earth is not to be considered as the centre of their orbits. Sometimes they appear to *proceed* in these orbits from west to east, sometimes they seem *stationary* or

or without motion, and at other times they appear *retrograde*, or to go backwards from east to west: and these irregularities, tho' different in the different planets, are exactly such, in all of them, as should appear to us in consequence of the motion of the earth in her orbit.

2. The motions of all the planets about the sun are constant and regular. They all move round him from west to east, almost in the same plane, in elliptic orbits that have the sun in one of the *foci*, but of which some approach very near to circles. *Mercury* possesses the lowest place; where moving with the greatest velocity of them all, and in the least orbit, he finishes his revolution in two months and 28 days. The planet *Venus*, which is called by us sometimes the evening star, sometimes the morning star, according as it appears to us eastward or westward from the sun, and consequently sets later or rises earlier, is next to *Mercury* in the system, and revolves in about seven months and 15 days. Above these next in order revolves the *earth*, with her satellite the *moon*, in the space of a year. *Mars* is above the earth, and is the first which includes the earth, as well as the sun, in his orbit; which he describes in one year, ten months and 22 days. Higher in the system and at a great distance *Jupiter* revolves, with his four satellites, in eleven years, ten months and 15 days. Last of all, *Saturn*, with five satellites, and a ring peculiar to him, moves in a vast orb with the slowest motion, and finishes his period in 29 years, 5 months and 27 days.

3. Suppose the earth's mean distance from the sun to be divided into 100 equal parts, then the mean distances of *Mercury*, *Venus*, *Mars*, *Jupiter* and *Saturn*, from the sun, shall consist of nearly 38,

72, 152, 520 and 954 such parts, respectively. Or if they be required with greater exactness, let the earth's mean distance be represented by 100000, and the distances of those several planets shall be represented by the numbers 38710, 72333, 152369, 520096, 954006, respectively.

The distances of *Mercury* and *Venus* are determined by their greatest elongations from the sun. Let *s* (*Fig. 54.*) represent the sun, *t* the earth, and supposing *A v B* the orbit of *Venus* to be perfectly circular, draw *t v* a tangent; then shall *v* represent the place of *Venus* where her elongation from the sun is greatest, and the triangle *s v t* being right angled at *v*, it follows that *s t*, the distance of the earth from the sun, is to *s v*, the distance of *Venus* from the sun, as the radius to the sine of the angle *s t v* the greatest elongation of *Venus* from the sun. In this manner, the distances of the inferior planets are compared with the distance of the earth from the sun. The distances of the superior planets are determined from their retrogradations, and, in such as have satellites, by the eclipses of those satellites. For example, let *i* (*Fig. 55.*) represent the planet *Jupiter*, and if the right line *s i*, joining the centres of the sun and *Jupiter*, be produced to *m*, then shall *i m* be the axis of his shadow, the position of which is determined by the eclipses of the satellites, and shews the *heliocentric* place of *Jupiter*, *i. e.* his place viewed from the sun. Produce the line *t i*, which joins the centres of the *Earth* and *Jupiter*, to *n*, and *n* shall represent the *geocentric* place of *Jupiter*, *i. e.* his place when viewed from the earth. The difference of those places gives the angle *n i m* or *t i s*; the angle *i t s*, the elongation of *Jupiter* from the sun as seen from the earth at *t*, is easily found by observation; consequently all the angles of the

the triangle $\tau i s$ are known, with the proportion of its sides, which is the same as of the sines of those angles; and thus the proportion of $s i$, the distance of *Jupiter* from the sun, to $s \tau$, the distance of the earth from the sun is discovered. The angle $\tau i s$ is that under which $s \tau$ the semi-diameter of the earth's orbit would appear if viewed from i , or the elongation of the earth from the sun as it would appear to a spectator at *Jupiter*.

4. In the first chapter of this book, we explained at length how the distances of the celestial bodies are discovered by what is called the diurnal parallax, that is, the angle under which the semi-diameter of the earth would appear at those distances. By this method the distance of the moon from the earth is compared with its semi-diameter. When *Venus* and *Mars* are at their least distances from the earth, it is of use likewise for estimating those distances. But in most other cases, the distances of the celestial bodies are so great, and the semi-diameter of the earth bears so small a proportion to them, that the angle under which it would appear, viewed at so great distances, cannot be discovered by our instruments, with any tolerable accuracy. Therefore astronomers have been obliged to have recourse to other inventions. The method proposed by *Aristarchus* for determining the distance of the sun, by observing the time when the moon's disk appears to be half illuminated by the sun, may be considered as an attempt to substitute the semi-diameter of the moon's orbit in place of the semi-diameter of the earth. Let s and τ (*Fig. 56.*) represent the sun and earth, L the moon's place when τL is perpendicular to $s L$, at which time her disk ought to appear to us to be bisected by the boundary of light and darkness upon her surface; and it is manifest that τs , the distance
of

of the earth from the sun, is then to TL , the distance of the moon from the earth, as the radius to the sine of the angle LST , the complement of the angle STL the elongation of the moon from the sun at that time. But this method, though very ingenious, has proved unsuccessful; astronomers finding it impracticable to determine the time of this bisection of the lunar disk with sufficient exactness for this purpose. We learn from it, however, that the distance of the sun is vastly greater than that of the moon; for it is obvious, that the nearer the angle STL approaches to a right one, the greater must the distance ST be in proportion to TL ; and that if this distance ST was infinite, then STL would be a right angle. Now astronomers find it very difficult to discover any difference between the angle STL and a right angle, or between the time when the lunar disk appears to be bisected and the quadrature; from which it follows that ST is vastly greater than TL .

5. Astronomers finding the diurnal parallax of no use for determining or comparing the greater distances in the celestial spaces, the semi-diameter of the earth being too small a base for this purpose, have had recourse to what they call the annual parallax. In place, therefore, of the semi-diameter of the earth, they substituted the semi-diameter of the orbit described by the earth annually about the sun; or, in place of two stations or spectators, one of which was supposed to be at the surface and the other at the centre of the earth, they substituted one at the earth and another at the sun. In this manner they obtained a base that bears a considerable proportion to any distances within the solar system, and with which they were able to compare them by accurate observations. As, in the former case, they compared the distances in the heavens with the semi-diameter of

the earth, by finding under what angle it would appear at those distances; so in this case, they compare the vast distances of the planets from the sun with the semi-diameter of the earth's orbit, by finding under what angle this semi-diameter appears at those distances. This angle is greater at the distance of *Mars* than at that of *Jupiter*, and is greater there than at the distance of *Saturn*; decreasing always with the distance, till at length it become too small to be discernible by the exactest instruments we have. Let *i* (*Fig. 55.*) represent any remote object in the system, *A* the point where the earth passes betwixt the sun *s* and that object *i*, *i T* a tangent from the point *i* to the earth's orbit, supposed to be circular: and when the earth is at *A*, the object *i* will appear in the same place to the earth and sun; but when the earth comes to *T*, if we suppose *i* to have no motion, it will appear to the earth in the right line *T i*, and will appear to have gone backward by the arc that measures the angle *T i s*, the same which the semi-diameter of the earth's orbit *s T* subtends at *i*; and this angle being determined by observation, its sine will be to the radius, as *s T* to *s i*; that is, as the distance of the earth from the sun to the distance of the object *i* from the sun; which proportion, therefore, is easily computed by trigonometry. When the object *i* has a proper motion, an allowance must be made for this motion, after it is determined by observation.

The appearances, in this case, may be explained in the following manner. Let *s i* produced meet the sphere in which the fixed stars are apparently disposed in *M*, let the two tangents *T i* and *t i* meet the same in *N* and *n*, and supposing the object *i* to vibrate continually between *N* and *n* like a pendulum, imagine this arc *N n* itself to be carried along
T
the

the arc DME with the proper motion and direction of the object i . If i represent a planet, the arc Nn which measures the angle Nin or Tit , will shew how much the planet is retrograde, the half of which angle is sIT ; which being known, the proportion of sI to sT is computed as above.

6. We ascribe the annual motion to the earth and not to the sun, according to the *Pythagorean* system revived by *Copernicus*, for many reasons; some of which were briefly mentioned in § 1. and 2. By comparing the periodic times of the primary planets and their distances from the sun, and by comparing the periodic times of the satellites that revolve about *Jupiter* and *Saturn* with their respective distances from their primary planets, it appears to be a general law in the solar system, that when several bodies revolve about one centre, the squares of the periodic times increase in the same proportion as the cubes of the distance from that centre; that is, the periodic times increase in a higher proportion than the distances, and not in so high a proportion as the squares of those distances, but accurately as the power of the distance whose exponent is $1\frac{1}{2}$, or as the number which is a mean proportional between those numbers that represent the distance and its square. The earth is the centre of the motion of the moon, in all the systems. If the sun likewise revolved round the earth, we should expect that the same general law would take place in their periodic times and distances compared together; or that the square of 27 days, 7^h, 43' would be to the square of 365 days, 6, 9, as the cube of the moon's distance from the earth to the cube of the sun's distance from the same: from which it is easy to compute that the sun's distance ought to be little more than $5\frac{3}{5}$ times greater than the moon's distance; whereas it is evident,

4

from

Fig. 42.

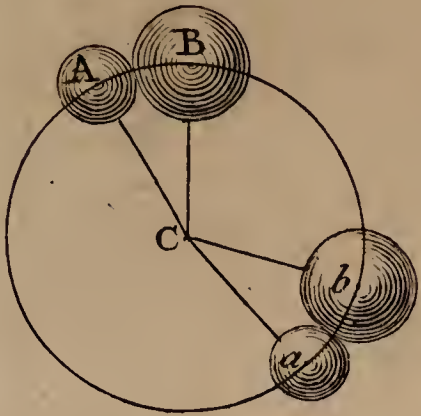


Fig. 43.

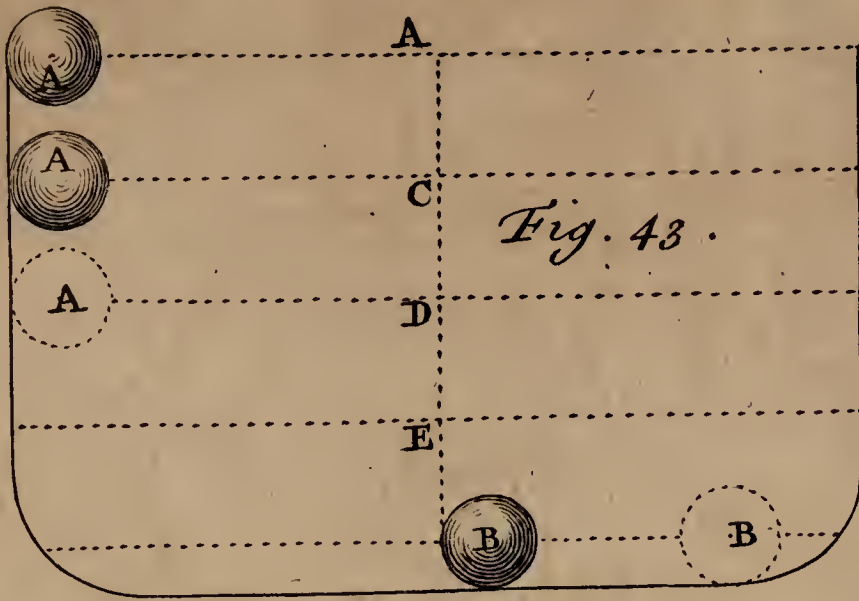


Fig. 44.

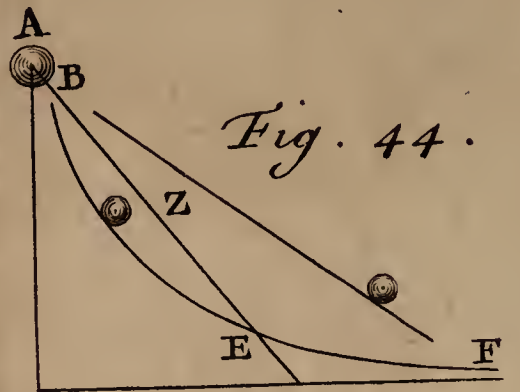


Fig. 45.

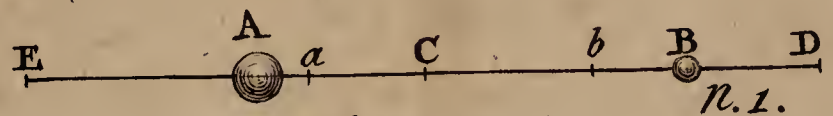
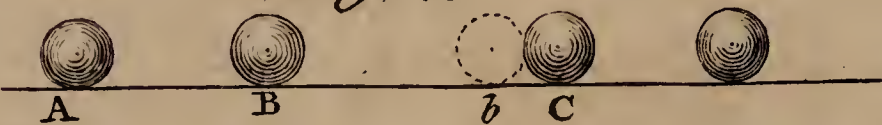
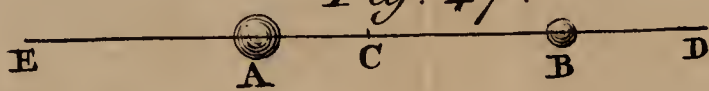


Fig. 48.

Fig. 47.



n.2.

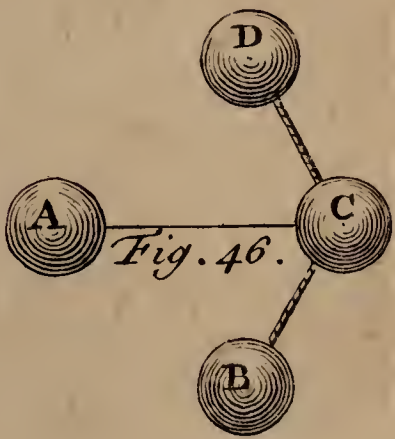
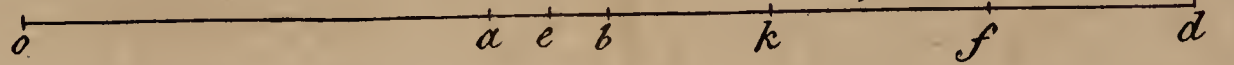


Fig. 46.

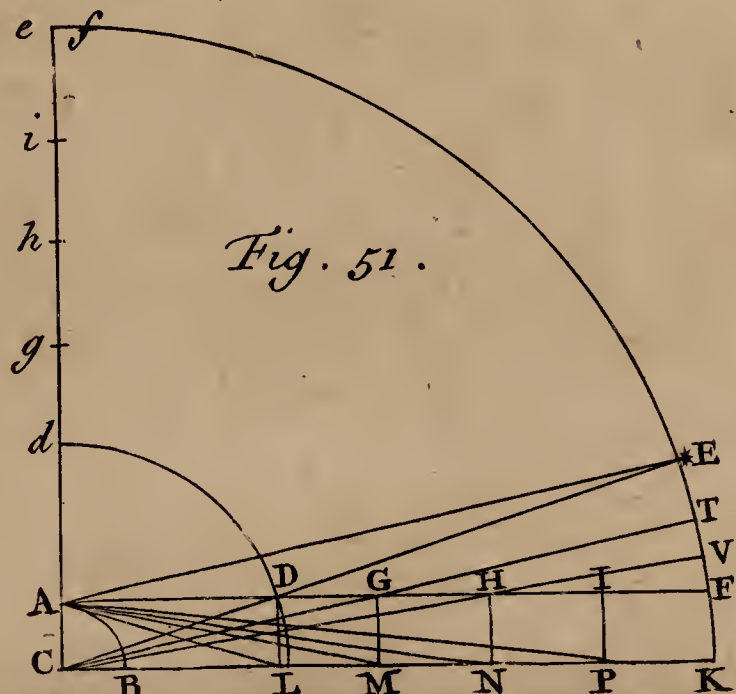


Fig. 51.

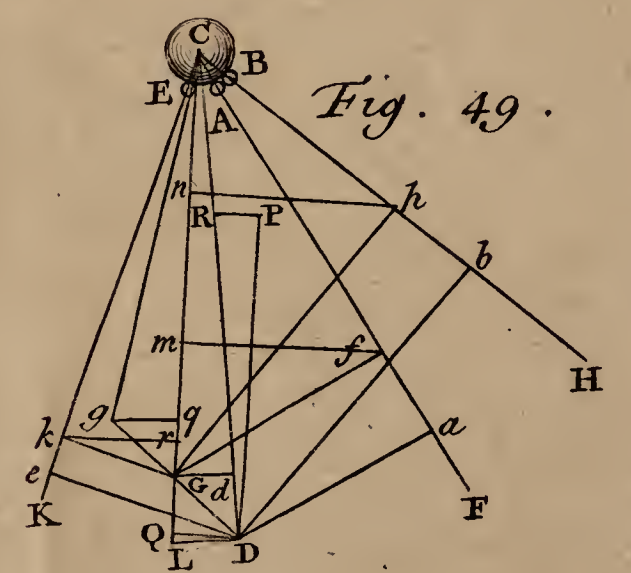


Fig. 49.

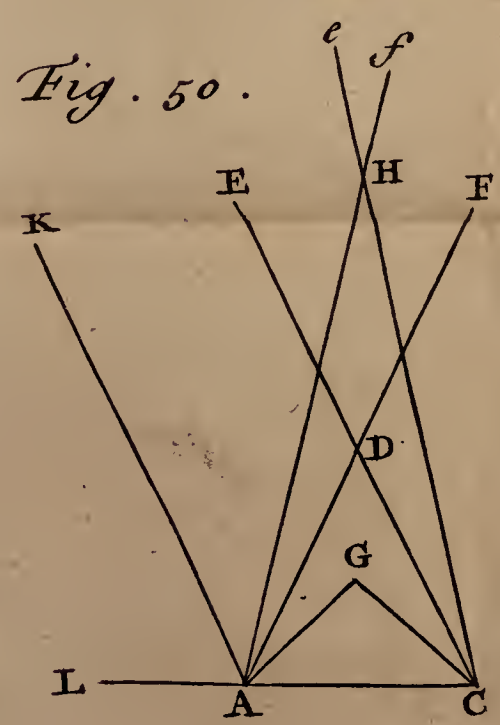


Fig. 50.

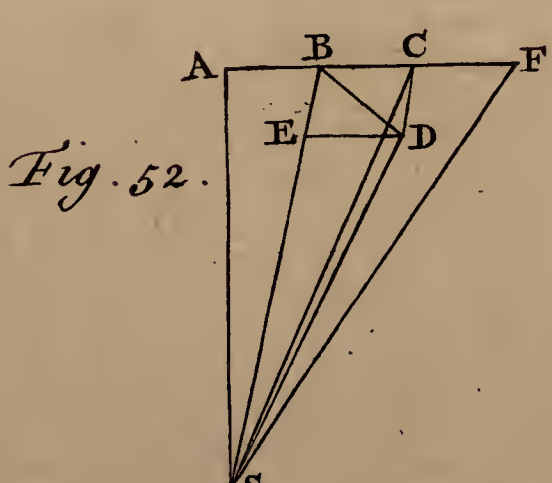


Fig. 52.

Fig. 53.

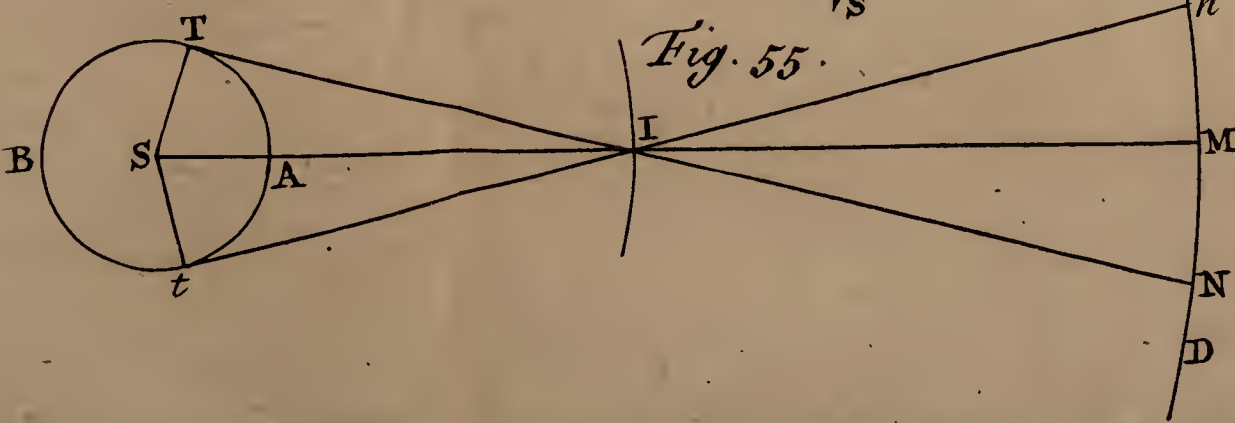
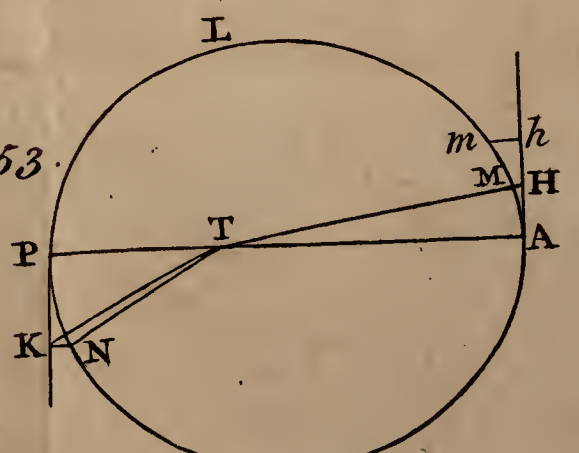


Fig. 55.

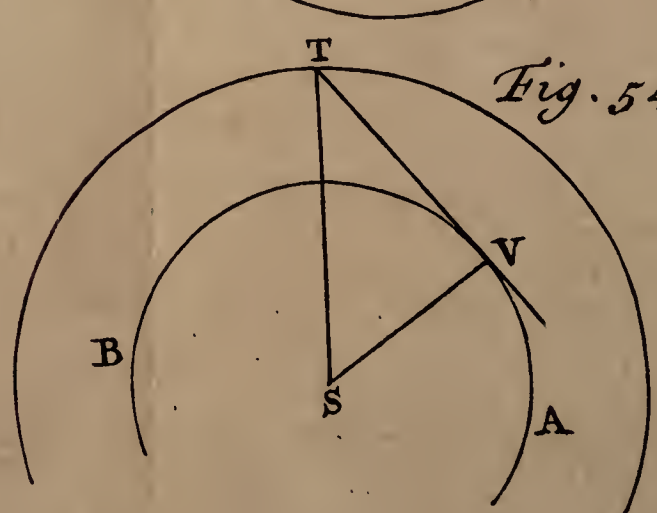


Fig. 54.

from the minuteness of the sun's diurnal parallax, that the sun's distance is some hundred times greater than the moon's distance from the earth. But if, with *Copernicus*, we suppose the *earth* to revolve about the sun in an orbit placed betwixt those of *Venus* and *Mars*, this law will be found to obtain between the periodic times and distances of the earth and any of the planets from the sun compared together; and the harmony of the system will appear complete. The retrogradations and stations of the planets, and the many apparent irregularities in their motions and distances from the earth, furnish us with so many arguments against the *Ptolemaic* system, according to which those appearances are explained by a number of perplexed solid orbs and epicycles, in a manner unworthy of the noble simplicity and beauty of nature. It is likewise to be remarked, that those inequalities are different in the different planets, but in each of them are such as ought to arise from the annual motion of the earth. The arguments derived from the magnitude of the sun, and its great usefulness to all the bodies in the system, which seem to entitle it to the most centric place, are too obvious to require our insisting on them. The earth and planets revolve about the sun, in order to enjoy the benefits of his light and heat; but no reason appears why the sun and planets should revolve around the earth.

7. There is but one argument against the annual motion of the earth that deserves any notice, *viz.* The want of an annual parallax in the fixed stars. Let $T A t$ (*Fig. 57.*) represent the earth's orbit about the sun s , $T x$ the axis of the earth, and $t x$, parallel to $T x$, shall represent the position of the same axis at the opposite point t . Suppose $T x$ to be directed

rected towards the star P ; and it is manifest that the axis of the earth will not be directed to the same star when it comes to the situation t , but will contain an angle $\angle t P$ with the line $t P$ joining the earth and star, equal to the angle $\angle t P T$, under which the diameter $T t$ of the earth's orbit appears to a spectator, viewed from the star P . It might be expected, therefore, that by observing the fixed star P from the different parts of the earth's orbit T, t . (which may be considered as two stations in this problem, the most sublime of all that can be brought into practical geometry,) we ought to be able to judge, from its different appearances at those stations, of the angle $\angle T P t$, and consequently of the proportion of $T P$, the distance of the star, to $T t$, the diameter of the earth's orbit, or double distance of the sun. Yet it is certain that astronomers, hitherto, have not been able to discover any difference in the apparent situations of the fixed stars, with respect to the axis of the earth or to one another, that can arise from the motion of the earth; though, since the restoration of the *Pythagorean* doctrine, they have taken great pains to examine this matter. In answer to this objection, it is observed, that the distance of the fixed stars is so very great, that the diameter of the earth's orbit bears no sensible proportion to it; so that the angle $\angle T P t$ is not to be discovered by our exactest instruments. Nor is this immense distance of the fixed stars advanced by the *Copernicans* as an hypothesis, merely for the sake of solving this objection; for, as they had reason to suppose the fixed stars like to our sun, they had ground to conclude their distance to be vastly great, since they appear to us with so faint a light, and of no sensible diameter, even in the largest telescopes. If we should suppose the distance between us and a fixed star to be divided into 300 equal parts, and a spectator, after passing
over

over 299 of those parts, should view it from the last division, or at $\frac{1}{300}$ th part of the whole distance, the star, indeed, would appear brighter to him, but not sensibly magnified in diameter; because it would appear of the same magnitude to him at that distance, as it was in a telescope that magnified 300 times. The immense distance of the fixed stars likewise appears from hence, that when the moon or any other planet covers them from us, this is done in an instant; they disappear at once, and not gradually as the more remote planets when covered by the nearer ones. If we join these observations together, they will rather appear to confirm one another and the motion of the earth, than to make against it. The immense distance of the fixed stars, that arises from them jointly, rather strengthens the evidence of the *Copernican* system; because the more remote the stars are, the more absurd it must appear to suppose so immense a space to revolve about our earth, so inconsiderable a point! that to our neighbouring planets it is seen but as a small spark of light; to others of them is hardly known; and to some of the fixed stars, neither it nor the whole solar system to which it belongs is visible. How can it be imagined that those immense bodies, sunk so deep in the abyss of space, describe daily such vast rounds about so mean a centre; especially if it be considered that it is highly probable some of the fixed stars are immensely farther distant than others, and that all the system of the fixed stars, visible to the naked eye in a clear night, form but a small corner of the universal system?

8. But this is not all we learn from the diligence and accuracy of late astronomers, in confirmation of the motion of the earth about the sun, and that serves to resolve this the only material objection

against it. An instrument was contrived by the famous Mr. *Graham* (for a description of which we refer the reader to Dr. *Smith's* excellent treatise of *Optics*) and executed with surprizing exactness, which being placed in the vertical line, a star in the constellation *Draco* that passed near the zenith was observed by this instrument for a number of years, with a view to discover its parallax, by Messrs. *Molyneux*, *Bradley*, and *Graham*. They soon discovered that the star did not appear always in the same place in the instrument, but that its distance from the zenith varied, and that the difference of its apparent places amounted to 21 or 22 seconds. This star is near the pole of the ecliptic. They made similar observations on other stars, and found a like apparent motion in them; proportional to the latitude of the star. This motion was by no means such as was to have been expected as the effect of a parallax; and it was some time before they discovered any way of accounting for this new phænomenon: but at length Mr. *Bradley* resolved all its variety in a satisfactory manner, by the motion of light and the motion of the earth compounded together.

Let $A D$ (*Fig. 58.*) represent a small portion of the earth's orbit, $c D$ a ray of light moving from the star with the direction $c D$; and if the earth was at rest, the telescope would be directed to the star, by placing it in the right line $A E$ parallel to $D C$. Let $A D$ be to $D C$, as the velocity of the earth in its orbit to the velocity of light, and it is manifest that the telescope must now be placed in the situation $A C$, that the ray of light may run along its axis, and, after entering the middle of the object glass at c , may issue at the middle of the eye-glass at A ; because, while the ray describes the right line $c D$, the point A is carried forwards to D , and the telescope
by

by moving parallel to itself is carried into the situation Dc . But the apparent place of the star is determined by the position of the telescope, and consequently the star will appear in the right line Ac , and not in its true situation Ae . Thus a star in the pole of the ecliptic will appear to have its latitude diminished by the angle EAc or ACD ; which will be found to exceed 20 seconds, if the velocity of light be to the velocity of the earth as 8000 to 1: and this star will in appearance describe a small circle round the pole of the ecliptic at a distance from it of about 20". In other cases, the star will appear to describe a small ellipsis having its centre in the true place of the star, (*i. e.* the place where it would appear if the earth was at rest) its transverse axis parallel to the ecliptic, and its second axis perpendicular to it; the former of which gives its greatest aberration in longitude, and the latter its greatest aberration in latitude. If the star be in the plane of the ecliptic, the aberration then is only in longitude. In this case, if the rays from the star touch the earth's orbit in G and H , and be perpendicular to it in A and B , the motion of the earth, at G and H , being in the direction of the ray, the star will appear in its true place, and there will be no aberration at those points; but the aberration in longitude will be greatest at A and B . He has explained all the appearances of the stars observed by Mr. *Molyneux* and himself, in this manner; and tho' he has not discovered any parallax by these observations, he has produced from them a new argument for the motion of the earth, by a series of observations made on different stars in different places. He finds ground to conclude from these, that the parallax of the fixed stars can hardly exceed one second; from which their distance ought to be 400,000 greater than the distance of the sun.

The true motions in the system being established, we may now proceed safely with our *analysis*.

9. Each of the primary planets bend their way about the centre of the sun, and are accelerated in their motion as they approach to him, and retarded as they recede from him; so that a ray drawn from any one of them to the sun always describes equal spaces, or areas, in equal times: from which it follows, as in *Chap. 2.* § 5, 6, 7. that the power which bends their way into a curve line must be directed to the sun. This power always varies in the same manner as the gravity of the moon towards the earth. The same reasoning by which the gravity of the moon towards the earth at her greatest and least distances were compared together, in *Chap. 2.* § 8, 9, 10. may be applied in comparing the powers which act on any primary planet, as its greatest and least distances from the sun; and it will appear, that these powers increase as the square of the distance from the sun decreases. Our author shews this generally, from the nature of the elliptic curve in which each planet moves.

10. But the universality of this law, and the uniformity of nature, still farther appears by comparing the motions of the different planets. The power which acts on a planet that is nearer the sun is manifestly greater than that which acts on a planet more remote; both because it moves with more velocity, and because it moves in a lesser orbit, which has more curvature, and separates farther from its tangent, in arcs of the same length, than a greater orbit. By comparing the motions of the planets, it is found that the velocity of a nearer planet is greater than the velocity of one more remote, in proportion as the square root of the number which expresses the
greater

greater distance to the square root of that which expresses the lesser distance ; so that if one planet was four times farther from the sun than another planet, the velocity of the first would be half the velocity of the latter, and the nearer planet would describe an arc in one minute, equal to the arc described by the higher planet in two minutes : and tho' the curvature of the orbits was the same, the nearer planet would fall by its gravity as much in one minute as the other would fall in two, and therefore the nearer planet would describe by its gravity four times as much space as the other would describe in the same time, by the law of motion of falling bodies so often mentioned ; the gravity of the nearer planet would therefore appear to be quadruple, from the consideration of its greater velocity only. But besides, as the radius of the lesser orbit is supposed to be four times less than the radius of the other, the lesser orbit must be four times more curve, and the extremity of a small arc of the same length will be four times farther below the tangent drawn at the other extremity in the lesser orbit than in the greater ; so that, tho' the velocities were equal, the gravity of the nearer planet would, on this account only, be found to be quadruple. On both these accounts together, the greater velocity of the nearer planet, and the greater curvature of its orbit, its gravity towards the sun must be supposed sixteen times greater, tho' its distance from the sun is only four times less than that of the other ; that is, when the distances are as 1 to 4, the gravities are reciprocally as the squares of these numbers, or as 16 to 1. In the same manner, by comparing the motions of all the planets, it is found that their gravities decrease as the squares of their distances from the sun increase.

11. Thus, by comparing the motions of any one planet in the different parts of its elliptic orbit, and the motions of the different planets in their different orbits, it appears that there is a power like the gravity of heavy bodies so well known to us on the earth, extending from the sun to all distances, and constantly decreasing as the squares of these distances increase. If any one planet descended to the distance of another, it would be acted on in the same manner, and by the same power, as that other: and as gravity preserves the substance of the earth together, and hinders its looser parts from being dissipated by its various motions; so a like power, acting at the surface of the sun, and within its body, keeps its parts together and preserves its figure, notwithstanding its rotation on its axis.

12. In the same manner as this principle governs the motions of the planets in the great solar system, it governs also the motions of the satellites in the lesser systems of which the greater is composed. There is the same harmony in their motions compared with their distances, as in the great system: we see *Jupiter's* satellites bending their way round him and falling every moment from the lines that are the directions of their motions, or the tangents of their orbits, towards him; each describing equal areas in equal times by a ray drawn to his centre, to which their gravity is therefore directed. The nearer satellites move with greater celerity, in the same proportion as the nearer primary planets move more swiftly round the sun, and their gravity, therefore varies according to the same law. The same is to be said of *Saturn's* satellites. There is, therefore, a power that preserves the substance of these planets in their various motions, acts at their surfaces,

faces, and is extended around them, decreasing in the same manner as that which is extended from the earth and sun to all distances.

13. These secondary planets must also gravitate towards the sun. It is impossible they should move so regularly round their respective primaries, if they were not acted on by the same powers. If we suppose them to be acted on by the same accelerating power in parallel lines, there will no disorder or perplexity arise from thence ; for they will then accompany their primary planets in their motions round the sun, and move about them at the same time, with the same regularity as if their primary planets were at rest. It will be as in a ship, or in any space carried uniformly forward : in which the mutual actions of bodies are the same as if the space was at rest, being no way affected by that motion which is common to all the bodies. As every projectile, while it moves in the air, gravitates towards the sun, and is carried along with the earth about the sun, while its own motion in its curve is as regular as if the earth was at rest ; so the moon, which we have shewed to be only a greater projectile, must gravitate toward the sun, and, while it is carried along with the earth about the sun, is not hindered by that motion from performing its monthly revolutions round the earth. *Jupiter's* satellites gravitate toward the sun as every part of *Jupiter's* body, and *Saturn's* satellites gravitate toward the sun as if they were parts of *Saturn*. Thus the motions in the great solar system, and in the lesser particular systems of each planet, are consistent with each other, and are carried on with a regular harmony without any confusion, or mutually interfering with one another, but what necessarily arises from small inequalities in the gravities of primary and secondary planets, and
the

the want of exact parallelism in the directions of those gravities ; of which we are to treat afterwards.

14. Nor is there any body that comes, tho' rarely and as a stranger, into the lower parts of our system, exempted from this universal gravitation toward the sun. When a *comet* appears, we see the effect of the same power acting on it ; since it descends with an accelerated motion as it approaches the sun, and ascends with a retarded motion, bending its way about the sun, and describing equal areas in equal times by a ray drawn from it to his centre. This power that acts on the comets varies according to the same law as the gravity of the planets, as appears from their describing either *parabolas* *, or very eccentric *ellipses* having one of their *foci* in the centre of the sun : our author having demonstrated, that the power which makes a body describe a parabola about its focus, must likewise vary according to the law so often mentioned. If a body was projected from our earth in a line perpendicular to the horizon, with a certain force, (*viz.* that which would carry it over about 420 miles with an uniform motion in a minute), it would rise in that line for ever and return to the earth no more. Its gravity would, indeed, retard its motion continually, but never be able to exhaust it, the force of gravity upon it decreasing as it rises to a greater height. If the body was projected with the same force in any other direction, it would go off in a *parabola* having its *focus* in the centre of the earth, and never return to the earth again. A force a little less would make it move in a very eccentric *ellipse*, in which it would return after a long period to its first place ; if it was

* *Princip.* Lib. III. Prop. 40.

not diverted in its course by approaching too near to some celestial body. In the same manner, a planet projected with a certain force would go off for ever in a parabolic curve having the sun in its focus ; and if it was projected with a force a little less would revolve in a very eccentric ellipsis having its focus in the sun. All these motions, therefore, proceed from the same principle, acting in a various but most regular manner in different circumstances, and are all analogous to the motions of heavy bodies projected from our earth. Effects so similar are to be resolved into the same cause, and there is hardly more evidence for supposing that it is the same power of gravity that acts upon terrestrial bodies in *Europe* and in *America*, at the equator and at the poles, than that it is the same principle which acts over the whole system, from the centre of the sun to the remote orb of *Saturn*, or to the utmost altitude of the most eccentric comet.

15. From several phænomena we have reason to conclude, that there is an atmosphere environing the sun and extended from it to a considerable distance. The ring of light observed around the moon, in a total eclipse of the sun, in 1605, mentioned by *Kepler*, and of late in 1706 and 1724, when it was observed to extend to 9 or 10 degrees distance from the moon, seems rather to have proceeded from the reflection of that atmosphere, while the solar direct rays were intercepted by the moon, than from the refraction of any atmosphere about the moon. The matter of this atmosphere appears to gravitate towards the sun, from the effect it has upon the vapour which arises in the tails of comets from their *Nucleus* and atmosphere, with a direction opposite to that of their gravity towards the sun. For this vapour,

vapour, being highly rarified, seems to arise with this direction in consequence of the greater gravity of the solar atmosphere towards the sun; in the same manner as a column of vapour rises in the air, in consequence of the air's greater gravity towards the earth; the rather that this vapour rises with more rapidity, as well as in greater plenty, in proportion as the comet is nearer the sun. Thus there is no sort of matter in the solar system but what we have ground to conclude gravitates towards the sun.

16. As to the fixed stars, they are removed to such an immense distance, that their gravity toward the sun can have no sensible effect upon them in many ages, and cannot appear to us by the phenomena. The power of gravity decreases in proportion as the square of the distance increases; the nearest fixed star seems to be several hundred thousand times farther distant from us than the earth is from the sun; and therefore their gravity must be some 100000×100000 times less than the gravity of the earth toward the sun. It is not therefore from phenomena, but from analogy only, that we can extend the power of gravity to the fixed stars. There is no influence but their light only which is able to traverse that vast abyss of space that is between us and them, so as to have any sensible effect. However, as their light is every way the same as that of our sun, our author thinks the argument from analogy may have its weight in this case. If they also gravitate toward the sun, and toward each other, then we may suppose that the unfathomable void that intervenes between the systems of which they are probably the centres, as the sun is of our system, may serve to hinder them from disturbing each other's motions, and from coming together into
one

one vast unformed mass of matter. It will not seem strange that where the sun itself is scarcely visible, the gravity toward it should be insensible; and that we should here find no effects of any gravitation toward the fixed stars.

17. As *action* and *reaction* are always equal and in opposite directions, so that the earth, for example, gravitates toward every mountain as well as every mountain toward the earth, and gravitates toward every projectile while it is moving in the air, as well as the projectile gravitates towards it; and without this law nothing would be steady or constant in nature: hence it follows, that the sun gravitates toward all the bodies in the system, and that the primary planets gravitate toward their satellites. The primary planets also gravitate toward one another; some minute irregularities in their motions, especially in those of *Jupiter* and *Saturn*, the two greatest planets, when they are in conjunction and come nearest to each other, are evidences of this. The motions of the satellites of *Jupiter* and *Saturn* are also said to be subject to irregularities that proceed from their mutual actions. From so many indications we may at length conclude, that all the bodies in the solar system gravitate toward each other; and tho' we cannot consider gravitation as essential to matter, we must allow that we have as much evidence, from the phenomena, for its universality, as for that of any other affection of bodies whatsoever.

C H A P. IV.

Of the general gravitation of matter.

I. **H**ITHERTO we have considered only the accelerating force of gravity at different distances, to which the velocity generated by it in a given time, is always proportional. It remains to shew that the motion produced by this power, at equal distances from a given centre, is always proportional to the quantity of matter in the heavy body; that the gravity of bodies arises from the mutual gravitation of their parts; and to ascertain the law of the gravitation of the particles of bodies. It is allowed as to terrestrial bodies, and was confirmed from many accurate experiments by Sir *Isaac Newton*, that bodies of the same bulk and figure, though of very different kinds, suspended by lines of the same length, performed their vibrations, when moving as pendulums, exactly in the same time; from which it follows, that the force of their gravity is exactly proportional to their quantity of matter: nor would there be any difference in the times of their vibrations though their figure and bulk were different, the distances between their centres of suspension and of oscillation being equal, if it was not for the resistance of the air. It has been already shewed, that the moon would fall toward the earth with the same velocity as any other heavy body, if she was at the same distance from its centre; and it is plain that the forces of bodies moved with equal velocities are as their quantities of matter: so that the weight of the moon would be to the weight of any heavy body at the same distance from the centre of the earth, in the same proportion as the matter of the moon is to the

the

the matter of that heavy body. The primary planets are acted on variously in their different distances, but according to the law which shews that if they were at equal distances they would descend with equal velocities toward the sun, so that their motion would be proportional to their quantity of matter. In the same manner it appears, that if the satellites of *Jupiter* and *Saturn* were at equal distances from the centres of their respective primary planets, they would descend towards them with equal velocities. The earth and moon, at equal distances from the sun, are acted upon by equal accelerating forces, and would descend with equal velocities toward it. *Jupiter* and his satellites would descend with the same velocity toward the sun, if their projectile motions were destroyed. The same is to be said of *Saturn* and his satellites. A very small inequality in the accelerating forces that act upon the primary planet and its satellites would produce very great irregularities in their motion. In all these cases, equal velocities being generated in equal times, the motions of the bodies, and consequently the gravities that produce these motions, must be proportional to the quantities of matter in the bodies; from which it follows, that all equal portions of matter, at equal distances from the centre of gravitation, are equally heavy; without regard to figure, bulk, or the texture of their parts: and that the gravitation of bodies arises from the gravitation of the particles of which they are composed.

2. Because *action* is always equal to *reaction*, if you still suppose the planets at equal distances from the sun, and therefore gravitating toward the sun with forces proportional to their quantities of matter, the sun will gravitate towards each of the planets with forces in the same proportion. In general,

the same body gravitates towards any other bodies, at equal distances from them, with forces proportional to their quantities of matter; because it gravitates toward them with the same forces with which they gravitate towards it, which are as their quantities of matter. The power, therefore, that is extended from the centre of the sun and of each of the planets, to all distances around them, is, at equal distances from their centres, proportional to their quantities of matter: and, in general, it appears that the weight or gravity of a body is the greater, in proportion as its quantity of matter is greater, as the quantity of matter in the body to which it gravitates is greater, and as the square of the distance from it is less. By compounding these three proportions together, the weight, and motion, of bodies, arising from their gravitation, may always be determined.

3. Gravity being found, by so many experiments and observations, to affect all the matter of bodies equally, we have hence more reason still to conclude its universality; since it appears to be a power that acts not only at the surfaces of bodies, and on such bodies as are removed at a distance from them, but to penetrate into their substance, and into that of all other bodies, even to their centres; to affect their internal parts with the same force as the external, to be obstructed in its action by no intervening body or obstacle; and to admit of no kind of variation in the same matter, but from its different distances only from that to which it gravitates.

4. The action of gravity on bodies arises from its action on their parts, and is the aggregate of these actions; so that the gravitation of bodies must arise from the gravity of all their particles towards each other.

other. The weight of a body toward the earth arises from the gravity of the parts of the body : the gravity of a mountain toward the earth arises from the gravitation of all the parts of the mountain towards it. The gravitation of the northern hemisphere toward the southern arises from the gravitation of all its parts towards it ; and if we suppose the earth divided into two unequal segments, the gravitation of the greater toward the lesser arises from the gravitation of all the parts of the greater toward the lesser. In the same manner, the gravity of the whole earth, one particle being excepted, toward that particle, must arise from the quantity of gravitation of all the other particles of the earth toward that particle. Every particle, therefore, of the earth gravitates toward every other particle of it ; and, for the same reason, every particle of matter in the solar system gravitates toward every other particle in it.

5. We now proceed to an important part of this doctrine, to determine the law according to which the particles of bodies gravitate towards each other ; after having discovered the law which is observed by bodies composed of those particles. To a superficial enquirer, at first sight, the former might possibly appear to be necessarily the same with the latter : but it is easily shewn, that the law which is observed in the attractions of the minute particles of matter is often very different from that which is observed by spheres composed of such particles. If, for example, the gravitation of the particles decrease in the same proportion as the cubes of their distances increase, or in any higher proportion, the spheres composed of such particles will not gravitate towards each other with forces that decrease in the same proportion as the cubes of the distances of their centres increase, or in that higher proportion ; for spheres

in contact shall attract each other, in those cases, with a force infinitely greater than when they are removed to the least distance from contact, tho' there be very little difference betwixt the distances of their centres in those two cases. This made it necessary for Sir *Isaac Newton* to treat of this subject fully; and as it is a very useful part of the theory of gravity, but not to be understood, as he has delivered it, without a profound skill in geometry and prolix computations, we shall endeavour to describe it in a more easy manner, by chusing (as on other occasions) the most simple cases. Suppose, first, that the gravitation towards any particle decreases in the same proportion that the square of the distance from it increases, let $P A E a$, $P B F b$ (*Fig. 59.*) be similar cones consisting of such particles, terminated by spherical bases $A E a$, $B F b$ that have their centre in P ; and the gravitation at P toward the solid $P A E a$, will be to the gravitation at P towards $P B F b$, as $P A$ to $P B$, or in the same ratio as any homologous sides of these similar solids. For let $M N m$ be any surface similar to $A E a$, having its centre likewise in P ; and the gravitation towards the surface $A E a$ will be to that towards $M N m$, in the ratio compounded of the direct ratio of the surface $A E a$ to $M N m$ (or $P A^2$ to $P M^2$) and of the inverse ratio of $P A^2$ to $P M^2$, that is, in the ratio of equality; consequently, the gravitation towards the surface $A E a$ being represented by A , the gravitation towards the solid $P A E a$ will be represented by $A \times P A$, and that towards the similar solid $P B F b$ by $A \times P B$, which are in the ratio of $P A$ to $P B$. In the same manner, the gravitation towards the frustum that is bounded by the surfaces $A E a$, $M N m$, is represented by $A \times A M$. It is evident, likewise, that, tho' the surfaces $A E a$ and $M N m$ be of any other form, yet the ultimate ratio of the gravitations at P towards the conical or pyramidical solids

solids $P A E a$, $P M N m$, is that of $P A$ to $P M$; and that if $A Q$ and $M q$ be perpendicular to $P H$ in Q and q , these forces reduced to the direction $P H$ will be ultimately in the ratio of $P Q$ to $P q$. Whence it appears, that, if $P B$ be equal to $B A$, the attraction of the particle P by the cone $P B b$, with which the particle is in contact, will be equal to the attraction of the frustum of the cone terminated by the surfaces $A E a$, $B F b$, when the attraction of the particles is supposed to increase as the square of the distance decreases; and that, in this case, the attraction of a portion of matter is not much greater when it is in contact with the particle attracted, than when it is removed to a small distance from it.

6. But it is otherwise when we suppose the attraction of the particles to decrease as the cubes of their distances increase. For, in this case, the particle P will tend to the surface $M N m$ with a force that is as the surface, or the square of $P M$ directly, and the cube of $P M$ inversely; that is, with a force which is as $P M$ inversely, or directly as $M v$ the ordinate of the æquilateral hyperbola $K v I$, described between the asymptotes $P A$ and $P H$. Therefore the attraction of the frustum $M N m A E a$ will be measured by the hyperbolic area $M v I A$ bounded by the ordinates at A and M ; and the attraction of the cone $P M N m$, by the infinite hyperbolic area that is conceived to be formed betwixt the ordinate $M v$ and the asymptote $P H$. It follows then, that, if such a law could take place, the particle P would tend towards the least portion of matter in contact with it, with a greater force than towards the greatest body at any distance, how small soever, from it. The same is easily shewn when the attraction of the particles decreases as any powers of the distances, higher than their cubes, increase. It appears, therefore, that the at-

traction of a particle in contact with a body is not sensibly increased by the addition or diminution of new matter, at any distance, how small soever, from the contact; whether this addition or diminution be made to the body or particle; and, in such cases, the less the particle is, the motions produced in it at infinitely small distances, by such attractions, must be the more violent; because the same force acting on a particle generates a velocity in it that is always greater in proportion as the particle itself is less.

7. The same things may be demonstrated without having recourse to the property of the hyperbolic area. Let PA (*Fig. 60.*) be to PB , as PB to PD ; let AB and BD be conceived to be divided into an infinite number of similar equal parts Ak , kl , &c. and Bm , mn , &c; then Ak will be to Bm as AB to BD , and the matter between the surfaces whose radii are PA and Pk , shall be to the matter between the surfaces whose radii are PB and Pm , as $PA^2 \times Ak$ to $PB^2 \times Bm$; that is, as PA^3 to PB^3 . The attractive powers of equal particles placed betwixt the surfaces of the radii PA and Pk , and the surfaces of the radii PB and Pm , are in the inverse proportion, or as PB^3 to PA^3 , by the supposition; and these two proportions compounded together give a ratio of equality. Therefore, because the attractive powers of the matter bounded by two such surfaces are in the compound ratio of the attractions of equal particles, and of the number of particles, it follows that the attraction of the matter contained by the surfaces of the radii PA and Pk must be equal to the attraction of the matter contained by the surfaces of the radii PB and Pm . In the same manner the attraction of the matter contained by the surfaces whose radii are Pk and Pl , is equal to the attraction of the matter between the surfaces whose radii are Pm and pn ; and

and the attraction of the frustum $A E a B F b$ is equal to the attraction of the frustum $B F b D G d$. In the same manner, if $P B$ be to $P D$, as $P D$ to $P H$, the attraction of the frustum $D G d H R b$ appears to be equal to the attraction of the frustum $A E a B F b$; and if this series of decreasing geometrical proportionals be continued, the attraction of the frustum contained by surfaces whose radii are any two subsequent terms of the progression, must be equal to the attraction of the first frustum $A E a B F b$. But in this decreasing progression continued from $P B$ the number of terms is infinite; and in the solid $P B F b$ there is an infinite number of frustums, the attraction of each of which is equal to the attraction of the first frustum terminated by the surfaces $A E a, B F b$; therefore the attraction of the solid $P B F b$, which is in contact with the particle P , is infinitely greater than the attraction of the frustum bounded by the surfaces $A E a, B F b$, which is the greater solid, but is removed from the contact of the particle P . We have taken this opportunity to illustrate and demonstrate this theorem here, because it will be of use to us afterwards, and serves to shew the advantages of the law of gravity which takes place in the solar system above other laws; tho' these, on other occasions, may be preferable.

8. The gravitation of the particles being supposed to decrease as the squares of their distances increase, the forces with which particles, similarly situated with respect to similar homogeneous solids, gravitate towards these solids, are as their distances from any points similarly situated in the solids, or as any of their homologous sides. For such solids may be conceived to be resolved into similar cones, or frustums of cones, that have always their vertex in the particles, and the gravitation towards these cones, or

U 4
frustums,

frustums, will be always in the same ratio by § 5. But if the gravitation of the particles decrease as the cubes of the distance increase, the forces, with which particles, similarly situated with respect to similar homogeneous solids, tend toward those solids, shall be equal. For such solids being resolved into similar frustums of cones that have always their vertex in the particles, and are similarly situated with respect to them, the gravitation towards these frustums will be always equal, by what was shewn in the last article; in the same manner as the forces with which the particle P tends toward similar frustums $A E a B F b$, $D G d H R b$ were demonstrated to be equal.

9. The gravitation of the particles being supposed to decrease as the squares of their distances from each other increase, if a particle be placed within the hollow solid generated by the annular space terminated by two concentric circles, or similar concentric ellipses, $A D B E$ and $a d b e$, (*Fig. 61.*) revolving about the axis $A B$, it shall have no gravity towards this solid. For let p be any such particle, $p r$ any right line from p that meets the internal circle or ellipse in any points f and q , and the external figure in x and r ; then if $x r$ be bisected in z , $f q$ will be likewise bisected in z , because the figures are similar and similarly situated; consequently $f x$ is equal to $q r$; and the gravitations of p towards opposite frustums of the solid that have their vertex in p , and are terminated by the same right lines produced from p , with opposite directions, will be always equal, by § 5. and mutually destroy each other's effect. It follows from this, that the gravity of any point Q in the semi-diameter $c p$, towards the sphere or spheroid, is to the gravity at P , as $c Q$ to $P c$, supposing the point Q to be within the solid: because the gravitation

tion towards the solid generated by the annular space, which is included between $A P B$ and $a Q b$, has no effect upon a particle at Q ; so that the gravity at Q towards the whole solid $A D B E$ is the same as the gravity at Q towards the solid $a d b e$, which is to the gravity at P towards the solid $A D B E$ as $C Q$ to $C P$, by the last article. It appears, therefore, that when a sphere or spheroid, of an uniform density, consists of particles that attract with a force decreasing as the square of their distance increases, the gravitation towards the solid decreases from the surface to the centre, in any given semidiameter, in the same proportion that the distance from the centre decreases.

10. Suppose now the particle P (*Fig. 62.*) to be placed without the sphere $A D B E$, at the distance $P C$ from the centre C ; and this particle shall be attracted towards the sphere with a force that decreases as the square of the distance $P C$ increases. For let $P N M$ be any right line from P meeting the generating semicircle $A D B$ in N and M , and the arc $C H$, described from the centre P with the radius $P C$, in L ; let $P n m$ be another such right line from P , constituting an infinitely small angle with $P M$, meeting the semicircle in n , m , and the arc $C H$ in l ; draw $L R$, $l r$, perpendicular to $P C$ in R and r , and $C V$ perpendicular to $P M$ in V . Suppose another circle $A d B e$ to intersect the circle $A D B E$ in the axis $A B$, and to constitute with it an infinitely small angle; and let $L u$ and $l x$, perpendicular to the plane $A D B$, meet $A d B$ in u and x . Then the gravitation of the particle P , towards the matter in the physical surface $L u x l$, shall be measured by $\frac{L l \times L u}{P L^2}$ or $\frac{L l \times L u}{P C^2}$; consequently the gravitation of P towards the pyramidical frustum, terminated by the circular

circular planes $A D B$ and $A d B$, and by planes perpendicular to $A D B$ in $N M$ and $n m$, shall be measured by $\frac{L l \times L u}{P C^2} \times N M$, by § 5. of this chapter. But, the angle contained by the planes $A D B$, $A d B$, being given, $L u$ is to $L R$, as $D d$, the arc intercepted by these circular planes at the distance $c D$, to $c D$ (or $c A$;) and $L l$ being to $R r$, as $P L$, or $P C$, to $L R$, so that $L l \times L R$ is equal to $P C \times R r$; it follows that the gravitation of P towards that frustum shall be measured by $\frac{L l \times L R \times 2 V M \times D d}{P C^2 \times C D}$ or $\frac{R r \times 2 V M \times D d}{P C \times C A}$.

This gravitation is reduced to the direction $P C$ by diminishing it in the ratio of $P V$, or $P R$, to $P C$;

and is then measured by $\frac{D d \times R r \times P R}{C A \times P C^2} \times 2 V M$; or (the simultaneous increment of $V M$ being represented by $V o$, and $P R^2$, or $P V^2$, being equal to $V M^2 + N P M$, by *Eucl.* 2. 6. or to $V M^2 + A P B$, so that $A P B$ being constant, the increments of $P R^2$ and $V M^2$ must be equal, and $R r \times P R$ equal to $V o \times V M$)

by $\frac{D d \times 2 V M^2 \times V o}{C A \times P C^2}$; which is the simultaneous increment of $\frac{D d \times 2 V M^3}{C A \times 3 P C^2}$, in the same manner as the increment of $V M^3$, while $V M$ acquires the infinitely small augment of $V o$, is $3 V M^2 \times V o$. Therefore the attraction of the part of the slice of the sphere terminated by the circular planes $A D B$, $A d B$, which is cut off by a plane perpendicular to $A D B$ in the right line

$N M$, is as $\frac{D d}{C A} \times \frac{2 V M^3}{3 P C^2}$; and the attraction of the portion of the sphere which is generated by the revolution of the segment $M D N$ about the axis $A B$ bearing the same proportion to the attraction of that slice, as the circumference of the whole circle to the

arc $D d$, it is measured by $\frac{c}{r} \times \frac{2 V M^3}{3 P C^2}$, where $\frac{c}{r}$ expresses the ratio of the circumference of a circle to the

the radius; and consequently is directly as the cube of the chord $M N$, and inversely as the square of $P C$, the distance of the particle P from the centre of the sphere. Hence the gravity at P towards the whole sphere is as the cube of its diameter, or its quantity of matter (the density being given) directly, and the square of the distance $P C$ inversely, the chord $M N$ coinciding with the diameter $A B$, when the attraction of the whole sphere is considered; so that this attraction is measured by $\frac{c}{r} \times \frac{2 C A^3}{3 P C^2}$.

11. It appears from what has been shewn, that any particle P , without the sphere, is attracted by it with the same force as if the whole matter of the sphere was collected in the centre, and attracted as one particle from that centre. For the circumference of the circle $A D B E$ is expressed by $\frac{c}{r} \times C A$, its area by $\frac{c}{r} \times \frac{C A^2}{2}$, the surface of the sphere by $\frac{c}{r} \times 2 C A^2$, and its solid content by $\frac{c}{r} \times \frac{2 C A^3}{3}$; so that the attraction of this solid content acting from the centre c , at the distance $P C$, is measured by $\frac{c}{r} \times \frac{2 C A^3}{3 P C^2}$, the very same which measures the attraction of the sphere at that distance, by the last article. The same is to be said of the gravity towards the aggregate of any number of such spheres that have a common centre; from which it follows, that however variable the density of a sphere may be at different distances from the centre, provided the density be always the same at the same distance from it, the gravity of a particle (that is not within the sphere) towards it will be as the quantity of matter in the sphere directly, and the square of the distance of the particle

particle from its centre inversely. If the attraction of the particles increased or decreased in the same proportion as their distances increase or decrease, the sphere would act, in this case likewise, in the same manner as if all its matter was lodged in the centre as one particle; but the case is different when the attraction of the particles observes other laws. Suppose that the attraction of the particles is inversely as the power of the distance of any exponent n less than 3, and the attraction of a sphere consisting of such particles, at its surface, will be to the force with which the whole matter of the sphere collected in its centre would attract at the same distance, as

$3 \times 2^{2-n}$ to $3 - n \times 5 - n$. If, for example, the attraction of the particles be the same at all distances (in which case we suppose $n = 0$) this ratio is that of 4 to 5; and if the attraction of the particles be inversely as their distance, it is that of 3 to 4; as we have shewn elsewhere*.

12. Having shewn that when the particles gravitate towards each other with forces that are inversely as the squares of their distances, the action of a sphere upon a particle placed without it observes the same law as that of the particles themselves, and decreases in the same proportion as the square of the distance of the particle from the centre of the sphere increases; it follows, because *action* and *reaction* are equal, that the particle will attract the sphere by a force varying in the same proportion; and if, in place of the particle, a second sphere be substituted consisting of such particles, since the total action of this second sphere will be the same as if all its matter was lodged in its centre, therefore the two spheres

* Treatise of Fluxions, § 902.

must observe the same law, in acting upon each other, as two particles placed in their centres; that is, their attraction must decrease in proportion as the square of the distance betwixt their centres increases.

13. The gravitation of bodies having been resolved by Sir *Isaac Newton* into the gravitation of their particles, and the law which is observed by the gravity of bodies having been discovered from the phænomena described at length above; it appears from the preceding conclusions, that the gravity of the particles of which the bodies are compounded observes the very same law. He was likewise enabled, by the same steps, to determine the progress of gravity from the centre of any sphere to the greatest distance from it. At the centre a particle can have no gravity at all, being equally attracted every way by the matter of the sphere about it. If it is placed within the sphere at some distance from the centre, its gravity will be the greater, the greater this distance is, by § 9; for these parts of the sphere only having an effect upon it that are at a less distance from the centre than itself, and its gravity being as the attracting matter directly and the square of the distance from the centre reciprocally, since the matter is as the cube of the same distance, the gravity must be as the distance itself. From the centre to the surface, its gravity increases in proportion as its distance from the centre increases; at the surface, its gravity is greatest; and from the surface upwards, its gravity decreases in proportion as the square of its distance from the centre increases; regularly observing this law to the utmost limits of space. Here we speak of the accelerating power of gravity, which is proportional to the velocity that it is able to generate in any given small moment of time; and since

it generates the same velocity in the same time in all bodies whatsoever at the same distance, it follows that their weight or motion arising from it, must be proportional to their quantities of matter. In general, to estimate the weight or motion of any sphere that is attracted by another whose parts are equally dense at equal distances from its centre, we are to measure it by compounding three proportions, that of the matter in the heavy bodies that gravitate, that of the matter in the attracting spheres to which they gravitate, and the reciprocal proportion of the squares of the respective distances betwixt the centres of the spheres that tend towards each other; and this is the law which we found from the phenomena to take place in the system. See *Art. 2.* of this chapter.

14. Thus Sir *Isaac Newton* discovered and fully described, from undisputed observations and unexceptionable calculations, this simple principle of the gravitation of the particles of matter towards each other; which being extended over the system to all distances, and diffused from the centre of every globe, is the chain that keeps the parts of each together, and preserves them in their regular motions about their proper centres. The same gravity, which is so well known to us on the earth, affects them all; the whole mass of the system is, in this respect, of a piece; and this one principle, so regularly diffused over the whole, shews one general influence and conduct, flowing from *one cause* equally active and potent every where. Several observations have been made of late that greatly confirm his doctrine, and particularly serve to shew that the gravitation towards bodies arises from the gravitation towards their particles. Of this kind are the measures of a degree on the meridian made lately, with
great

great accuracy, by the *French* mathematicians ; and the declination of the plumb-line from the true vertical, in consequence of the attraction of a great mountain in the neighbourhood.

C H A P. V.

Of the quantity of matter, and density, of the sun and planets.

1. **T**HUS far our author ascends by way of *analysis*, tracing the causes from their effects, and from the coincidence, or perfect similarity of many effects, shewing the cause to be more general. But in order to descend by the *synthesis*, and to determine the effects from the cause now known, it was not sufficient to establish the general gravitation of the particles of matter ; it was requisite to determine, as far as possible, the quantities of the powers which act in the system. We have seen that there is a gravity extending from each body in the system on all sides, at equal distances from their centres proportional to their quantities of matter. We know, from experience, the force of this power at the surface of our own earth, and have seen how to estimate its efficacy at any other distance. In order to be able to estimate all the powers in the system directed to their different bodies, it is necessary to determine the proportion of their quantities of matter to that of our earth. If this is once obtained, all the powers that operate in the system being known, it will require no more but a skilful application of geometry and mechanics to determine the motions and phænomena of the celestial bodies, which all flow from them.

2. To

2. To measure the matter in the sun and planets was an arduous problem, and, at first sight, seemed above the reach of human art. But the principles of this philosophy afforded a natural and easy solution of it in the most important cases, and Sir *Isaac Newton* has determined the proportions of the matter that is in the *Sun*, *Jupiter*, *Saturn*, and the *Moon*, to that in our *Earth*; that is, he has shewed how many earths might form a *Sun*, a *Jupiter*, or a *Saturn*. To understand how he was able to discover this, we are to recollect that the matter in each of these is in the same proportion as the force of gravity toward them, at equal distances from their centres. We know the force of gravity towards our earth from the descent of heavy bodies, and also by calculating how much the moon falls below the tangent of her orbit in any given time. We have no experience of any rectilineal descent of heavy bodies toward the *Sun*, *Jupiter*, or *Saturn*; but as the primary planets revolve about the sun; and their satellites revolve about *Jupiter* and *Saturn*, by computing from their motions how much a primary planet falls below its tangent in a given time, and how much any of *Jupiter's* and *Saturn's* satellites fall below their tangents in the same time, we are able to determine the proportion which the gravity of a primary planet to the sun, and of a satellite towards its primary, bears to the gravity of the moon towards the earth, in their respective distances: then from the general law of the variation of gravity, the forces that would act upon them at equal distances from the *Sun*, *Jupiter*, *Saturn*, and the *Earth* are computed; which give the proportion of the matter contained in these different bodies.

3. That the quantity of matter in *Jupiter* is greater than the quantity of matter contained in the earth, we may easily learn from the motion of his satellites; all of which revolve about his centre in less time than the moon revolves about the earth, and are all, excepting the first, at a greater distance from his centre than the moon is from the earth. The second satellite is farther distant from *Jupiter* than the moon is from the earth in the proportion of 3 to 2 nearly; and moves in an orbit greater in the same proportion. But this satellite finishes its revolution in 3 days, 13 hours, which is less than a seventh part of the moon's periodic time about the earth; consequently its motion must be much more swift than that of the moon. A satellite nearer *Jupiter* would move still more swiftly than this satellite: so that if a satellite revolved about *Jupiter* at a distance from his centre equal to the distance of the moon from the earth, it would move much more swiftly than the moon moves about the earth, and therefore would be acted on by a much greater centripetal force; for it requires always a greater force to bend into the same orbit a body that moves with a greater velocity. But the quantities of matter in the central bodies are proportional to their attractive powers at equal distances, and therefore the matter in *Jupiter* must very much exceed the matter in the earth. In like manner, we may easily observe that *Mercury* revolves about the sun in very little more than thrice the time in which the moon revolves about the earth, and yet moves in an orbit about 140 times greater, being so many times farther distant from the centre of his motion; from which it is easy to see that if a satellite revolved about the earth as far distant from it as *Mercury* is from the sun, this satellite would move vastly slower than *Mercury*:

whence it follows that the attractive power of the sun must be vastly superior to that of the earth, and therefore that the sun must contain vastly more matter than the earth. The matter in *Saturn* is also found to be greater than that in the earth. From our author's calculations, founded on these principles, it follows that the quantities of matter in the *Sun*, *Jupiter*, *Saturn*, and the *Earth* are to each other as the numbers 1, $\frac{1}{1067}$, $\frac{1}{3021}$, $\frac{1}{169282}$.

4. The quantities of matter in these bodies being thus determined, and their bulk being known from astronomical observations, it is easy to compute what matter each of them contains in the same bulk; which gives the proportion of their densities. Thus our author finds the densities of the *Sun*, *Jupiter*, *Saturn*, and the *Earth*, to be as the numbers 100, $94\frac{1}{2}$, 67 and 400.

From which it appears that the earth is more dense than *Jupiter*, and *Jupiter* more dense than *Saturn*; that is, those planets which are nearer the sun are found to be more dense, by which they are enabled to bear the greater heat of the sun. This is the result of our most subtle enquiries into nature, that all things are in the best situations, and disposed by perfect wisdom. If our earth was carried down into the orb of *Mercury*, our ocean would boil and soon be dissipated into vapour, and the dry land would become uninhabitable. If the earth was carried to the orb of *Saturn*, the ocean would freeze at so great a distance from the sun, and the cold would soon put a period to the life of plants and animals. A much less variation of the earth's distance from the sun than this would depopulate the torrid zone if the earth came nearer the sun, and the temperate zones, if it was carried from the sun. A less heat
at

at *Jupiter's* distance is adapted to the greater rarity of his substance: the consequences might be as fatal in *Jupiter*, if he was carried into the orb of the earth, as it would be to us to be carried into the orb of *Mercury*. The still greater rarity of *Saturn* is fitted to his more remote orb; so that tho' he is the last of the planets, and receives 90 times less light and heat from the sun than we do, he may nevertheless be in the best situation that could possibly be assigned him in the system; and there the situation of *Jupiter*, and of all the lower planets, may appear as terrible as that of *Mercury* does to us. *Saturn* terminates the planetary revolutions; and, as if the heat of the sun was too weak in the higher orbs, we find no bodies revolving higher, but such as descend in some part of their orbit nearer to this great centre of light and heat. Upon the whole we have reason to conclude, that they are all disposed in such order, and in such situations, from which any considerable variation would produce fatal effects. The hypothesis of *Des Cartes* led him to place the more dense planets at a greater distance from the sun; but a philosophy founded on the observation of nature corresponds better with the final causes of things, and proves, on every occasion, the wisdom of the author.

5. As astronomers have found no satellites revolving about *Mercury*, *Venus*, or *Mars*, we are deprived of the like opportunities of comparing their attractive powers and proportional quantities of matter. But it is highly probable, from what we have said of the *Earth*, *Jupiter*, and *Saturn*, that the densities of the other planets correspond to their distances from the sun, and are greater in the nearer planets. Our author has also computed the proportion of the attractive powers of the *Sun*, *Jupiter*, *Saturn*, and

the *Earth*, at their respective surfaces, and finds them to be in proportion as these numbers, 10000, 943, 529, 435, respectively. From which it appears that the force of gravity towards these very unequal bodies approaches surprisingly to an equality at their surfaces; so that tho' *Jupiter* be several hundred times greater than the earth, the force of gravity at his surface is very little more than double what it is at the surface of the earth; and the force of gravity at the surface of *Saturn* is but about $\frac{1}{4}$ greater than that of terrestrial bodies.

6. The most considerable powers that act in the system being thus determined; before we proceed to consider their effects, it is necessary, first, to enquire whether they act in a *void*, or if there is any medium that resists the motions produced by them. We find that the air makes a considerable resistance to the motion of projectiles near the earth; which, if it extended unto the planetary regions, would also very considerably affect their motions. But experiments shew that the density of the air is proportional to the force that compresses it, and that the weight of the superincumbent atmosphere is the force which compresses the air in every altitude; so that the higher any portion of air is, having a less weight of air above it to compress it, it must have less density in the same proportion: and from this it follows, that if we abstract from the diminution of gravity, and the altitudes from the surface of the earth be taken in arithmetical progression, the densities of the air at these altitudes will decrease in geometrical progression*. Since therefore, it appears from several experiments, made in *France* and *England*, that the

* See Dr. *Halley* in *Phil. Trans.* N^o 181. and *Schol. Prop.* 22. Lib. II. Princip.

density of the air decreases in such a manner, that at the height of seven perpendicular miles it is about $\frac{1}{4}$ of the density it has at the level of the sea, at 14 miles it must be $\frac{1}{16}$ of it, at 21 miles $\frac{1}{64}$, at 28 miles $\frac{1}{256}$, at 35 miles $\frac{1}{1024}$, at 42 miles $\frac{1}{4096}$, at the height of 49 miles $\frac{1}{16384}$ part of it, and at the height of a semidiameter of the earth altogether insensible. It appears from the laws of motion, and from many accurate experiments, that the resistance of fluids arising from the *inertia* of their matter, is proportional to their density; and therefore the resistance of the air, tho' sensible at the surface of the earth, would be 16384 times less at the height of 49 miles, and could not be sensible in the greatest number of ages at the height of a semidiameter of the earth: it must be still less at the distance of the moon, which therefore, meeting with no resistance, continues to revolve for ever in her orbit, without any impediment or diminution of motion. As for a more subtile medium than the air, no experiments nor observations shew that there is any here, or in the celestial spaces, from which any sensible resistance can arise.

B O O K IV.

The effects of the general power of gravity deduced synthetically.

C H A P. I.

Of the centre of the solar system.

I. **S**IR *Isaac Newton* having established the general principle of the gravitation of the particles of matter, and having determined the chief powers that act in the system, viz. those which tend to the *Sun*, *Jupiter*, *Saturn*, and the *Earth*; and having found that the celestial motions are performed in free spaces, where the resistance is insensible; he has now prepared the way for proceeding synthetically in his account of the system of the world, and enquiring into the various effects that arise from a power so evidently established. Any general principle ascertained in nature is a great acquisition to philosophy, especially when the variations of this power, with its direction and force, are clearly determined; and the fertility of this principle will appear from the various phænomena resolved by it synthetically, of which we are now to treat. Sir *Isaac Newton* begins with enquiring into the centre of the system. The *Pythagoreans* ascribed this place to the centre of the sun, the followers of *Aristotle* and *Ptolemy* to the earth. But Sir *Isaac*, having found that these gravitate towards each other and towards all the other bodies in the system, neither

of them, nor indeed any body in the system, can be supposed to be void of all motion.

2. It is the centre of gravity of the whole system that is the only point which can be supposed quiescent in it; the same point about which all the matter of the system would soon be accumulated, if the progressive motions of the bodies in it were destroyed, and their gravity was permitted to bring them together. The mutual actions of bodies on each other never affect the state of this centre; their attracting or repelling each other produces no effect upon it; and it must either be quiescent, or proceed uniformly in a right line. All seem agreed that the centre of the system is at rest, and no reason or observation argues for our ascribing any motion to it. The centre of gravity of the system is, therefore, the only immoveable point, while all the bodies in the system move round it with various motions.

3. As we have our knowledge of gravity, and the laws of nature, from what passes on the surface of the earth, we cannot illustrate the motions of the bodies of the solar system, arising from their mutual gravity, better than by some images we find of them on the earth, after having shewn so fully the similarity of the powers that act on the parts of the earth and on the celestial bodies. We know that when, by any power or machine, a body is projected in the air, the power reacts on the earth with an equal force, and that if the power was sufficient to project a mountain or a much larger part of the earth, it would act on the remainder of the earth with an equal force, in an opposite direction; so that while the projected part began to move in its curve, the remainder of the earth would begin at the same time to move in an opposite direction, with

an equal quantity of motion, but with a velocity so much less as the matter in it is greater than in the projected part; and both would revolve in certain orbits about the common centre of gravity, which would continue in the same state as before the projection. If, by the resistance of the medium, the motions of these parts of the earth came to be destroyed, they would come together again and be accumulated in one mass about the same centre. If there were more such parts of the earth projected, the centre of gravity of all would be no way affected by such projections, but they would move round it, so that the sum of the motions on one side of it should be equal to the sum of the motions on the other side: and this obtains even in those small motions that are every day produced by powers and agents on the earth.

4. The motions of the great bodies in the solar system are analogous to these: the different parts of the solar system gravitate to each other, as the parts of the earth gravitate towards one another; and the different parts of the system move in the same manner about their common centre of gravity, as the parts into which we supposed the earth to be divided, if projected in any direction, would all move about their common centre of gravity; or as the earth, and all the bodies that are actually projected every day on its surface, revolve about the common centre of gravity of the earth and these projectiles. Only there is this difference, that the bodies of the great system were projected at great distances from each other, and in such a manner that the planets revolve in orbits almost circular, so as not to come too near to the sun, or to be carried too far from him, in their revolutions. The Creator of the world had in vain made them of densities adapted to certain distances,
if

if he had not projected them with the forces that were requisite to preserve them revolving at those distances, or near to them; and as the greatness of the force impressed on those vast bodies, some of which are many times greater than our earth, shews the power, its just quantity, varied regularly in the different distances of the planets, and its proper direction, shew the skill of the *first mover*.

5. We may suppose that all the matter of which the system consists was formed first in one mass, where now the centre of gravity of the whole system is found; that of this mass various bodies were formed and separated from each other to proper distances, where they received their projectile motions; and that the powers which separated and moved them observed the law of nature that requires an equality between action and reaction, and is observed in all the actions of powers at present: and thus these motions would begin, and continue for ever, without producing any motion in the centre of gravity of the system.

6. When the bodies were thus moved in their just orbits, we may conceive some of them to have been subdivided again, by actions observing the same laws, into several other bodies, which in like manner were formed into lesser systems; as that of the earth and moon, those of *Jupiter* and *Saturn* and their satellites. There is not any of these quiescent in its particular system; the earth and moon move about their common centre of gravity, while it is carried with a regular motion round the centre of gravity of the whole system. The same is to be said of *Jupiter* and *Saturn* and their satellites; and it is certain from the laws of nature, that the motions in any lesser system about its centre of gravity, and the motion
of

of that centre about the centre of gravity of the whole system, interfere not with each other. A lesser system being thus formed, one of the bodies that compose it might be subdivided into lesser bodies that might form a system of an inferior order. But we do not find that nature carries this subordination so far, unless we would consider the motion of projectiles, near the surfaces of the secondary planets, as an example of this kind.

7. It is next to be considered, where this point of rest of the common centre of gravity of the system is to be found; and it is plain from what we have already seen, that it can never be far removed from the sun, because the matter in the sun vastly exceeds the matter in all the planets taken together: and, from what we said of the centre of gravity above, it appears that it is always nearer the greater body in proportion as it is greater. *Jupiter* is the largest of the planets, and yet is but $\frac{1}{1067}$ of the sun, so that their centre of gravity must be 1067 times nearer the sun than *Jupiter*; and as the distance of *Jupiter* is little more than 1067 semidiameters of the sun, it follows that the centre of gravity of the sun and *Jupiter* cannot be much above the surface of the sun. *Saturn* is less than *Jupiter* both in bulk and density, and the centre of gravity of the *Sun* and *Saturn* falls within the body of the sun: and thus it easily appears, that tho' all the planets were on one side of the sun in one line, the centre of gravity of the sun and them all could scarcely be above a semi-diameter of the sun from his surface: and this is the farthest that the sun is ever removed from that centre. It appears therefore, that tho' the sun is in perpetual agitation about this centre, yet, being always so near it, he may very well be considered by astronomers as the centre of the solar system. Thus, though the

ter-

terraqueous globe receives an impresson from every power that move projectiles in the air, and is, to speak accurately, agitated a little by these powers with a very complex motion, yet we consider it as at rest, neglecting such exceeding minute actions and their effects.

C H A P. II.

Shewing how gravity produces some small irregularities in the motions of the planets.

I. **I**F the planets were acted on by a power directed to the centre of the sun only, varying according to the general law of gravity, and that centre was quiescent, their motion about it would be perfectly regular. But we found that each of the planets was acted on by a power directed to every body in the system. In order to judge of the effects of these actions, our author first supposes two bodies equally gravitating towards each other, and revolving about their common centre of gravity: and, since the direction of their mutual gravitation passes always from the one to the other through their centre of gravity, and their distances from it vary always in the same proportion as their distances from each other: it follows, that they must describe equal areas in equal times about that centre, and about each other, and describe similar figures about that point, and about each other*. So that in the motions of two bodies no irregularities arise in their motions about each other from their mutual attractions; whatever the law of their gravity be supposed to be: only they will finish their revolutions about

* *Princip. Lib. I. Prop. 58.*

the centre of gravity in less time than if the one was to revolve about the other quiescent, at the same distance, and with the same centripetal force; because the orbit described about the centre of gravity being less than that which is described by any one of them about the other quiescent (their distance from each other being equal in both cases) and being also similar to it, it must be described in less time.

2. If three or more bodies mutually attract each other, the gravitation of any one, arising from the actions of the rest, may be determined by the rule for the composition of motion; and if the law of gravity be such as we find to obtain in the solar system, its gravitation will not be always directed to the centre of gravity of the other bodies, or indeed to any fixed point, but sometimes to one side of that centre, and sometimes to the other; and therefore, equal areas will not be described in equal times about any point in the system, and several irregularities will necessarily arise in the motions of the bodies. But if you suppose one of these bodies to be vastly greater than the rest, so that the actions of the other bodies may be neglected if compared with its action, and the centre of gravity of the system be always found near it, then the irregularities in the motions in such a system will be very small. The areas described in equal times, about the centre of that great body, will be nearly equal, and the orbits described will be nearly elliptic, having that centre in their focus. That this is the case of the sun and planets, appears from what we have shewn concerning their quantities of matter: and thus we see that not only the regular motions of the planets are to be derived from the principle of gravity, but also how their minute errors and irregularities are accounted for from it. The same is the case of *Jupiter* and *Saturn* and

and their *satellites*. As for the *Earth* and the *Moon*, though there be a less disproportion in their magnitudes, and their common centre of gravity be sensibly removed from the earth, yet as there are only two in their system, no irregularities arise from their mutual actions in their motions about their common centre of gravity, or they are easily determined when the position of their centre of gravity is known. These lesser systems of the *Earth*, *Jupiter*, and *Saturn*, are carried about the centre of gravity of the general solar system, without receiving any disturbance from any action of the sun or planets, which is equal on all their parts and in the same direction. When a fleet of ships is carried away by a current that affects them equally, it has no effect on their particular motions amongst themselves, nor is the motion proceeding from the current discovered by them, if they have no body in sight that is not affected by it. In the same manner, if the gravity towards the sun acted equally, and in the same direction, on the parts of these lesser systems, it would have no effect on their motions amongst one another, and could only be discovered by comparing their motions with the fixed stars, or with some body foreign to that lesser system, which is acted on in a different manner by the sun. But as there is some variation in the actions of the sun upon the parts of these systems, and in the directions of these actions, from hence some irregularities necessarily arise.

3. Though the actions of the sun and of the inferior planets, compounded together, do not always produce in a superior planet a gravitation exactly directed towards their centre of gravity; yet, as upon the whole it is more nearly directed to that point than to any other, the motions of a superior planet will be found more regular by supposing that point
to

to be the centre of its attraction, rather than any other, and its ellipse will be just by placing its *lower focus* there. A planet that is higher than this will, by its attraction, have some effect on the motion in this ellipse, but as it also acts on the inferior planets at the same time, there will no irregularity arise from that part of its action which is equal and in the same direction on them all, but from the differences of its actions only; which being exceedingly minute, and having contrary effects in the opposite situations of that higher planet, can produce effects scarcely sensible in many revolutions.

4. The action of *Jupiter* on *Saturn* when greatest (that is in their conjunction when their distance is least) is found to be $\frac{1}{204}$ of the action of the *Sun* upon *Saturn*, by comparing the matter of *Jupiter* with the matter in the *Sun*, and the square of the distance of the *Sun* from *Saturn*, with the square of the distance of *Jupiter* from *Saturn*. The effect of this action on *Saturn* is not altogether insensible. But the elliptic orb of *Saturn* will be found to be more just, if you suppose its focus not to be in the centre of the *Sun*, but in the centre of gravity of the *Sun* and *Jupiter*, or rather in the centre of gravity of the *Sun* and of all the planets below *Saturn*. In the same manner, the elliptic orb of any other planet will be found more accurate, by supposing its focus to be in the centre of gravity of the *Sun* and all the planets that are below it.

5. The whole action of *Jupiter* disturbs the motion of *Saturn* in their conjunction, because *Jupiter* acts upon *Saturn* and upon the *Sun* with opposite directions, at that time. But, because *Saturn* acts then in the same direction on *Jupiter* and on the *Sun*, if it acted also with the same force on both, it
would

would have no effect on the motion of *Jupiter* about the *Sun*, and it is by the excess of its action on *Jupiter* above its action on the *Sun* that it disturbs the motion of *Jupiter*. This excess is found to be $\frac{1}{1923}$ of the action of the *Sun* on *Jupiter*, and therefore is much less than the force with which *Jupiter* disturbs the motion of *Saturn*. The actions of the other planets on each other are incomparably less than these, and the irregularities proceeding from those actions are always less in any planet as it is nearer the sun. Only the orbit of the earth may appear a little more irregular than that of its neighbouring planets, because it revolves about the centre of gravity of the earth and moon, while that centre annually revolves about the sun.

6. If the planets were attracted by the sun and by one another, but the sun was not reciprocally attracted by them, the centre of gravity of the system, because of the deficiency of this reaction, would necessarily be in motion; and this would be a new source of errors and irregularities. If the primary planets were not attracted by their satellites, as well as the satellites by their primary planets, other irregularities would necessarily arise. If the great planets, *Jupiter* and *Saturn*, had moved in the lower spheres, their influences would have had much more effect to disturb the planetary motions. But while they revolve at so great distances from the rest, they act almost equally on the sun and on the inferior planets, and have the less effect on their motions about the sun, and the motions of their satellites are at the same time less disturbed by the action of the sun. The earth and moon move in a lower sphere, but their motions are the less irregular because there are only two in their system. We shall afterwards see that the comets continue for a very small time among

among the planetary spheres, and that in the far greater part of their revolutions they are carried to such vast distances that their actions can have very little effect on the motions of the planets. Such is the law of gravity, and the manner of its operation, and such is the disposition of the bodies in the system, as seems well adapted for preserving their motions with great regularity; but this will appear still more fully from the following chapter.

C H A P. III.

Of the approach and recess of the planets to and from the sun, in every revolution.

I. **T**HUS far we have considered the powers that act in the system of the sun, and have found that those which produce the regular motions of the planets vastly exceed those that disturb them. We are next to consider how the motions in their orbits proceed from the action of those powers; and how the planet is made to ascend and descend by turns, at the same time that it revolves about the centre of its gravitation. This requires an illustration, the rather, because we have nothing similar to it in the motion of heavy bodies at the earth's surface; for these are always made to fall to the earth by their gravity: in whatever direction they are projected, upwards, perpendicularly, or obliquely, their gravity soon brings them down to the earth again. Hence many find it hard to conceive how a planet after approaching to the sun can recede from it again, especially since its gravity is increased as its distance decreases. They imagine that it ought to continue to approach to the sun, and at length fall upon his body, as heavy bodies fall to the earth.

2. But

2. But we are to remember, that the force with which heavy bodies are projected, from our most powerful engines, is inconsiderable, compared with the motions which their gravity could generate in them in a few minutes; and they move over such small spaces, when compared with their distance from the centre of the earth, that their gravity is considered as acting in parallel lines, without any sensible error, so that the centrifugal force arising from the rotation about that centre is altogether neglected. But when we examine the motion of a projectile in larger spaces, and trace it in its orbit, we must consider the action of gravity as directed to a centre, and take in the centrifugal force arising from its motion of rotation about that centre; and it will appear, that there are indeed some laws of gravity which would make the body approach to the centre continually, till it fall into it; but that there are other laws which make bodies approach to the centre, and suffer them to recede from it, by turns. How to distinguish these we shall now consider.

In the first place, it will be easily understood that if s (*Fig. 63.*) be the centre of attraction, and a body is projected with a certain force in the line AE , perpendicular to As , it will describe the circle ALa with an equable motion, and after a complete revolution return to its first place A , with its first motion. The same gravity that acted at A upon it, and carried it below the tangent AE , acts upon it at any other point L , at an equal distance from the centre s , and brings it from the tangent at L thro' the same length in the same time. The centrifugal force, arising from its rotation, being equal to its gravity, neither of them prevails, and the body therefore neither approaches to the centre nor recedes from it.

Y

If

If you suppose the motion of projection at A to be increased, the gravity necessary to keep it in the same circle must be increased also; so that if the velocity of the projection be double, the gravity requisite to retain the body in the same circle must be quadruple; because $A K$ being double of $A L$, the point K falls four times farther below the tangent than the point L , as we shewed above: in general, the gravity necessary to retain a body in the same circle is in the duplicate proportion of the motion of projection; and the velocity, therefore, in the subduplicate proportion of the gravity; so that when the gravities are as 1 to 4, the velocities are as 1 to 2.

3. If the body is projected at a less distance from the centre of attraction, as at D , with the same velocity, the gravity must be greater to retain it in a circle; because the curvature being greater, the extremity P of the arc $D P$, equal to $A L$, falls farther below the tangent at D , than L falls below the tangent at A , in proportion as the arc $D P$ is more curve, that is, in proportion as the distance $S D$ is less than $S A$. If the velocity of projection is increased at D , so that the body describe a greater arc $D Q$ in the same time, than the force of gravity, necessary to retain the body in a circle there, must be increased in a duplicate proportion; because $Q T$ is to $P R$ in the duplicate proportion of $D Q$ to $D P$. If the velocity at D , for example, is greater than that at A in proportion as $S A$ is greater than $S D$, then $Q T$ will be to $P R$ as the square of $S A$ is to the square of $S D$, and $Q T$ will be to $L M$ as the cube of $S A$ is to the cube of $S D$; that is, the force requisite to retain bodies in circles must be reciprocally as the cubes of the semidiameters, when the velocities in these circles are reciprocally as the semidiameters themselves; and *con-*
versely,

versely, if the gravities increase as the cubes of the distances from the centre decrease, the velocities necessary to carry bodies in circles, at different distances from the centre of attraction, must increase in proportion as the distances decrease.

4. In general, as the gravities of bodies that describe circles about the same centre increase in proportion as the squares of the velocities increase, and as the distances decrease; it follows *conversely*, that, in order to compare the velocities of projection that are necessary to carry bodies in circles at these different distances, we must compound the proportion of the gravities and the proportion of these distances together, for this compounded proportion will give that of the squares of the requisite velocities. So in the solar system, if the distances of two planets were as 1 to 4, the gravities being as 16 to 1, these proportions compounded give that of 16 to 4, or of 4 to 1, which is that of the squares of the velocities, and therefore the velocities themselves are as 2 to 1. In like manner we can determine the law according to which the velocities, necessary to carry bodies in circles about s , vary at any distances, in any given law of gravity.

5. If a body is projected at A (*Fig. 64.*) with a velocity less than that which is necessary to carry it in a circle there, it must fall within the circle, the centrifugal force, arising from the motion of rotation about s , is less than that which it would have in the circle AL , in proportion as the square of its velocity is less, and is therefore less than its gravity in the same proportion: the body, therefore, by the excess of its gravity above its centrifugal force, is made to approach to the centre. The motion of the body, as it descends in the orbit AMB , must be

accelerated so as to describe equal areas in equal times about s , and the velocity of its motion at m must be greater than its velocity at A , in proportion as sA is greater than sP , the perpendicular from s on the tangent to its orbit at m ; because if the arcs AK , MN , be described in the same time, the triangular spaces ASK , MSN , being equal, the bases AK , MN must be reciprocally as their altitudes sA , sP , and the velocities are as the arcs AK , MN , described in the same time, and therefore reciprocally as sA , sP . The velocity, therefore, in the orbit from A to m , increases in a higher proportion than that in which the distances sA , sM decrease, because sA is to sP in a higher proportion than sA is to sM : only if the direction of the body ever become perpendicular again to the ray drawn from s , at any point, as B , there sM and sP will coincide, and the proportion of the velocities will be the same as the reciprocal of the distances sA , sB .

6. If a body is projected at B in a direction perpendicular to sB , with a velocity greater than that which is necessary to carry it in the circle BCH about the centre of attraction, at the distance sB , it must be carried without that circle, and recede from the centre s . The centrifugal force, in this case, arising from its motion of rotation, is greater than that which would arise from its motion in the circle BCH , and therefore greater than its gravity; and by the excess of its centrifugal force above its gravity, it recedes from s the centre of attraction. The motion of the body decreases as it rises, being retarded by the action of its gravity, so that the velocity is always less than the velocity at B , in proportion as sB is less than sp , the perpendicular from s on the direction of its motion.

7. A planet descends from A, which is called its higher *apsis*, to B, which is called its lower *apsis*, and reascends again from B to A. It descends from A, approaching to the centre of attraction, because its velocity at A is less than that which would be able to carry it in a circle about S, at the distance SA. As it descends to lesser distances, its velocity in its orbit increases in a higher proportion than the velocities, which would be sufficient to carry bodies in circles at these distances, increase. For the velocity in the orbit at B is greater than that at A, in proportion as SA is greater than SB; whereas the velocity in a circle at B is greater than the velocity in a circle at A, as \sqrt{SA} is greater than \sqrt{SB} . If SA were to SB as 4 to 1, the first proportion would be that of 4 to 1, but the second that of 2 to 1 only. Hence it appears how the velocity in the orbit at B, exceeds that in a circle at the same distance, though the velocity in the orbit at A was exceeded by the velocity that was able to carry it in a circle at the distance SA. In the higher part of the orbit, the velocity of the body is less than that which would carry it in a circle there about S; but the velocity in the orbit increases more, by the approach of the body to the centre of attraction, than the velocities requisite for carrying bodies in circles do, and so gets the better of them in the lower part of the orbit. Of these two each prevails over the other by turns, in the two apses; the velocity in the circle in the higher apsis, and the velocity in the orbit in the lower apsis. After the body is carried off at B by its superior velocity, the velocity in a circle afterwards gets the better, because it does not decrease so quickly as the velocity in the orbit, and the body is made to move, in its ascent, in a semi-ellipse equal and similarly situated to that which it described in its descent.

8. The gravity indeed at B is greater than the gravity at A, in proportion as the square of the distance is less. But the centrifugal force arising from the circular motion about S increases in a higher proportion, *viz.* as the cubes of the distances decrease; for these centrifugal forces are in the direct proportion of the squares of the velocities and their inverse proportion of the distances, compounded together: the first of these is the inverse proportion of the squares of the distances, and the two together compound the inverse proportion of the cubes of the distances. The centrifugal forces, therefore, increase more quickly than the gravities; and though the gravities prevail in the higher part of the orbit, the centrifugal forces get the better in the lower part of it. The gravity prevailing in the higher apsis makes the body approach to S, the centrifugal force prevailing in the lower apsis makes the body recede from it; and, by their actions, the body for ever revolves from the one to the other.

9. It is easy to see from what we have said, that the body can descend from the higher apsis to the lower, and ascend again from the lower apsis to the higher, when the velocities necessary to carry bodies in circles about the centre of attraction increase, in approaching to that centre, in a less proportion than the velocity of a body moving in an orbit A M B increases. For though the velocity in a circle in the greater distances exceed the velocity in the orbit, this latter, by increasing more quickly as the distance decreases, gets the better of the other in the lower part of the orbit, and carries the body off again. But if the velocities by which circles can be described about the centre of attraction increase, in approaching to that centre, in a higher proportion, or in the same

same proportion, as the velocity in the orbit increases, then this latter having been supposed at A less than the former, it must always continue less than it, and never get the better of it, so as to be able to carry off the body; and therefore, in all such cases, the body can never recede from the centre after it has once begun to approach to it, but must descend to distances less and less, till it fall into the centre. It approaches at A, because its velocity is less than that which is requisite to carry it in a circle there: its velocity indeed increases as it descends to lesser distances, but the velocities which would carry bodies in circles at these distances about s, increasing also in as great a proportion, the velocity in the orbit must still continue to be less than in these circles, and the body must still continue to approach to the centre.

10. To fix the limit of these two cases, we are to consider, that the velocities in an orbit, at A and B, are in the inverted proportion of the distances there from the centre of gravitation; and that, if the gravity increase as the cubes of the distances decrease, the velocities necessary to describe circles at A and B are in the same inverted proportion of the distances at A and B from s. In this case, therefore, the velocities in circles, and in the orbit at A and B, vary in the same proportion, and the same which exceeds at the one distance must exceed at the other; so that, for the same reason for which the body approached to s at A, it would approach to it at B, and if it receded from it at B, it must recede from it at A; that is, if it once begin to approach, it must always approach to s, and if it once begin to recede, it must always recede from it. This also appears from what we said of the centrifugal force, which, in the same orbit, increases as the cube of the distance decreases; and consequently in the same proportion

in which the gravity is supposed to increase in this case; so that of these two, which ever is supposed to prevail in any one apsid, the same must prevail in any other apsid, if such could be assigned; and the body must either descend continually to the centre, or rise from it for ever.

11. If the gravity increase in a higher proportion than as the cubes of the distances from the centre of attraction decrease, then the velocities necessary to carry bodies in circles about that centre, in approaching to it, will increase in a higher proportion than the distances decrease; that is, in a higher proportion than the velocity in an orbit increases from A to B; so that as the velocity in a circle at A exceeded the velocity in the orbit there, it will much more exceed it at B, and therefore the body, acted on by a gravity varying in such a manner, must approach to the centre till it fall into it, if it once begin to approach to it at A; and if it once begin to recede from it, it must continue to recede from it for ever. The higher the power of the distance is to which the gravity is reciprocally proportional, the body will descend in a less number of revolutions to the centre, in like circumstances. If the gravity is reciprocally proportional to the cubes of the distances, the body will descend after an infinite number of revolutions. If the gravity increase as the 4th power of the distance decreases, and the body is projected at A with a velocity less than that which would carry it in a circle about s in proportion as $\sqrt{2}$ is less than $\sqrt{3}$, the body will describe a certain *epicycloid* about s, and fall into it after half a revolution. If the gravity increase as the 5th power of the distance decreases, and the velocity of the projection be to that which would carry it in a circle about the centre s as 1 is to $\sqrt{2}$, it will descend in a semicircle described on the

the diameter s A , and fall into the centre in a quarter of a revolution. If the gravity increase as the 7th power of the distance decreases, and these velocities be as 1 to $\sqrt{3}$, it will fall into the centre in $\frac{1}{8}$ of a revolution. In general, if the gravity increase as the $n + 3$ power of the distance decreases, and the velocity of projection at A be to the velocity which would carry the body in a circle there, about s , as 1 to $\sqrt{1 + \frac{n}{2}}$, it will fall into the centre in the $\frac{1}{2n}$ part of a revolution. If the gravity increase as the $3\frac{1}{100}$ power of the distance decreases, and the velocities be as 1 to $\sqrt{1 + \frac{1}{200}}$, the body must fall into the centre after 50 revolutions. We cannot pretend to demonstrate these things here, and have mentioned them only to illustrate this theory *.

12. If the gravity increase in a less proportion than that in which the cubes of the distances decrease, the velocities, necessary to carry bodies in circles about the centre s , will increase, in approaching to it, in a less proportion than the simple proportion in which the distances decrease, and therefore in a less proportion than the velocity in the orbit from A to B ; so that, tho' the former exceed in the greater distances, the latter may exceed in the lesser distances, and the body may consequently descend from the higher apsis to the lower, and ascend from the lower apsis to the higher by turns. The gravity may prevail over the centrifugal force in the higher parts of the orbit, but, increasing more slowly in descending to the lesser distances than the centrifugal force, it is overcome by it in the lower parts of the orbit, and the body is made to recede again to its first distance. If the gravity increase as the cubes of the distances decrease, the body never can arrive at the

* See *Treatise of Fluxions*, Art. 437.

lower apsis B. If the gravity increase as the squares of the distances decrease, the body will descend in a semi-ellipse from the higher to the lower apsis in half a revolution.

13. If the gravity increase in the reciprocal proportion of some power of the distance betwixt the square and cube, the body will take more than half a revolution to descend from the higher to the lower apsis, the more the increase of gravity approaches to the reciprocal proportion of the cubes of the distances; for the velocity in the orbit will find the more difficulty to get the better of the motion that would carry the body in a circle, or the centrifugal force will with more difficulty get the better of the gravity. But if the gravity increase in proportion as some power of the distance less than the square decreases, the velocities in circles increasing less in approaching to the centre, the velocity in the orbit will the more easily prevail, and the centrifugal force will sooner exceed the gravity; and therefore the body will descend to the lower apsis in less than half a revolution, and return to the higher apsis in less than a complete revolution. From which it appears, that as the apsides are fixed in the regular course of gravity, that is, while it increases as the squares of the distances decrease, they must be carried forwards, in the direction of motion of the body, when gravity varies in a higher proportion than that, and must be carried backwards with a contrary motion when gravity varies less than in that proportion. As a change from the proportion of the squares to that of cubes gives an infinite motion to the apsides, so that the body never arrives at either of them again; a very small change in the course of gravity will produce a sensible motion in the apsides, and the least change from the regular course

course of gravity must become very sensible, in a great many revolutions, by the motion of the apsides. From which we learn, that since the apsides of the planets have so small a motion that some astronomers neglect it altogether, and doubt if there is indeed any such motion at all, we may conclude that their gravity must observe very accurately, in its variations, the law of the squares of the distances.

14. Our author, to reduce to a computation the motion of the apsides arising from a variation from the regular course of gravity, supposes with astronomers, that the body moves in an ellipse that is carried at the same time with a regular motion about s , which, in an entire revolution, gives the motion of the apsides. In a quiescent ellipsis, (*Fig. 65.*) the curvature at A and B being the same, the centripetal forces there were found, above, to follow the inverse proportion of the squares of the distances sA , sB . Supposing that the body moves in the ellipse a/b , while this ellipse itself is carried about s with an angular motion, so that s/b in the moveable orbit being equal to s/L in the fixed orbit, the angle Asb may be to AsL in a constant invariable proportion, suppose that of G to F ; then the increments of these angles, while sL and s/b decrease equally, will observe the same constant proportion; and the angular motions about s of two bodies b and L , revolving in the same time in these orbits, will be in the same proportion, as also the areas described by rays drawn from these bodies to s : so that if the bodies be projected together at A with velocities in the same proportion, and are acted on by the necessary centripetal forces, they will move in these orbits, and approach equally towards s , and arrive at b and L in the same time. The motion of approach to the centre being the same at equal distances from it,
and

and this motion being caused by the excess of their gravities above the centrifugal forces arising from their circular motions about s ; the gravity will exceed the centrifugal force in the one orbit by the same excess as in the other, and therefore the difference of the centrifugal forces must be the same as the difference of their gravities; so that, to find the gravity in the moveable orbit, we are to add to the gravity in the fixed orbit, at the same distance, the excess of the centrifugal force in the moveable orbit there, above the centrifugal force in the fixed orbit at the same distance. These centrifugal forces are in a given proportion to each other, *viz.* in that of the squares of the angular motions, or in the proportion of G^2 to F^2 , and their difference must be in a given proportion to either; the same centrifugal forces, at different distances, are reciprocally as the cubes of the distances, as we shewed above, and their differences must vary in the same proportion: so that the difference of the gravities in the moveable and immoveable, must vary in the reciprocal proportion of the cubes of the distances.

15. If the ellipse is carried about s with a progressive motion, that is in the direction of the motion of the body, the angular motion of the body in the moveable orbit is greater than in the fixed orbit, and the centrifugal force, and consequently the gravity is greater. But if the ellipse is carried about s with a retrograde motion, the angular motion in the moveable orbit, and consequently the gravity, is lesser. In the first case, the difference of the centrifugal forces is to be added to the gravity in the fixed orbit, to find the gravity in the revolving orbit at the same distance from s . In the latter case, the difference of the centrifugal forces is to be subtracted from the gravity in the fixed orbit, to find the gravity

gravity in the revolving orbit at the same distance from s .

16. The force in the fixed ellipse increases as the square of the distance decreases; add to this a force that increases as the cube of the distance decreases, and the sum must increase in a higher proportion than that of the squares of the distances, but never in so high a proportion as their cubes. A body therefore that moves in an ellipse that has itself a progressive motion about s , must be acted on by a force that varies according to some power of the distance higher than the square, but less than the cube. The greater this motion of the ellipse is, the greater is the excess of the centrifugal force in the moveable ellipse above that in the fixed ellipse, at the same distance from s ; and the greater is the quantity that varies as the cube of the distance in the aggregate, in proportion to that which varies in it as the square of the distance only; and the more does the proportion of the aggregate vary from that of the squares towards that of the cubes of the distances. In such a moveable ellipse, the gravity, which is as the aggregate, cannot be said to vary in the proportion of any one power of the distance accurately; but if the ellipse is very near to a circle, the proportion of the aggregate will be found to vary very nearly as a certain power of the distance, and the motion of the ellipse may be adjusted so as that the aggregate may vary, very nearly, as any power of the distance that can be assigned betwixt the squares and the cubes.

17. If from a force that increases as the square of the distance decreases, you subduct a force that increases in a higher proportion, *viz.* as the cube of the distance decreases, the remainder must increase
in

in a less proportion than that in which the square of the distance decreases. A body, therefore, that moves in an ellipse which revolves itself at the same time with a retrograde motion about s , must be acted on by a gravity that varies in a less proportion than the square of the distance ; and the greater the motion of the ellipse is, the gravity will vary in a less proportion, so that if the motion of the ellipse be sufficiently great, the gravity may decrease instead of increasing as the distance decreases. By supposing the orbit near to a circle, the motion of the ellipse may be adjusted, that the remainder may vary according to any proportion less than that of the squares of the distances.

18. Our author has made an improvement of this, to judge of the motion of the apsides in any law of gravity : for, by supposing the gravity in the moveable ellipse, when near to a circle, computed from the foresaid principles, to vary according to any given law, he determines what must be the motion of the ellipse, or of the apsides, in consequence of this supposition ; or, the motion of the ellipse being given, he determines what is the power of the distance according to which the gravity varies, nearly, when the ellipse revolves with that given motion *.

19. We have said as much as our design will allow us, of the motions arising from gravity, that are performed in regular revolutions from the one apsis to the other ; where the distance from the centre of gravitation varies indeed, but so as to keep within certain limits, betwixt which the body constantly revolves ; and we have shewn that the motion

* See *Princip.* Lib. I. Sect. 9.

of the body may be of this kind, if the gravity decrease in a less proportion than that in which the cubes of the distances from the centre increase. But the motion of the body is not always of this kind, in these cases; for if the velocity of projection at B is sufficiently great, the body will, in some of these cases, recede for ever from the centre of gravitation, and never arrive at the higher apsis A. We have already shewed that if the gravity decrease as the cubes, or any higher powers, of the distance increase, and the velocity at B exceed, in the least, that which would carry the body in a circle there, about the centre of gravitation, it will recede from s for ever. If the gravity decrease in a less proportion than that of the cubes of the increasing distances, it may be projected at B with a motion which will still carry it for ever from the centre, provided the gravity decrease in a proportion greater than that in which the distances increase: for the limit here is the inverse simple proportion of the distances. If gravity vary more, the body may be carried off for ever from the centre by a finite motion of projection; but if the gravity varies in that proportion, or in any less proportion, then no finite force will be able to make the body move in such a manner, as to recede from the centre s for ever: but the body in these cases must always revolve betwixt the two *apsides*.

20. In order to see this, we may first suppose a body to be projected perpendicularly to the horizon, that is acted on by a gravity decreasing in a higher proportion than that of the increasing distances; and if the force of projection be sufficiently great, it will rise for ever with a motion continually retarded by the action of its gravity, but that shall never be altogether destroyed by these actions; because they
decrease

decrease in ſuch a manner that the ſum of an infinite number of them amounts to a finite quantity.

21. The ſame law of gravity is the limit betwixt the caſes of infinite aſcents, in curvilinear motions and in rectilinear: for our author has ſhewn, that if one body move in a curve, and another aſcend or deſcend in a right line, acted on by the ſame gravity, and their velocities be equal in any equal altitudes, they will be equal in all other equal altitudes *: and ſince the gravity of the body projected upwards in a vertical line, with a certain aſſignable force, is not able to bring it back again; it will not be able to make it return, if it was projected with the ſame force obliquely upwards, ſo as to move in a curve. For the centrifugal force, ariſing from the motion of rotation about *s*, leſſens the effect of the gravity, and makes it leſs capable to deſtroy the motion of

* Suppose the velocities of bodies *L* and *P* (*Fig. 66.*) to be equal at *L* and *P*, at equal diſtances *s L*, *s P*; and let them deſcribe the very ſmall lines *L l*, *P p*, ſo that *s p* being equal to *s l*, and *p n l* a circular arc deſcribed from the centre *s* meeting *s l* in *n*, *L n* muſt be equal to *P p*. The gravity of *L* toward *s* may be reſolved into two forces, one of which may be represented by *L R*, and acts in the direction of the tangent *L R*, the other in a direction *R s* perpendicular to the tangent or the direction of the body's motion. The latter has no effect in accelerating its motion, being perpendicular to it, and the former is to the gravity, as *L R* is to *s l*, or as *L n* is to *L l*. The motion of the body *P* is accelerated by its whole gravity, ſo that the forces which accelerate the bodies *L* and *P* are to each other, as *L n* (or *P p*) to *L l*; but the velocities at *L* and *P* having been equal, the times in which *L l* and *P p* are deſcribed are in the proportion of the ſpaces *L l* and *P p*; ſo that tho' the body *L* is accelerated by a leſs force in deſcending to *l*, the time of its acceleration is greater in the ſame proportion: from which it appears that their accelerations are equal in deſcribing theſe ſpaces, and their velocities conſequently equal at *l* and *p*. The velocities therefore of theſe bodies muſt be equal in all equal altitudes. See *Princip. Math.* Lib. I. Prop. 40.

aſcent

ascent in this case, than in the case of a perpendicular ascent. Therefore if gravity varies in the reciprocal proportion of some power of the distance higher than unit, a body may run out to infinity in its orbit, if it be projected with a certain force.

22. If this force is the same which it would acquire by falling from an infinite height, it will go off in a curve of the *parabolic* kind. But if it is projected with a greater force than that which would be acquired even by an infinite descent, the curve will be of the *hyperbolic* kind. If it is projected with the same velocity which it would acquire by falling from an infinite height (assuming different laws of gravity, but other circumstances similar) it will go off to infinity after a greater or less part of a revolution, or after a greater or smaller number of revolutions, according as the power of the distance, which is reciprocally proportional to the gravity, is greater or less. The limit here is a quarter of a revolution from the apsis, or the place where the direction of the body's motion is perpendicular to the line drawn to the centre; for it must always take more than that to get off from the apsis to an infinite distance. If gravity observe the reciprocal sesquiplicate proportion of the distance, then the body will go off in $\frac{1}{2}$ of a revolution. If it observe the reciprocal duplicate, it will go off for ever in a parabola, in half a revolution. If it observes the reciprocal $\frac{5}{2}$ power of the distance, it will go off in a complete revolution. But if gravity observe the reciprocal triplicate proportion of the distance, and the body be projected oblique to the radius, it will go off in an infinite number of revolutions*.

23. If

* In general, if gravity vary as the m power of the distance reciprocally, and the body is projected obliquely upwards with
Z
a force

23. If gravity decrease in a less proportion than the reciprocal simple proportion of the distances, and a body is projected from the apsis with any finite force whatsoever, it cannot rise for ever; but will have the same velocity at any distance, as it would have had at the same distance, supposing it had been projected at A directly upwards with the same force of projection: and since any finite force would have been destroyed in the perpendicular, if the body move in a curve it must return again, and after passing the higher apsis, descend again to the lower apsis, tho' that apsis be not in the same place as before. If gravity increase as the distance increases, *a fortiori* the body will never be able to ascend to an infinite distance. These observations shew the limits of the various sorts of motions, that can proceed from various laws of gravity.

C H A P. IV.

Of the motion of the moon.

1. **W**E have explained the motions of the bodies in the solar system, from gravity, and have taken notice of some inequalities or errors in their motions, that arise from the same principle.

a force that is to that which would carry it in a circle as 1 to $\sqrt{\frac{m-1}{2}}$, it will rise for ever from the centre, and go off in the $\frac{1}{6-2m}$ part of a revolution or in the $\frac{1}{2}n$ part of the revolution. Supposing $\frac{1}{n}$ to be the excess of 3 above the number m . If the gravity follow the reciprocal proportion of the $2\frac{22}{108}$ power of the distance, the body will go off in 50 revolutions. See *Fluxions*, § 416, & seq.

But

But the manifold irregularities that are produced by it in the motion of the moon deserve particularly to be considered, as she is the nearest to us in the system, and as great advantages might be deduced from her motions, if they could be subjected to exact computation. Formerly they who built systems had great difficulties to reconcile their principles with the phenomena: our author anticipates observations, and the more perfect our knowledge of the motions in the system shall become, the more will this philosophy be esteemed. Posterity will see its excellence yet more fully than we do, when the celestial motions shall be determined more accurately, by a series of long-continued exact observations.

2. To give the principles of our author's computations on this perplexed subject, in as plain a manner as possible, we must recollect what has been already observed; that if the sun acted equally on the earth and moon, and always in parallel lines, this action would serve only to restrain them in their annual motions round the sun, and no way affect their actions on each other, or their motions about their common centre of gravity. In that case, if they were both allowed to fall directly towards the sun, they would fall equally, and their respective situations would be no way affected by their descending equally towards it. We might then conceive them as in a plane, every part of which being equally acted on by the sun, the whole plane would descend towards the sun, but the respective motions of the earth and moon would be the same in this plane as if it was quiescent. Supposing then this plane, and all in it, to have the annual motion imprinted on it, it would move regularly round the sun, while the earth and moon would move in it, with respect to each other, as if the plane was at rest, without any

Z 2

irregu-

irregularities. But because the moon is nearer the sun in one half of her orbit than the earth is, and in the other half of her orbit is at a greater distance than the earth from the sun, and the power of gravity is always greater at a less distance; it follows, that in one half of her orbit the moon is more attracted than the earth towards the sun, and in the other half less attracted than the earth; and hence irregularities necessarily arise in the motions of the moon, the excess, in the first case, and the defect, in the second, of the attraction, becoming a force that disturbs her motion: add to this, that the action of the sun on the earth and moon is not directed in parallel lines, but in lines that meet in the centre of the sun.

3. To see the effects of these powers, let us suppose that the projectile motions of the earth and moon were destroyed, and that they were allowed to fall freely towards the sun. If the moon was in conjunction with the sun, or in that part of her orbit which is nearest to him, the moon would be more attracted than the earth, and fall with greater velocity towards the sun; so that the distance of the moon from the earth would be increased in the fall. If the moon was in opposition, or in the part of her orbit which is farthest from the sun, she would be less attracted than the earth by the sun, and would fall with a less velocity towards the sun than the earth, and the moon would be left behind by the earth; so that the distance of the moon from the earth would be increased, in this case also. If the moon was in one of the quarters, then the earth and moon being both attracted towards the centre of the sun, they would both directly descend towards that centre, and by approaching to the same centre, they would necessarily approach at the same time to each

each other, and their distance from one another would be diminished, in this case. Now, wherever the action of the sun would increase their distance, if they were allowed to fall towards the sun, there we may be sure the sun's action, by endeavouring to separate them, diminishes their gravity to each other; wherever the action of the sun would diminish their distance, there the sun's action, by endeavouring to make them approach to one another, increases their gravity to each other: that is, in the conjunction and opposition, their gravity towards each other is diminished by the action of the sun; but in the two quarters it is increased by the action of the sun. To prevent mistaking this matter, it must be remembered, it is not the total action of the sun on them that disturbs their motions, it is only that part of its action by which it tends to separate them, in the first case, to a greater distance from each other; and that part of its action by which it tends to bring them nearer to each other, in the second case, that has any effect on their motions with respect to each other. The other, and the far more considerable, part has no other effect but to retain them in their annual course, which they perform together about the sun.

4. In considering, therefore, the effects of the sun's action on the motions of the earth and moon with respect to each other, we need only attend to the excess of its action on the moon above its action on the earth, in their conjunction; and we must consider this excess as drawing the moon from the earth towards the sun in that place. In the opposition, we need only consider the excess of the action of the sun on the earth above its action on the moon, and we must consider this excess as drawing the moon from the earth, in this place, in a direction

opposite to the former, that is, towards the place opposite to where the sun is; because we consider the earth as quiescent, and refer the motion, and all its irregularities to the moon. In the quarters, we consider the action of the sun as adding something to the gravity of the moon towards the earth.

5. Suppose the moon setting out from the quarter that precedes the conjunction, with a velocity that would make her describe an exact circle round the earth, if the sun's action had no effect on her; and because her gravity is increased by that action, she must descend towards the earth, and move within that circle: her orbit, there, will be more curve than otherwise it would have been; because this addition to her gravity will make her fall farther at the end of an arc below the tangent drawn at the other end of it; her motion will be accelerated by it, and will continue to be accelerated till she arrives at the ensuing conjunction; because the direction of the action of the sun upon her, during that time, makes an acute angle with the direction of her motion. At the conjunction, her gravity towards the earth being diminished by the action of the sun, her orbit will be less curve there, for that reason; and she will be carried farther from the earth as she moves to the next quarter; and, because the action of the sun makes then an obtuse angle with the direction of her motion, she will be retarded by the same degrees by which she was accelerated before.

6. Thus she will descend a little towards the earth, as she moves from the first quarter towards the conjunction, and ascend from it, as she moves from the conjunction to the next quarter. The action which disturbs her motion will have a like, and almost equal, effect upon her, while she moves
in

in the other half of her orbit, I mean that half of it which is farthest from the sun: she will proceed from the quarter that follows the conjunction with an accelerated motion to the opposition, approaching a little towards the earth, because of the addition made to her gravity, at that quarter, from the action of the sun; and receding from it again, as she goes on from the opposition to the quarter from which we supposed her to set out. The areas described in equal times by a ray drawn from the moon to the earth will not be equal, but will be accelerated by the conspiring action of the sun, as she moves towards the conjunction or opposition from the quarters that precede them; and will be retarded by the same action, as she moves from the conjunction or opposition to the quarters that succeed them.

7. Our author has computed the quantities of these irregularities from their causes. He finds, that the force added to the gravity of the moon in her quarters, is to the gravity with which she would revolve in a circle about the earth, at her present mean distance, if the sun had no effect on her, as 1 to $178\frac{2}{40}$. He finds the force subducted from her gravity, in the conjunctions and oppositions, to be double of this quantity, and the area described in a given time in the quarters to be to the area described in the same time in the conjunctions and oppositions, as 10973 to 11073. He finds, that, in such an orbit, her distance from the earth in her quarters, would be to her distance in the conjunctions and oppositions, as 70 to 69.

8. The moon does not move in the same plane, round the earth, in which the earth moves round the sun, but in a plane that is inclined to it in an angle

of about 5 degrees : and hence it is that the centre of the moon appears to us to trace a different circle from the *ecliptic*, the circle which the centre of the sun appears to describe in the heavens. These circles cut each other in two opposite points, that are called by astronomers the *nodes* of the moon ; at the greatest distance from the nodes, these circles are separated from each other by about five degrees. The eclipses of the sun and moon depend on their distances from these nodes, at the time of the new and full moon ; for, if the change of the moon happen when she is near one of the nodes, she eclipses the sun ; and, if the moon is full, near one of the nodes, she must fall into the shadow of the earth, and there become eclipsed. Astronomers have at all times been very attentive to the situation of the nodes, in order to calculate these eclipses, which have been always a phænomenon much considered by them. The nodes are not fixed in the same part of the heavens, but are found to move over all the signs in the *ecliptic*, with a retrograde motion, in about eighteen or nineteen years.

9. Sir *Isaac Newton* has not only shewed that this motion arises from the action of the sun, but has calculated, with great skill, all the elements and varieties in this motion, from its cause. We called these points the moon's nodes, in which her orbit cuts the plane in which the earth revolves about the sun, and the line that joins the points we call the line of the nodes. We say the motion of the nodes is direct when they proceed in the same way as the moon moves in her orbit, *viz.* from west to east, according to the order of the signs *Aries*, *Taurus*, &c. in the *ecliptic* ; and we say their motion is retrograde, when they move with a motion contrary to that of the moon, or from east to west, contrary to

to the order of the signs. We conceive the plane of the moon's motion to pass always through the centre of the earth and the centre of the moon, and to be a plane in which the right line joining their centres, and the right line that is the direction of the moon's motion, or the tangent of her orbit, are always found. It is certain, that if the earth and moon were always acted on equally by the sun, they would descend equally toward the sun; the plane determined always by these two lines, would descend with them, keeping always parallel to itself, so that the moon would appear to us to revolve in the same plane constantly, with respect to the earth. But the inequalities in the action of the sun, described above, will bring the moon out of this plane, to that side of the plane on which the sun is, in the half of her orbit that is nearest the sun, and toward the other side, in the half of her orbit that is farthest from the sun.

10. From which we have this general rule for judging of the effect of the sun on the nodes; that while the moon is in the half of her orbit that is nearest the sun, the node towards which she is moving is made to move towards the conjunction with the sun; and while the moon is in the half of her orbit which is farthest from the sun, the node towards which she is moving is made to move towards the opposition: but when the nodes are in conjunction with the sun, its action has no effect upon them. In the first case, the moon is brought into a direction which is on the same side, as the sun is, of that direction which she would follow of herself: and the intersection of a plane passing through this direction, and through the centre of the earth, will cut the ecliptic, on that side towards which the moon moves, in a point nearer the conjunction,

junction, than if there was no action of the sun to disturb her motion. In the other case, the action of the sun has a contrary direction, and for the same reason makes the ensuing node move towards the opposition. When the line of the nodes produced passes thro' the sun, then the sun, being in the plane of the moon's motion, has no effect to bring her to either side; and therefore, in that case, the nodes have no motion at all.

11. From this general rule, it appears, that if you suppose the nodes to be in the quarters A and c, (*Fig. 67.*) after the moon sets out from the node A, that is, in the quarter preceding the conjunction B, the ensuing node c moves towards the conjunction B, and is therefore retrograde; because it moves in a direction opposite to that in which the moon moves; and, in all this revolution of the moon, the nodes are manifestly retrograde; for, after the moon passes the quarter c that succeeds the conjunction, then the ensuing node A moves towards the opposition D, so that the nodes are, in that half of her orbit also, retrograde.

12. Suppose the nodes in the situation N n, so as one of them may be between the quarter A and the ensuing conjunction B, while the other node n falls on the opposite point of the moon's circle, between the subsequent quarter c and the opposition D. In this case, while the moon moves from A to N, the node N moves towards the conjunction B (by the general principle in § 10.) and therefore its motion is direct. While the moon moves from N to c, the ensuing node n moves towards the conjunction B, and therefore is retrograde; and because the arc N c exceeds A N, the retrograde motion exceeds the direct

direct motion. While the moon moves from c to n , the ensuing node n moves towards the opposition D , and the motion of the nodes is then direct. But while the moon moves from n to A , the ensuing node N moves towards the opposition D , and then the motion of the nodes being contrary to the motion of the moon, their motion is retrograde; and because the arc nA exceeds nc , it is apparent that the motion is more retrograde than direct.

13. When (*Fig. 68.*) one node N is between the conjunction B and the ensuing quarter c , while the moon moves from A to N , the ensuing node N moves towards the conjunction B , and therefore is retrograde: while the moon moves from N to c , the ensuing node n moves towards the conjunction, and is direct. But as the arc AN exceeds Nc , the retrograde motion of the nodes must exceed the direct motion. While the moon moves from c to n , the motion of the ensuing node is towards the opposition D , and is therefore retrograde. While the moon moves from n to A , the ensuing node N moves towards the opposition D , and therefore is direct. But, as the arc cn exceeds AN , it follows, that the retrograde motion exceeds the direct motion.

It appears, therefore, that in every revolution of the moon, the retrograde motion of the nodes exceeds the direct motion, excepting only when the line of the nodes passes through the sun, in which case there is no motion of the nodes at all. We see then, how, from the principle of gravity, the nodes of the moon are made to recede every year. Our author has determined the quantity * of this retrograde motion in every revolution of the moon, and

* *Princip. Lib. III. Prop. 32.*

in every year; and it is no small pleasure to see how exactly the theory of these motions, drawn from their causes, agrees with the observations of astronomers. He finds, from the theory of gravity, that the nodes ought to move backward about $19^{\circ} 18' 1''$ in the space of a year, and the astronomical tables make this motion $19^{\circ} 21' 21''$; whose difference is not $\frac{1}{308}$ of the whole motion of the nodes in a year. By a more correct computation of this motion from its cause, the theory and observation agree within a few seconds.

14. The inclination of the moon's orbit to the ecliptic, is also subject to many variations. When the nodes are in the quarters A and c, while the moon moves from the quarter A to the conjunction B, the action of the sun diminishes the inclination of the plane of her orbit; the inclination of this plane is least of all when the moon is in the conjunction B; it increases again as she moves from the conjunction B to the next quarter at c, and is there restored to its first quantity nearly. When the nodes of the moon are in B and D, so that the line of the nodes passes through the sun, the inclination of the moon's orbit is not affected by the action of the sun; because, in that case, the plane of her orbit produced passes through the sun: and therefore the action of the sun can have no effect to bring the moon out of this plane to either side. It is in this last case that the inclination of the moon's orbit is greatest; it decreases as the nodes move towards the quarters; and it is least of all when the nodes are in the quarters, and the moon either in the conjunction or opposition. Our author calculates these irregularities from their causes, and finds his conclusions agree

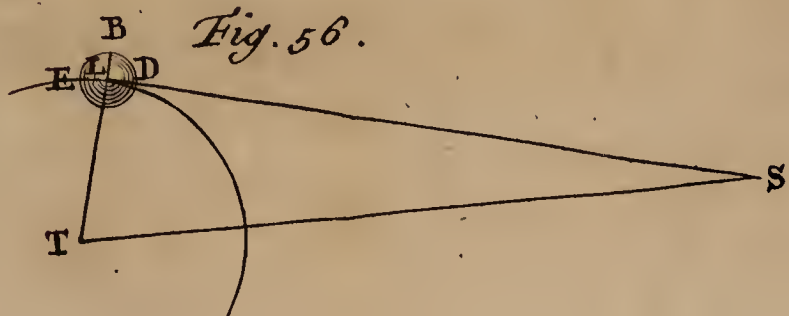


Fig. 57.

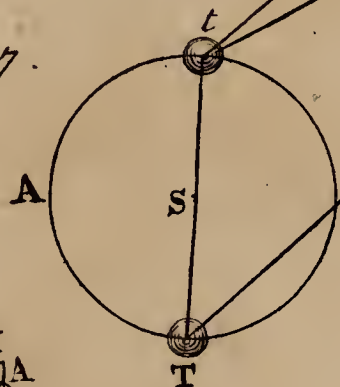


Fig. 58.

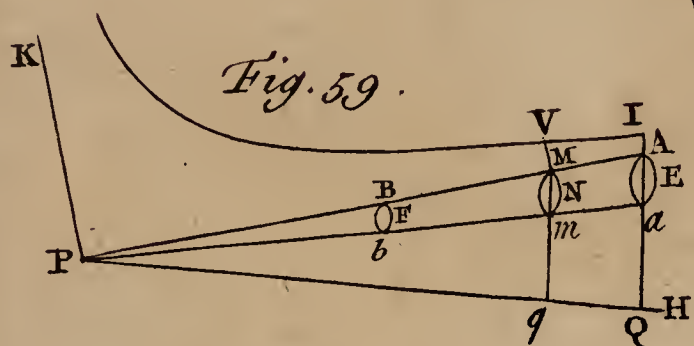
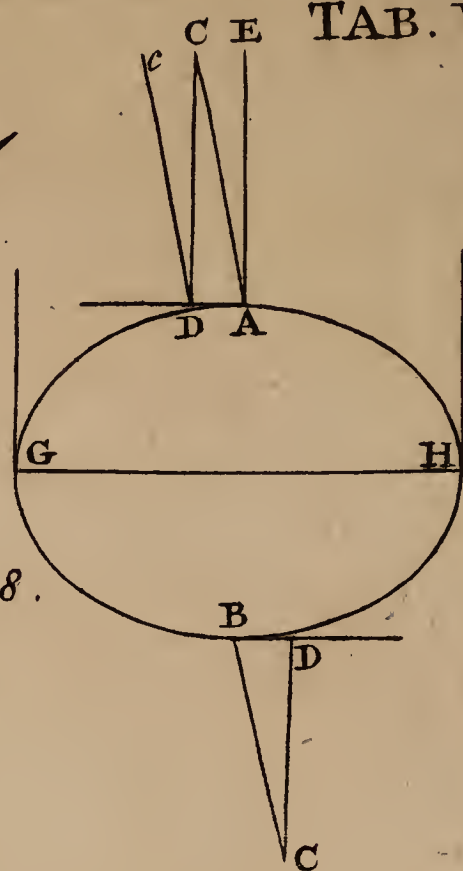


Fig. 60.

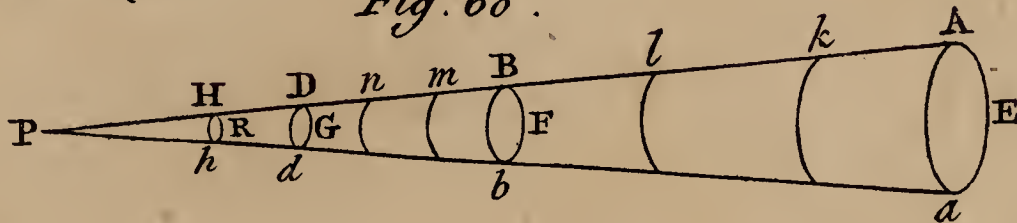
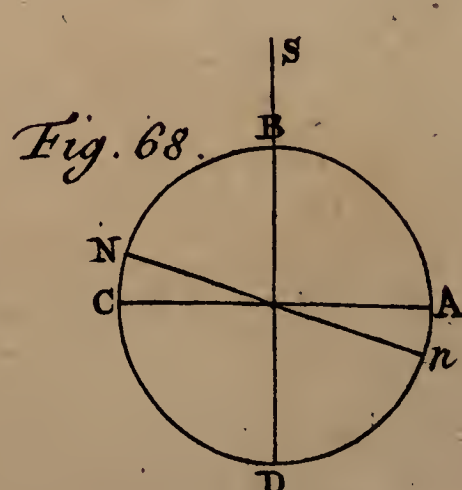
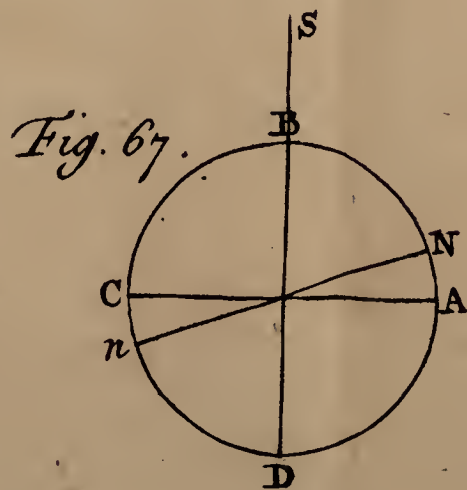
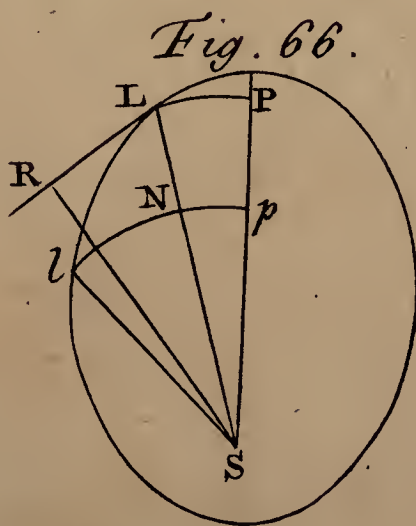
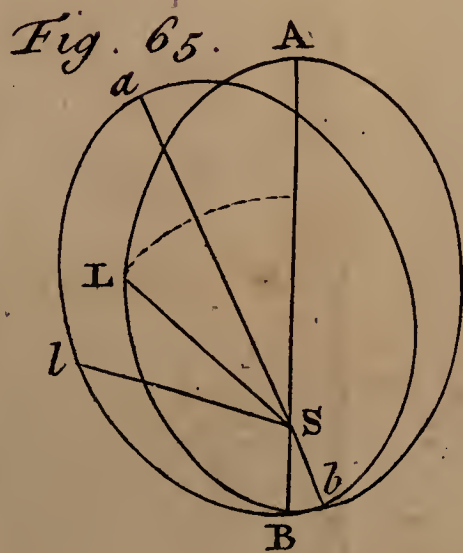
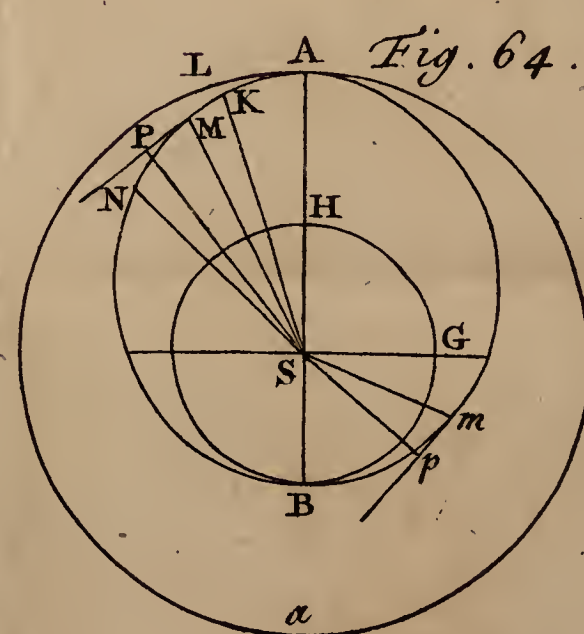
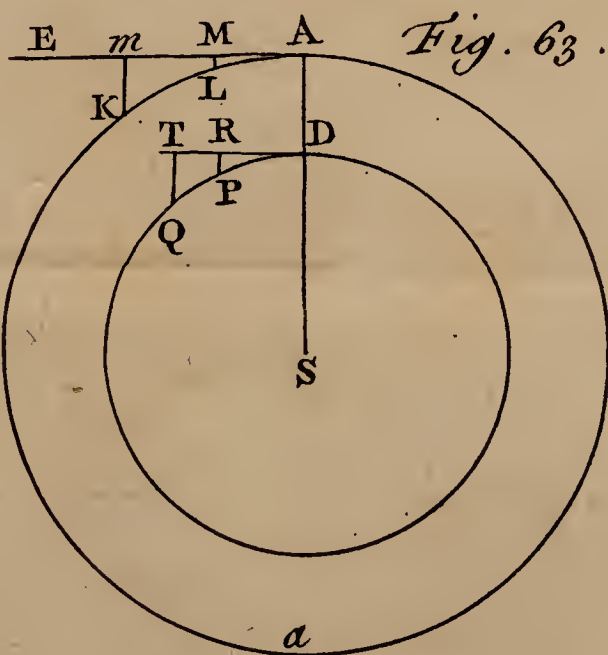
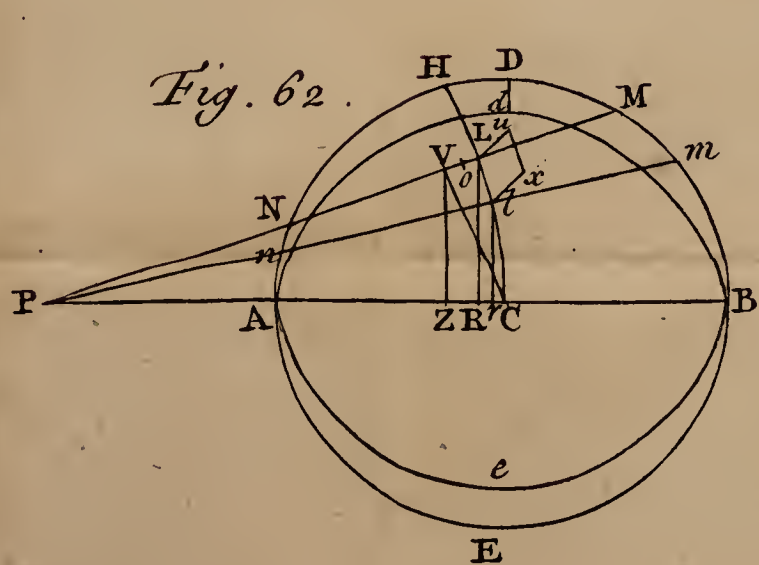
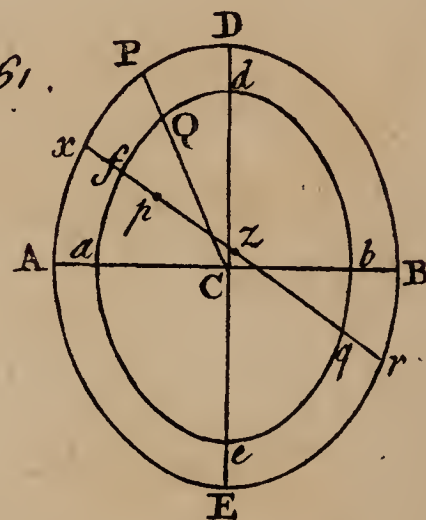
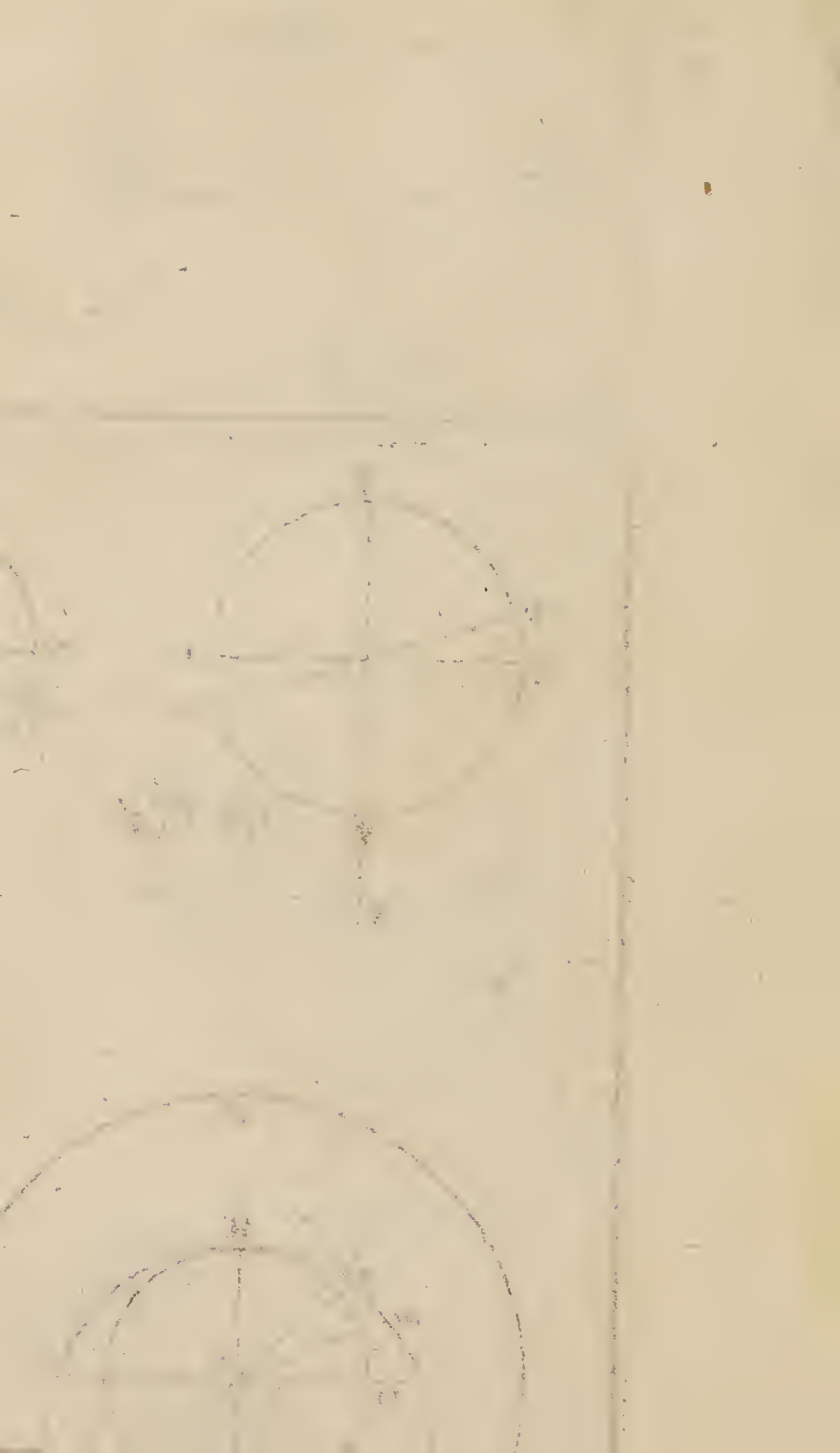


Fig. 61.





agree very well with the observations of astronomers *.

15. The action of the sun diminishes the gravity of the moon towards the earth, in the conjunctions and oppositions, more than it adds to it in the quarters, and, by diminishing the force which retains the moon in her orbit, it increases her distance from the earth and her periodic time : and because the earth and moon are nearer the sun in their perihelium than in their aphelium, and the sun acts with a greater force there, so as to subduct more from the moon's gravity towards the earth ; it follows, that the moon must revolve at a greater distance, and take a longer time to finish her revolution in the perihelium of the earth, than in the aphelium ; and this also is conformable to observation.

16. There is another remarkable irregularity in the moon's motion, that also arises from the action

* To make the foregoing account of the motion of the moon's nodes still clearer, we have added *Fig. 69, (Plate VI.)* in which, the plane of the scheme representing that of the ecliptic, *s* is the sun, *t* the centre of the earth, *L* the moon in her orbit *D N d n* ; *N n* is the line of the nodes passing between the quadrature *Q* and the moon's place *L*, in her last quarter. Let now *L P*, any part of *L s*, represent the excess of the sun's action at *L*, above his action at *t*, and this being resolved into the force *L R*, perpendicular to the plane of the moon's orbit, and *P R* parallel to it, 'tis the former only that has any effect to alter the position of the orbit, and in this it is wholly exerted. Its effect is twofold ; (1.) It diminishes its inclination, by a motion which we may conceive as performed round the diameter *D d*, to which *L t* is perpendicular. (2.) Being compounded with the moon's tangential motion at *L*, it gives it an intermediate direction *L i* ; thro' which, and the centre of the earth, a plane being drawn must meet the ecliptic nearer the conjunction *c* than before : and in the same manner, the other cases are explained.

of the sun : I mean, the progressive motion of the apsidal. The moon describes an ellipse about the centre of the earth, having one of the foci in that centre. Her greatest and least distances from the earth are in the apsidal, or extremities of the longer axis of the ellipse. This is not found to point always to the same place in the heavens, but to move with a progressive motion forwards, so as to finish a revolution round the earth's centre in about nine years.

To understand the reason of this motion of the apsidal, we must recollect what was shewed above, that if the gravity of a body decreased less as the distance increases, than according to the regular course of gravity, the body would descend sooner from the higher to the lower apsis, than in half a revolution ; and therefore the apsis would recede in that case, for it would move in a contrary direction to the motion of the body, meeting it in its motion. But if the gravity of the body should decrease more as the distance increases than according to the regular course of gravity, that is, in a higher proportion than as the square of the distance increases, the body would take more than half a revolution to move from the higher to the lower apsis ; and therefore, in that case, the apsidal would have a progressive motion in the same direction as the body.

In the quarters, the sun's action adds to the gravity of the moon, and the force it adds is greater, as the distance of the moon from the earth is greater ; so that the action of the sun hinders her gravity towards the earth, from decreasing as much while the distance increases, as it ought to do according to the regular course of gravity ; and therefore, while the moon is in the quarters, her apsidal must recede.

In the conjunction and opposition, the action of the sun subducts from the gravity of the moon towards the earth, and subducts the more the greater her distance from the earth is, so as to make her gravity decrease more as her distance increases, than according to the regular course of gravity; and therefore, in this case, the apsides are in a progressive motion. Because the action of the sun subducts more in the conjunctions and oppositions from her gravity, than it adds to it in the quarters, and, in general, diminishes more than it augments her gravity; hence it is that the progressive motion of the apsides exceeds the retrograde motion; and therefore, the apsides are carried round according to the order of the signs.

17. Thus the various irregularities of the moon's motion are explained from gravity: and from this theory, with the assistance of a long series of accurate observations, her motion may be at length reduced so exactly to computation, and her appulses to the fixed stars, over which she passes in her course, may be predicted with so much accuracy, as to afford, on many occasions, an opportunity to navigators, to discover their longitude at sea.

C H A P. V.

Of the path of a secondary planet upon an immoveable plane; with an illustration of Sir Isaac Newton's account of the motions of the satellites, from the theory of gravity.*

IN describing the motions of the solar system, it is usual to consider the primary planets, as revolving in immoveable planes, but to refer the motions of the satellites to planes that are carried along with their primaries about the sun. Sir *Isaac Newton* follows the same method, in accounting for their motions from the theory of gravity: by this analysis, the explication of the motions themselves, and of the powers that produce them, is rendered more simple and easy, than if we should refer the motion of the satellite to an immoveable plane, and contemplate only the path described by it, in consequence of so compounded a motion, in absolute space.

The properties, however, of this path are more simple than perhaps will be expected on a superficial consideration of it; and the referring of the motion of the satellite to it, may be of use on some occasions, particularly for resolving the difficulties some have found to understand Sir *Isaac Newton's* account of the motions of the satellites, from gravity. This path is, in some cases, concave towards the sun throughout; in other cases, the part of it nearest

* The following chapter, as belonging properly to this place, is inserted from a letter of the author, to his learned friend Dr. *Benjamin Hoadley*, physician to his Majesty's household.

the sun is convex towards the sun, and the rest is concave. An instance of the former we have in the moon, of the latter in the satellites of the superior planets.

The force that bends the course of the satellite into a curve, when the motion is referred to an immoveable plane, is, at the conjunction, the difference of its gravity towards the sun, and of its gravity towards the primary. When the former prevails over the latter, the force that bends the course of the satellite tends towards the sun; consequently, the concavity of the path is towards the sun: and this is the case of the *Moon*, as will appear afterwards. When the gravity towards the primary exceeds the gravity towards the sun, at the conjunction, then the force that bends the course of the satellite tends towards the primary, and therefore towards the opposition of the sun; consequently the path is there convex towards the sun: and this is the case of the satellites of *Jupiter*. When these two forces are equal, the path has, at the conjunction, what mathematicians call a point of *rectitude*; in which case, however, the path is concave towards the sun throughout.

Because the gravity of the moon towards the sun is found to be greater, at the conjunction, than her gravity towards the earth, so that the point of equal attraction, where those two powers would sustain each other, falls then between the moon and earth, some * have apprehended that either the parallax of the sun is very different from that which is assigned by astronomers, or that the moon ought necessarily to abandon the earth. This apprehension may be

* See *Cosmotheoria puerilis*.

easily removed, by attending to what has been shewn by Sir *Isaac Newton*, and is illustrated by vulgar experiments concerning the motions of bodies about one another, that are all acted upon by a third force in the same direction. Their relative motions, not being in the least disturbed by this third force, if it act equally upon them in parallel lines; as the relative motions of the ships in a fleet, carried away by a current, are no way affected by it, if it act equally upon them; or as the rotation of a bullet, or bomb, about its axis, while it is projected in the air, or the figure of a drop of falling rain, are not at all affected by the gravity of the particles of which they are made up, towards the earth. It is to the inequality of the actions of the sun upon the earth and moon, and the want of parallelism in the directions of these actions, only, that we are to ascribe the irregularities in the motion of the moon.

But it may contribute towards removing this difficulty, to observe, that if the absolute velocity of the moon, at the conjunction, was less than that which is requisite to carry a body in a circle there around the sun, supposing this body to be acted on by the same force which acts there on the moon, (*i. e.* by the excess of her gravity towards the sun, above her gravity towards the earth, then the moon would, indeed, abandon the earth. For, in that case, the moon having less velocity than would be necessary to prevent her from descending within that circle, she would approach to the sun, and recede from the earth. But tho' the absolute velocity of the moon, at the conjunction, be less than the velocity of the earth in the annual orbit, yet her gravity towards the sun is so much diminished by her gravity towards the earth, that her absolute velocity is still much superior to that which is requisite to carry a body in a circle,

circle, there, about the sun, that is acted on by the remaining force only. Therefore, from the moment of the conjunction, the moon is carried without such a circle, receding continually from the sun to greater and greater distances, till she arrive at the opposition; where, being acted on by the sum of those two gravities, and her velocity being now less than what is requisite to carry a body in a circle, there, about the sun, that is acted on by a force equal to that sum, the moon thence begins to approach to the sun again. Thus she recedes from the sun and approaches to it by turns, and in every month her path has two apses, a *perihelium* at the conjunction, and an *aphelium* at the opposition; between which she is always carried, in a manner similar to that in which the primary planets revolve between their apses. The planet recedes from the sun at the perihelium, because its velocity, there, is greater than that with which a circle could be described about the sun at the same distance, by the same centripetal force; and approaches towards the sun from the aphelium, because its velocity there is less than is requisite, to carry it in a circle, at that distance, about the sun. See my *Treatise of Fluxions*, Article 447.

Tho' the path of the moon be concave towards the sun throughout, its curvature is very unequal: it is least at each lower apside or conjunction, and greatest at each higher apside or opposition. The path of a satellite of *Jupiter* has likewise two apses, in the part which is described every synodic revolution: but in the lower apside, the convexity is towards the sun; and it has likewise two points of contrary flexure in every such part*.

* See the note to *Corol.* 1. Prop. II. below.

By considering this path, we shall arrive at the same conclusions which Sir *Isaac Newton* derived, more briefly from the laws of motion; that if the solar action was the same on the satellite and on the primary, and in the same direction, the motion of the satellite around the primary, would be the same as if the sun was away. This will appear from the following propositions, where we suppose the orbits of the primary about the sun, and of the satellite about the primary, to be both circular, and the motions in these orbits to be uniform and in the same plane.

PROPOSITION I. *Fig. 70. Plate VI.*

The path of the satellite, on an immoveable plane, is the epicycloid that is described by a given point in the plane of a circle, which revolves on a circular base, having its centre in the centre of the sun, and its diameter in the same proportion to the diameter of the revolving circle, as the periodic time of the primary about the sun, to the time of the synodic revolution of the satellite about the primary: the tangent of the path is perpendicular to the right line that joins the satellite to the contact of the two circles: and the absolute velocity of the satellite is always as its distance from that contact.

Let T denote the periodic time of the primary about the sun, t the periodic time of the satellite about the primary. Let s represent the sun, A the orbit of the primary; upon the radius As , take AE to As as t is to T . From the centre s describe the circle Eez , and from the centre A the circle EMF . Let this circle EMF revolve on the other Eez , as its base: then a point L , taken on the
plane

plane of the circle EMF , at the distance AL , equal to the distance of the satellite from the primary, shall describe the path of the satellite.

For suppose the circle EMF to move into the situation emf , the point A to a , L to l , and let AL and al , produced, meet EMF and emf , in M and m . Upon the arc em take $er = EM$, then the angle $ear = EAM$. Let ar meet the circle $cl d$, described from the centre a with the distance al , in q ; and because $eaq = EAL$, the angle eaq represents the elongation of the satellite from the sun at its first place L . Because $em (= er + rm) = ee + EM$ and $er = EM$, it follows that $rm = ee$; consequently the angle $ram : ese :: se : AE :: T - t : t$; or, as the angular velocity of the satellite from the sun, to the angular velocity of the primary about the sun. But ese is the angle described by the primary about the sun, consequently ram , or $qa l$, is the simultaneous increment of the elongation of the satellite from the sun; l is its place when the primary comes to a ; and the epicycloid described by l is the path of the satellite.

Because the circle EMF moves on the point E , the direction of the motion of any point L is perpendicular to EL ; or the tangent of the path at any point L is perpendicular to EL . The velocity of any point L is as its distance EL ; and, the motion of the primary A being supposed uniform and represented by EA , the velocity of the satellite shall be represented by EL .

PROPOSITION II.

Upon AS take $AB : AS :: tt : TT$ (or $AB : AE :: AE : AS$); upon the diameter EB describe the circle

$Aa3$

EKB

$E K B$ meeting $E L$ in K , take $L O$ a third proportional to $L K$ and $L E$, on the same side of L with $L K$; and O shall be the centre of the curvature at L of the path, and $L O$ the ray of curvature.

Because $E L$ and $e l$ are perpendicular to the path at the points L and l , let them be produced, and their ultimate intersection o shall be the centre of curvature at L . Produce $q e$ till it meet $L E$ in v , join $s v$, and the angle $s e v = q e a = L E A = s e v$; consequently the angle $e v e = e s e$, the angle $e v s = e s e$, and the angle $e v s = e e s$, and $s v$ is ultimately perpendicular to $E o$. Now the angle $E o e$ is ultimately to $E v e$ ($= E s e$) as $E v$ to $E o$, that is (because $E v : E K :: E s : E B :: A s : A e$) as $E K \times A s$ to $E o \times A e$. But the angular motion of $E L$ being equal to the angular motion of $E A$, while the circle $E M F$ turns on the point E , $L E l$ is therefore ultimately equal to $A E a$, which is to $E s e$ as $s A$ to $A e$; and $E o e$ being to $L E l$ as $E L$ to $L o$, it follows, that $E o e : E s e :: s A \times E L : A e \times L o :: E K \times s A : E o \times A e$. Therefore $E L : L o :: E K : E o$, and $E L : L K :: L o : E L$, or $L K$, $L E$ and $L o$ are in continued proportion. This theorem serves for determining the ray of curvature of *epicycloids* and *cycloids* of all sorts; only when the base $E e$ is a right line, $A B$ vanishes, and $E B$ becomes equal to $E A$.

Corol. I. When $A L$ or $A C$ is less than $A B$, then (because $L o$ is always on the same side of the point L with $L K$) the path is concave towards s throughout. When $A C = A B$, the curvature at the conjunction vanishes, or the path has there a point of *rectitude*. When $A C$ is greater than $A B$ (or $A s \times \frac{t}{T}$), a portion of the path near the conjunction is convex towards s , because a part of the circle $c L D$

falls

falls within the circle B K E ; and when L comes to either of the intersections of these two circles, the path has a point of contrary flexure *.

Corol. 2. In the case of the moon, $t t : T T :: 1 : 178$, and $A B = \frac{1}{178} \times A S$; but $A C$ is about $\frac{1}{337} \times A S$; consequently $A C$ is less than $A B$, and the path of the moon is concave towards the sun throughout.

PROPOSITION III.

Let $A B : A S :: t t : T T$, and the force by which the path of the satellite can be described on an immoveable plane, is always directed to the point B upon the ray AS ; and is always measured by B L the distance of the satellite from the point B, the gravity of the primary towards the sun being represented by B A.

We conceive the force by which this path could be described, on an immoveable plane, to be resolved into a force that acts in the direction L O, perpendicular to the path, and bends the path, but has no effect on the velocity of the satellite ; and a force perpendicular to L O that accelerates or retards the motion of the satellite. The former of these is measured by L K, the latter by B K, the gravity of the primary towards the sun being measured by A B. For the former is to the gravity of the primary towards s, as $\frac{E L^2}{L O}$ to $\frac{E A^2}{A S}$ (those forces being directly as the squares of the velocities, and inversely as the

* If $A C = A E$, these points meet again, and form a *cusp* : and if $A C$ is greater than $A E$, the path has a *nodus* : which last is the case of the innermost of the satellites of *Jupiter* and *Saturn*.

rays of the curvature) that is, as $L K$ to $A B$, by *Prop. II.* Therefore the gravity of the primary being represented by $A B$, the former force will be measured by $L K$.

The second force that acts on the satellite in the direction of the tangent of its path, and accelerates or retards its motion, is as the fluxion of the velocity $E L$ directly, and the fluxion of the time inversely. The fluxion of the time is measured by $\frac{A a}{E A}$ ($A a$ being the arc described by the primary, and $E A$ the velocity with which it is described) $= \frac{E e}{E B} = \frac{r m}{E B} = \frac{l q \times A E}{E B \times A C} =$ (supposing $a n$ and $q u$ to be perpendiculars to $e l$ in n and u , because $l q : l u :: a c : a n$, or $A C : A N$) $\frac{A E \times l u}{E B \times A N} = \frac{l u}{B K}$. Therefore the force which is measured by $l u$, the fluxion of the velocity $E l$, or $E L$, divided by the fluxion of the time or $\frac{l u}{B K}$, is measured by $B K$. The force, therefore, in the direction $L E$ being measured by $L K$, and the force in the perpendicular direction $K B$ by $K B$, the compounded force is measured by $L B$, and is directed from L to B .

It appears, from what has been demonstrated, that the path may be described by a force directed towards the point B , (which is given upon the ray $A s$, but revolves along with this ray about s) or by any forces which, compounded together, generate a force tending to B , and always proportional to $L B$, the distance of the satellite from B . Let $L H$ be equal and parallel to $A B$, and $A B H L$ shall be a parallelogram, and the force $L K$ may be compounded of $L H$ and $L A$, that is, the force $L K$ may be the result of a force $L H$ acting on the satellite, equal and parallel

parallel to AB , the gravity of the primary towards the sun, and of a force LA tending to the primary, and equal to the gravity by which the satellite would describe the circle CLD about the primary, in the same periodic time t , if the sun was away; because such a force is to the gravity of the primary towards the sun, (represented by AB) as $\frac{AL}{t^2}$ to $\frac{AS}{T^2}$ or as AL to $AS \times \frac{t^2}{T^2} = AB$.

Thus we arrive at the same conclusion which Sir *Isaac Newton*, more briefly, derived from an analysis of the motions of the satellite; that while the satellite gravitates towards the primary, if, at the same time, it be acted on by the same solar force as the primary, and with a parallel direction, it will revolve about the primary, in the same manner as if this last was at rest, and there was no solar action. These two forces, the gravitation towards the primary, and a force equal and parallel to the gravitation of the primary towards the sun, are exactly sufficient to account for the compounded motion of the satellite in its path, however complex a curved line it may appear to be. Nor is there any perturbation of the satellite's motion, but what arises from the inequality of the gravity of the satellite, and of the primary towards the sun, or from their not acting in parallel lines. If we should suppose them to move about their common centre of gravity, while this is carried round the sun, or if we suppose the orbits to be elliptical, the conclusions will still be found consonant to what was more briefly deduced by this great author.

C H A P VI.

Of the figure of the earth, and the precession of the equinoxes.

1. **I**F the earth was fluid, and had no motion on its axis, the equal gravitation of its parts towards each other would give it a figure exactly spherical, the columns from the surface to the centre mutually sustaining each other at equal heights from it. But, because of the diurnal rotation of the earth on its axis, the gravity of the parts at the equator is diminished by the centrifugal force arising from this rotation; the gravity of the parts on either side of the equator is diminished less, as the velocity of rotation is less, and the centrifugal force, arising from it acts less directly against the gravity of the parts; while the gravity at the poles is not at all affected by the rotation. The equilibrium that was supposed to be amongst the parts will not therefore now subsist in a spherical figure, but will be destroyed by the inequality of their gravitation, till the water rise at the equator and sink at the poles, so as by a greater height at the equator, to compensate the greater gravity at the poles; and till, by assuming an intermediate height in the intermediate places, the whole earth become of an oblate spheroidal form, whose diameter at the equator will be the greatest, and the axis the least, of all the lines that can pass through the centre.

2. If the gravity of a body at the equator was destroyed, the motion of rotation would there make it go off in a tangent to the earth; and by moving in the tangent it would rise, in a second of time,

from the spherical body of the earth, as much as one extremity of the arc which bodies describe there, in a second, falls below the tangent drawn at the other extremity: and this is found to be a space of about 7,54 lines, *French* measure. The effect of the centrifugal force of bodies at the equator, in a second of time, is proportional to this space. The effect of the centrifugal force at any place at a distance from the equator, for example, at *Paris*, is less for the reasons above mentioned; and, there, it is found by calculation, that it could only produce a motion of 3,267 lines in a second. Add this to what, by experiments, bodies are found to describe by their gravity at *Paris*, viz. 15 feet, 1 inch and 2 lines, and the sum 2177,267 lines will shew the space which bodies would describe by their gravity, in a second of time, if there was no centrifugal force there. By comparing this with the effect of the centrifugal force at the equator, in the same time, we shall find that the centrifugal force, there, is the $\frac{1}{289}$ part of the power of gravity, because 7,54 is to 2177,267 as 1 to 289.

3. From this it follows, that a body at the equator loses $\frac{1}{289}$, at least, of its gravity; and the equator must be at least, $\frac{1}{289}$ higher than the poles from the centre of the earth. But as the parts of the equator lose still of their gravity as they rise from the centre of the earth, and the regular course of gravity is altered by the change of figure, this is not the true proportion of the height of the earth at the equator, to its height at the poles.

Our author, who was never at a loss to find some expedient by which he might determine, accurately or near the truth, what he wanted; in order to take in these perplexed considerations, assumes, as an
hypo-

hypothesis, that the axis of the earth is to the diameter of the equator, as 100 to 101; he thence determines what must be the centrifugal force at the equator, that the earth might take such a form, and finds it must be $\frac{4}{505}$ of gravity, and therefore would exceed the present centrifugal force there, which is only $\frac{1}{289}$ of gravity. By the rule of proportion, he says, that if a centrifugal force equal to $\frac{4}{505}$ of gravity, would make the earth higher at the equator than at the poles, by $\frac{1}{100}$ of the whole height at the poles, a centrifugal force that is the $\frac{1}{289}$ of gravity, will make it higher by a proportional excess, which is found by calculation to be $\frac{1}{229}$ of the height at the poles; and thus our author discovers that the diameter at the equator is to the diameter at the poles, or the axis, as 230 to 229.

4. This computation supposes the earth to be of an uniform density every-where: but if the earth is more dense near the centre, then bodies at the poles will be more attracted by this additional matter being nearer to it; and, for this reason, the excess of the semidiameter of the equator above the semi-axis will be different. What we have said of a fluid earth must hold of the earth in its present state; for if it had not this figure in its solid parts, but a spherical figure, the ocean would overflow all the equatorial regions, and leave the polar regions elevated many miles above the level of the sea; whereas we find the one is no more elevated above the level of the ocean, than the other.

5. The planet *Jupiter* revolves on his axis with much more rapidity than our earth, and finishes his diurnal rotation in less than ten hours. The density of that planet is also less; and therefore his figure is more different from a sphere than the figure of
the

the earth, and his equatorial diameter exceeds his axis in a greater proportion. Their difference is so sensible that they are found, by the observations of astronomers, to be to one another as 13 is to 12.

6. The decrease of gravity from the poles towards the equator, is very manifest from the motion of pendulums. A pendulum that vibrates, in a second, in the northern regions, when carried to the equator, is always found to move too slow, and requires to be made shorter to vibrate truly in a second. This shews the gravity is less there: and this observation confirms the diurnal motion of the earth, and its oblate spheroidal figure at the same time. It is also a consequence of this figure of the earth, that the degrees in a meridian must increase from the equator to the poles; but the difference is so small that it cannot be discovered, from observation, but in latitudes that differ considerably from each other; and the variation of the degrees, that are near one another, appears, by our author's computations, to be incomparably less adapted for judging of the figure of the earth, than the motion of pendulums, in which the least variation becomes very sensible, in a great number of vibrations.

7. Some have imagined the slowness of the pendulums at the equator, may have proceeded from the rod of the pendulum being extended to a greater length, by the heat: but our author has shewed, that this could produce but a very small part of the effect. Mr. *Richer*, who was very careful in making his observations, found, that a pendulum vibrating in a second of time, in the island of *Cayenne*, was shorter than one that vibrated, in the same time, at *Paris*, by one line and a fourth part of a line. Our author, with good reason, thinks that a difference
of

of one sixth part of a line, may be allowed as the effect of the heat; and, subducting this from the difference observed by Mr. *Richer*, the remainder, 1 line and $\frac{1}{12}$ of a line, is the difference owing to the decrease of gravity, and is very consonant to what our author draws from his theory. This observation and our author's theory agree, in allowing seventeen miles for the excess of the height of the earth at the equator, above its height at the poles.

8. From the oblate figure of the earth, our author has accounted for the *precession* of the equinoxes. We commonly suppose, that, while the earth moves in her orbit round the sun, her axis continues always parallel to itself, so as to form an invariable angle with the ecliptic of about $66\frac{1}{2}^{\circ}$: from hence it is, that the plane of the equator is inclined to the ecliptic, in an angle of $23\frac{1}{2}^{\circ}$, and produced passes through the centre of the sun, twice only in every revolution. The points of the heavens, where the centre of the sun appears to be, in these two cases, are called the equinoctial points. In any other parts of the earth's orbit, the sun is on one side of the plane of the equator; being to the north of it in the summer half of the year, and to the south of it in the winter half. These equinoctial points are not fixed in the heavens, but have a slow motion, from east to west, among the stars, of about $50''$ in a year; and hence it is, that the interval of time betwixt any equinox and that same equinox, in the following revolution of the earth (which astronomers call the *tropical* year), is some minutes shorter than the *sidereal* year, or the period wherein the earth revolves from one point of her orbit, to the same point again: and, because the retrograde motion of the equinoctial points thus advances the time of every equinox a little sooner than it would otherwise have happened,

happened, this phænomenon is called the precession of the equinoxes. The philosophers, who maintained the *Ptolemaic* system ascribed this motion to the fixed stars; and, in their ordinary way, made no scruple to contrive a sphere for this purpose, which they supposed to revolve with a very slow motion on the poles of the ecliptic, and to carry all the fixed stars along with it; whereas this phænomenon is accounted for by a retrograde motion of the nodes of the equator and ecliptic, similar to the motion of the nodes of the moon's orbit.

It was shewn above, how the action of the sun produces the retrograde motion of the nodes of the moon; and it follows, from the same principles, that if a planet revolved about the earth near to its surface in the plane of the equator, its nodes would also go backward, though with a slower motion than those of the moon, in proportion as its distance from the earth's centre was less than that of the moon. Suppose the number of such planets to be increased till they touch each other, and form a ring in the equator, and the nodes of this ring would go backward in the same manner as the nodes of the orbit of any one planet revolving there. Suppose then this ring to adhere to the earth; and its nodes would still go backward, but with a much slower motion, because the ring must move the whole earth, to which it is supposed to adhere. The elevation of the equatorial parts of the earth has the same effect as such a ring would have; only the motion of the nodes of the equator, or of the equinoctial points, is slower, because the accumulated parts of the earth, above a spherical figure, are diffused over its surface, and have a less effect than if they were all collected in the place of the equator, in the form of a ring. The moon has a greater force on this ring than the sun,

fun, because of her less distance from the earth; and they both contribute to produce the retrograde motion of the equinoctial points: the motion, however, produced by both is so slow, that those points will not finish a revolution in less than 25000 years. Our author has determined the quantity of this motion, from its causes, and finds it, from the theory, to be perfectly consonant with the observations of astronomers.

There is another effect of the action of the sun and moon on this ring, which is too small to be sensible in astronomical observations: their action on the ring, makes its inclination to the ecliptic to decrease and increase, by turns, twice every year.

C H A P. VII.

Of the ebbing and flowing of the sea.

IT is not in the motions of the celestial bodies only, that the effects of their mutual gravitation are visible, for we are now to shew, that a phænomenon which passes on our earth, and is known to every body, proceeds from the same principle; I mean, the ebbing and flowing of the sea, the solution of which, from the bad success of those who attempted it before our author, had become a reproach to philosophy. But he has very plainly and fully accounted for it, from the unequal gravitations of the parts of the earth towards the sun and moon. It will be worth while, because it is a very celebrated question, to be the more particular in explaining his solution of it.

It is obvious, that, if the earth was entirely fluid, and quiescent, its particles, by their mutual gravity towards each other, would form themselves into the figure of an exact sphere. Suppose now, that some power acts on all the particles of this earth, with an equal force, and in parallel directions, the whole mass will be moved by such a power, but its figure will suffer no alteration by it; because all the particles being equally moved by this power, in parallel lines, they will still keep the same situation with respect to each other, and still form a sphere, whose centre will have the same motion as each particle. For, as a drop of water, while it falls towards the earth, retains its spherical figure; and, as the situation of bodies in a ship, that moves with an uniform motion forward, is no way affected by the motion which is common to the ship and all the bodies in it; so the situation of the parts of the earth, with respect to each other, can be no way affected by any power that acts with the same force, and in the same direction, on every part, and promotes each equally.

We have already shewed, that the particles of the earth gravitate towards the moon, and if the gravitation of the particles was every where the same, and acted in the same direction, it would have no effect on the figure of the earth; so that, if the motion of the earth round the common centre of gravity of the earth and moon was destroyed, and the earth was left to the influence of its gravitation towards the moon, the earth falling towards the moon would retain its spherical figure, all the parts being equally carried on, and retaining, therefore, the same situation with respect to each other.

But the actions of the moon, on different parts of the earth, are unequal; those parts, by the general law, being most attracted which are nearest the moon, and those being least attracted which are farthest from the moon; while the parts that are at a middle distance, are attracted by a mean degree of force: nor are all the parts acted on in parallel lines, but in lines directed towards the centre of the moon: and, on these accounts, the spherical figure of the earth must suffer some change from the moon's action.

Suppose the earth to fall towards the moon, as before, and let us abstract from the mutual gravitation of its parts towards each other, as also from their cohesion; and it will easily appear, that the parts nearest the moon would fall with the swiftest motion, being most attracted, and that they would leave the centre or greater bulk of the earth behind them in their fall; while the more remote parts would fall with the slowest motion, being less attracted than the rest, and be left a little behind the bulk of the earth, so as to be found at a greater distance from the centre of the earth than at the beginning of the motion. From which it is manifest, that the earth would soon lose its spherical figure, and form itself into an oblong spheroid, whose longest diameter would point at the centre of the moon. If the particles of the earth did not gravitate towards each other, but towards the moon only, the distances betwixt the parts of the earth that are supposed to be nearest to the moon, and the central parts, would continually increase, because of their greater celerity in falling; and the distance betwixt the central parts, and the parts that are farthest from the moon, would increase continually at the same time; these being
left

left behind by the central parts, which they would follow, but with a less velocity. Thus the figure of the earth would become more and more oblong, that diameter of it which pointed towards the moon continually increasing.

But this is not the only reason why the earth would soon assume an oblong spheroidal form, if its parts were allowed to fall freely by their gravity towards the moon's centre. The lateral parts of the earth (that is, those which are at the distance of a quarter of a circle from the point which is directly below the moon) and the central parts descending with equal velocities, towards the same point, *viz.* the centre of the moon, in approaching to it, would manifestly approach, at the same time, to each other, and, their distance growing less, the diameters of the earth passing through them would become less; so that the diameter of the earth that points towards the moon would increase, and those diameters of the earth that are perpendicular to the line joining the centres of the earth and moon, would decrease at the same time, and render the figure of the earth still more oblong for this reason.

Let us now allow the parts of the earth to gravitate towards its centre; and, as this gravitation far exceeds the action of the moon, and much more exceeds the differences of her actions on the different parts of the earth, the effect that will result from the inequalities of these actions of the moon, will be only a small diminution of the gravity of those parts of the earth which it endeavoured, in our former supposition, to separate from its centre, and a small addition to the gravity of those parts which it endeavoured to bring nearer to its centre; that is, those parts of the earth which are nearest to the

moon, and those which are farthest from her, will have their gravity towards the earth somewhat abated; whereas the lateral parts will have their gravity increased: so that, if the earth be supposed fluid, the columns from the centre to the nearest and to the farthest parts must rise, till, by their greater height, they be able to ballance the other columns, whose gravity is either not so much diminished, or is increased by the inequalities of the action of the moon: and thus the figure of the fluid earth must be still an oblong spheroid.

We have hitherto supposed the earth to fall towards the moon by its gravity. Let us now consider the earth as projected in any direction, so as to move round the centre of gravity of the earth and moon: it is manifest, that the gravity of each particle towards the moon will endeavour to bring it as far from the tangent, in any small moment of time, as if the earth was allowed to fall freely towards the moon; in the same manner as any projectile at our earth, falls from the line of projection as far as it would fall by its gravity in the perpendicular, in the same time. Therefore the parts of the earth nearest to the moon, will endeavour to fall farthest from the tangent, and those farthest from the moon will endeavour to fall least from the tangent, of all the parts of the earth; and the figure of the earth, therefore, will be the same as if the earth fell freely towards the moon: that is, the earth will still affect a spheroidal form, having its longest diameter directed towards the moon.

What must be carefully attended to here, is, that it is not the action of the moon, but the inequalities in that action, that produce any variation from the spherical figure; and that if this action was the
same

same in all the particles as in the central parts, and acted in the same direction, no such change would ensue. Our author, therefore, to account for this matter, conceives first the attraction of the central parts to be diffused with an equal force over all the parts in the same direction, and then conceives the inequalities as arising from a power superadded, and directed towards the moon where there is an excess, and directed in the opposite line where there is a defect, in the attraction of the parts, compared with the attraction of the central parts: for thus the sum of these forces, in the first case, will account for the attraction where it exceeds, and their difference will account for the attraction where it is less than in the central parts. And when the effects of these powers are considered as they affect the figure of the earth, it is manifest that they must produce such an oblong spheroid as we have described; the superadded force drawing the parts nearest the moon towards her, and therefore from the earth's centre, while it draws the parts farthest from the moon in an opposite direction; and therefore still draws from the centre of the earth also.

The action of the moon on the lateral parts is resolved into two, one equal and parallel in its direction to her action on the central parts, and another directed from those lateral parts towards the centre of the earth; the first of these can have no effect upon the figure of the earth, being considered as common to all the particles, and therefore to be neglected in this enquiry: it is the other that adds to the gravity of the lateral parts towards the centre of the earth, and, by adding to the weight of the lateral columns, it makes them sustain the other columns, whose gravity is diminished by the action of the moon to a greater height; and the power which

alters the spherical figure is to be estimated as the sum of two powers, that which is added to the gravity of the one, and is subducted from the gravity of the other.

Hitherto we have abstracted from the motion of the earth on its axis : but this must also be considered in order to know the real effect of the moon's actions on the sea. Was it not for this motion, the longest diameter of the spheroidal figure, which the fluid earth would assume, would point at the moon's centre ; but, because of the motion of the whole mass of the earth on its axis from west to east, the most elevated part of the water no longer answers precisely to the moon, but is carried beyond the moon towards the east in the direction of the rotation.

The water continues to rise after it has passed directly under the moon, tho' the immediate action of the moon there begins to decrease, and comes not to its greatest elevation till it has got half a quadrant further. It continues to descend after it has passed at 90 degrees distance from the point below the moon, tho' the force which the moon adds to its gravity begins to decrease there. For still the action of the moon adds to its gravity, and makes it descend till it has got half a quadrant farther : the greatest elevation, therefore, is not in the points which are in a line with the centres of the earth and moon, but about half a quadrant to the east of these points in the direction of the motion of rotation.

Thus it appears that the spheroidal form which the fluid earth would affect, will be so situated that the longest diameter of that figure will point to the east of the moon, or that the moon will always be

to the west of the meridian of the parts of greatest elevation. Suppose now an island in this fluid earth, and it will approach in every revolution to each elevated part of this spheroid, and the water on the shore of this island will necessarily rise twice every lunar day; and the time of high water will be when it approaches to these elevated parts, that is, when it has passed to the east of the moon, or when the moon is at some distance to the west of the meridian.

We have hitherto taken notice of the action of the moon only: but it is manifest, that, for the same reasons, the inequality of the sun's action on the different parts of the earth would produce a like effect, and that these alone would produce a like variation from the exact spherical figure of a fluid earth. Indeed the effect of the sun, because of his immense distance, must be considerably less, tho' the gravity towards the sun be vastly greater. For it is not their actions, but the inequalities in the actions of each, which have any effect; as we have often observed. The sun's distance is so great, that the diameter of the earth is a point compared to it, and the difference between the actions of the sun on the nearest and farthest parts becomes, on this account, vastly less than it would be if the sun was as near as the moon, whose distance from us is about 30 diameters of the earth. Thus the inequality of the action of the earth on the parts of a drop of water is altogether insensible, because the diameter of the drop is an insensible quantity compared with its distance from the centre of the earth.

However, the immense bulk of the sun makes the effect still sensible at so vast a distance; and therefore, though the action of the moon has the

greatest share in producing the tides, the action of the sun adds sensibly to it when they conspire together, as in the change and full of the moon, when they are nearly in the same line with the centre of the earth, and therefore unite their forces; so that then the tides are greatest, and are what we call the *spring-tides*. The action of the sun diminishes the effect of the moon's action in the quarters, because the one raises the water in that case where the other depresses it; and therefore the tides then are least; and these we call the *neap tides*. Though, to speak accurately, the spring and neap tides must be some time after; because, as in other cases, so in this, the effect is not greatest or least when the immediate influence of the cause is greatest or least. As the greatest heat, for example, is not on the solstitial day, when the immediate action of the sun is greatest, but some time after.

That this may be more clearly understood, let it be considered, that, though the actions of the sun and moon were to cease this moment, yet the tides would continue to have their course for some time: For the water where it is now highest would subside and flow down on the parts that are lower, till by the motion of descent, being there accumulated to too great a height, it would necessarily return again to its first place, though in a less measure, being retarded by the resistance arising from the attrition of its parts. Thus it would for some time continue in an agitation like to that in which it is at present. The waves of the sea that continue after a storm ceases, and every motion almost of a fluid, may illustrate this.

The high water does not always answer to the same situation of the moon, but happens sometimes sooner

fooner, and sometimes later, than if the moon alone acted on the sea. This proceeds from the action of the sun, which brings on high water sooner when the sun alone would produce a tide earlier than the moon, as the sun manifestly would in the first and third quarter; and retards the time of high water a little, when the sun alone would produce a tide later than the moon, as in the second and last quarters. The different distances of the moon from the earth, produce likewise a sensible variation in the tides. When the moon approaches the earth, her action on every part increases, and the differences of that action, on which the tides depend, increase. For her action increases as the squares of the distances decrease; and though the differences of the distances themselves be equal, yet there is a greater disproportion betwixt the squares of less than the squares of greater quantities. As for example, 3 exceeds 2 as much as 2 exceeds 1, but the square of 2 is quadruple of the square of 1, while the square of 3 (*viz.* 9.) is little more than double the square of 2 (*viz.* 4.) Thus it appears, that, by the moon's approach, her action on the nearest parts increases more quickly than her action on the remote parts, and the tides, therefore, increase in a higher proportion as the distances of the moon decrease. Our author shews that the tides increase in proportion as the cubes of the distances decrease, so that the moon at half her present distance would produce a tide eight times greater. The moon describes an ellipse about the earth, and in her nearest distance produces a tide sensibly greater than at her greatest distance from the earth: and hence it is that two great spring tides never succeed each other immediately; for if the moon be at her nearest distance from the earth at the change, she must be at her greatest distance at the full, having, in the intervening time, finished half a revolution; and there-

therefore the spring tide then will be much less than the tide at the change was : and for the same reason, if a great spring tide happen at the time of full moon, the tide at the ensuing change will be less.

It is manifest, that if either the sun or moon was in the pole, they could have no effect on the tides ; for their action would raise all the water at the equator to the same height ; and any place of the earth, in describing its parallel to the equator, would not meet, in its course, with any part of the water more elevated than another ; so that there could be no tide in any place. The effect of the sun or moon is greatest when in the equator : for then the axis of the spheroidal figure, arising from their action, moves in the greatest circle, and the water is put into the greatest agitation ; and hence it is, that the spring tides produced when the sun and moon are both in the equator, are the greatest of any, and the neap tides are the least of any, about that time. But the tides produced when the sun is in either of the tropics, and the moon in either of her quarters, are greater than those produced when the sun is in the equator, and the moon in her quarters ; because, in the first case, the moon is in the equator ; and, in the latter case, the moon is in one of the tropics : and the tide depends more on the action of the moon than that of the sun, and is therefore greatest when the moon's action is greatest. However, because the sun is nearer the earth in winter than in summer, hence it is, that the greatest spring tides are after the autumnal and before the vernal equinox.

When the moon declines from the equator towards either pole, one of the greatest elevations of the water follows the moon, and describes nearly the parallel on the earth's surface which is under that
which

which the moon, because of the diurnal motion, seems to describe: and the opposite greatest elevation, being *Antipodes* to that, must describe a parallel as far on the other side of the equator: so that while the one moves on the north side of the equator, the other moves on the south side of it, at the same distance. Now the greatest elevation which moves on the same side of the equator, with any place, will come nearer to it than the opposite elevation, which moves in a parallel on the other side of the equator; and therefore if a place is on the same side of the equator with the moon, the day tide, or that which is produced while the moon is above the horizon of the place, will exceed the night tide, or that which is produced while the moon is under the horizon of the place. It is the contrary if the moon is on one side and the place on the other side of the equator; for then the elevation which is opposite to the moon, moves on the same side of the equator with the place, and therefore will come nearer to it than the other elevation. This difference will be greatest when the sun and moon both describe the tropics; because the two elevations in that case describe the opposite tropics, which are the farthest from each other of any two parallel circles they can describe. Thus it is found, by observation, that the evening tides in the summer exceed the morning tides, and the morning tides in winter exceed the evening tides. The difference is found at *Bristol* to amount to fifteen inches, at *Plymouth* to one foot. It would be still greater, but that a fluid always retains an impressed motion for some time; so that the preceding tides affect always those that follow them*.

The

* See *Fig. 71*, (from *Sir Isaac Newton*) in which the spheroid *P A p E* represents the earth, *P, p*, the poles, *A E* the equator,

The phænomena of particular places agree with these general observations, if the situation and extent of the seas and shores, in which they are situated, are considered. It has been always known that the tides follow the motion of the moon, rising twice in one revolution of the moon to the meridian of any place; which exceeds a solar day by above $\frac{3}{4}$ of an hour, because the proper motion of the moon retards so much her appulse to the meridian of the place. All the effects of the sun's action, sometimes promoting, sometimes abating the effects of the action of the moon, as before mentioned, are also conformable to perpetual observation: and the tides in places that lie on a deep and open ocean, where the water can easily follow the influences of the sun and moon, are agreeable to this theory.

That the tides may have their full motion, the ocean in which they are produced ought to be extended from east to west 90° , or a quarter of a circle of the earth at least. Because the places,

tor, F any place not in the equator, Ff its parallel Dd a parallel on the other side of the equator, L the moon's place three hours before, H the place of the earth to which L is vertical, and b the opposite place, K, k , places 90° distant from these. Then will CH, cb , measure the greatest elevations of the water, and cK, ck , the least. cF, cf, cD, cd , will be the elevations at F, f, D, d . And if NM is a circle of the spheroid, meeting the equator and these parallels in s, r, t , cN will be the elevation of the water at s, r, t , or any other places in the circle NM . The highest tides at any place F , happen at F and f , three hours after the moon's passing the meridian, above or below the horizon; and the lowest at Q three hours after her setting or rising. And if F and L are on the same side of the equator, the day tide will rise higher than the night tide, cF being greater than cf . It is the contrary, when the moon's declination and the latitude of a place D are of opposite denominations, the one north and the other south; because then cD is greater than cd .

where

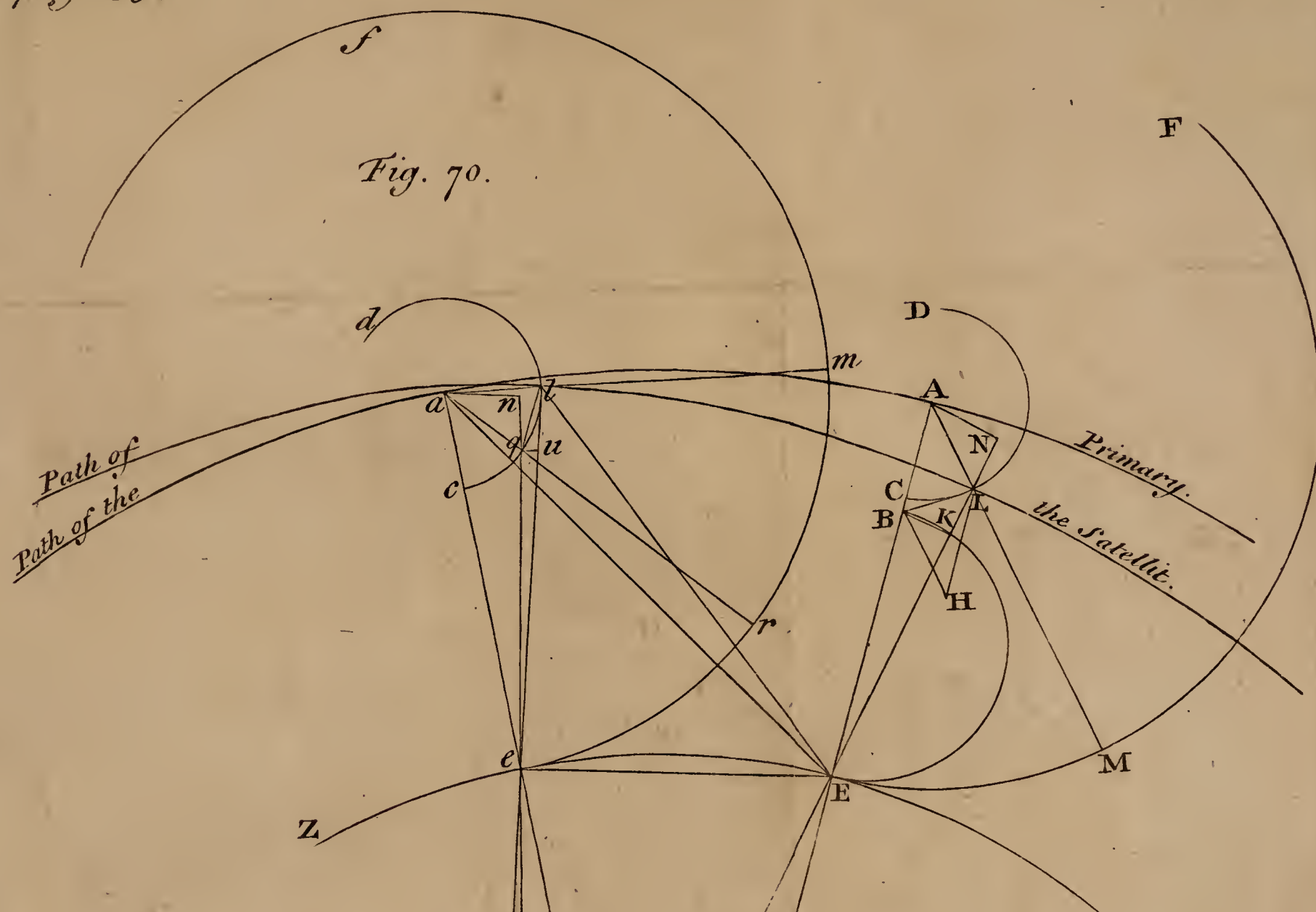
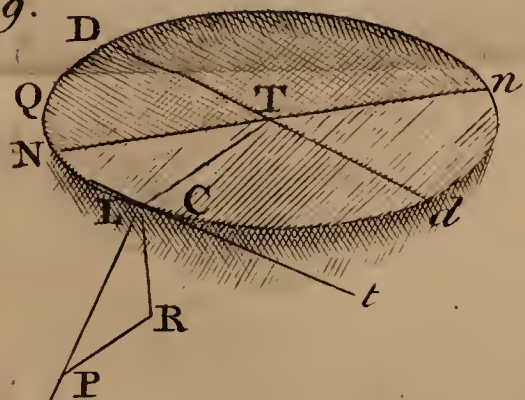
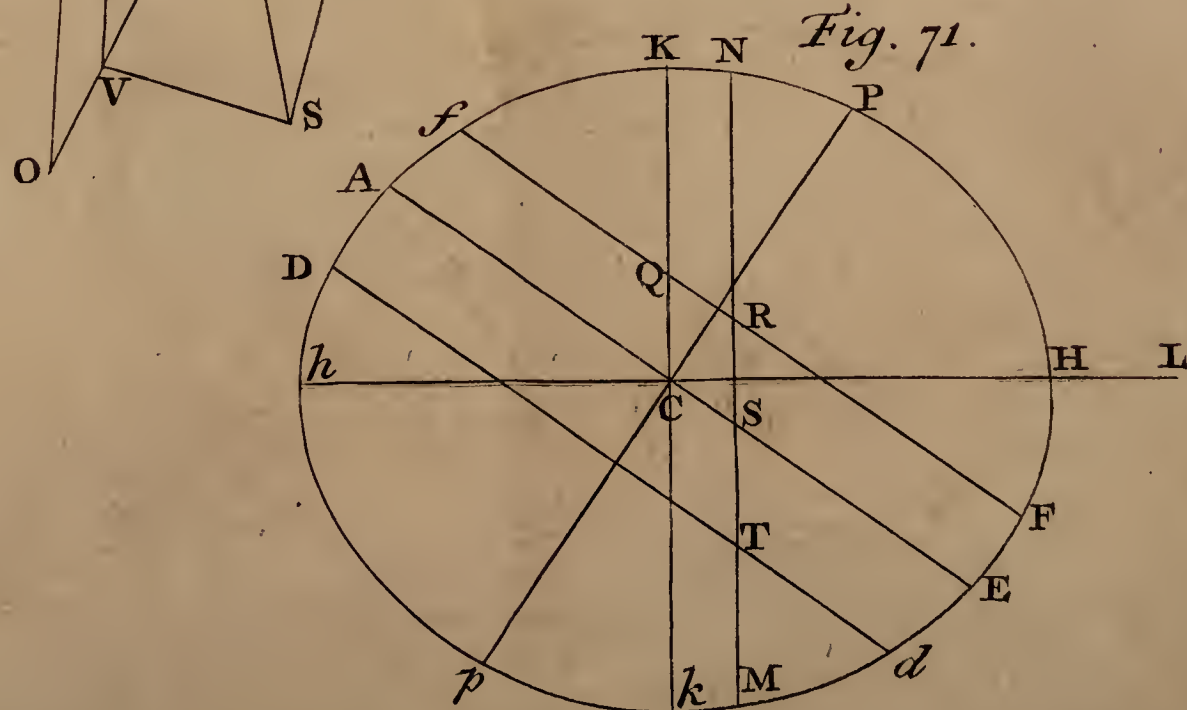


Fig. 69.



s



where the moon raises most, and most depresses, the water, are at that distance from each other. Hence it appears, that it is only in the great oceans that such tides can be produced; and why in the larger *pacific* ocean they exceed those in the *Atlantic* ocean. Hence also, it is obvious why the tides are not so great in the torrid zone, between *Africa* and *America*, where the ocean is narrower, as in the temperate zones on either side; and, from this also, we may understand why the tides are so small in islands that are very far distant from the shores. It is manifest, that, in the *Atlantic* ocean, the water cannot rise on one shore but by descending on the other; so that, at the intermediate distant islands, it must continue at about a mean height betwixt its elevation on the one and on the other shore.

As the tides pass over shoals, and run through straits into bays of the sea, their motion becomes more various, and their height depends on a great many circumstances. The tide that is produced on the western coasts of *Europe*, in the *Atlantic*, corresponds to the situation of the moon we described above. Thus it is high water on the coasts of *Spain*, *Portugal*, and the west of *Ireland*, about the third hour after the moon has passed the meridian. From thence it flows into the adjacent channels, as it finds the easiest passage. One current from it, for example, runs up by the south of *England*, another comes in by the north of *Scotland*: they take a considerable time to move all this way, and it is high water sooner in the places to which they first come; and it begins to fall at those places, while they are yet going on to others that are farther in their course. As they return, they are not able to raise the tide, because the water runs faster off than it returns, till, by a new tide propagated from the open ocean, the
return

return of the current is stopped, and the water begins to rise again. The tide takes twelve hours to come from the ocean to *London-bridge*, so that, when it is high water there, a new tide is already come to its height in the ocean; and, in some intermediate place, it must be low water at the same time. In channels, therefore, and narrow seas, the progress of the tides may be, in some respects, compared to the motion of the waves of the sea. Our author also observes, that when the tide runs over shoals, and flows upon flat shores, the water is raised to a greater height than in the open and deep oceans that have steep banks; because the force of its motion cannot be broke, upon these level shores, till the water rises to a greater height.

If a place communicates with two oceans, (or two different ways with the same ocean, one of which is a readier and easier passage) two tides may arrive at that place in different times, which, interfering with each other, may produce a great variety of phænomena. An extraordinary instance of this kind is mentioned by our author at *Batsha*, a port in the kingdom of *Tunquin* in the *East Indies*, of northern latitude $20^{\circ} 50'$. The day in which the moon passes the equator, the water stagnates there without any motion: as the moon removes from the equator, the water begins to rise and fall once a day; and it is high water at the setting of the moon, and low water at her rising. This daily tide increases for about seven or eight days, and then decreases for as many days by the same degrees, till this motion ceases when the moon has returned to the equator. When she has passed the equator and declines towards the south pole, the water rises and falls again, as before; but it is high water now at the rising, and low water at the setting, of the moon.

Our

Our author, to account for this extraordinary tide, considers that there are two inlets to this port of *Batsha*, one from the *Chinese* ocean betwixt the continent and the *Manillas*, the other from the *Indian* ocean betwixt the continent and *Borneo*. This leads him to propose, as a solution of the phænomenon, that a tide may arrive at *Batsha*, through one of these inlets, at the third hour of the moon, and another through the other inlet six hours after, at the ninth hour of the moon. For, while these tides are equal, the one flowing in as the other ebbs out, the water must stagnate: now they are equal when the moon is in the equator; but as soon as the moon begins to decline on the same side of the equator with *Batsha*, we have shewed that the diurnal tide must exceed the nocturnal, so that two greater and two lesser tides must arrive at *Batsha* by turns. The difference of these will produce an agitation of the water, which will rise to its greatest height at the mean time betwixt the two greatest tides, and fall lowest at a mean time betwixt the two least tides; so that it will be high water about the sixth hour at the setting of the moon, and low water at her rising. When the moon has got to the other side of the equator, the nocturnal tide will exceed the diurnal; and therefore, the high water will be at the rising, and low water at the setting, of the moon. The same principles will serve to account for other extraordinary tides, which, we are told, are observed in places whose situation exposes them to such irregularities.

Our author does not content himself with these general observations, but calculates the effects of the sun and moon upon the tides, from their attractive powers. The augmentation of the gravity of the lateral parts of the earth, produced by the action
of

of the sun, is a similar effect to an augmentation, estimated by him before, that is made to the gravity of the moon towards the earth by the same action, when the moon is in the quarters; only the addition made to the gravity of the lateral parts is * about $60\frac{1}{2}$ times less, because their distance from the earth's centre is so many times less than the distance of the moon from it. The gravity of those parts of the earth that are directly beneath the sun, and of those opposite to it, is diminished by a double quantity of what is added to the lateral parts; and as the diminution of gravity of the one, and augmentation of the gravity of the other, conspire together in raising the water under the sun, and the parts opposite to it, above its height in the lateral parts; the whole force that produces this effect is to be considered as triple of what is added to the gravity of the lateral parts: and is thence found to be to the gravity of the particles as 1 to 12868200, and to the centrifugal force at the equator as 1 to 44527. The elevation of the waters, by this force, is considered by our author as an effect similar to the elevation of the equatorial parts above the polar parts of the earth, arising from the centrifugal force at the equator; and, being 44527 times less, is found to be 1 foot and $11\frac{1}{30}$ inches, *Paris* measure. This is the elevation arising from the action of the sun upon the water.

In order to find the force of the moon upon the water, he compares the spring tides at the mouth of the river *Avon* below *Bristol* (which are the effect of the sum of the forces of the sun and moon when their actions almost conspire together,) with the neap tides there (which are the effect of the difference of these forces when they act almost against one ano-

* *Princip.* Lib. III. Prop. 36.

ther,) and finds their proportion to be that of 9 to 5; from which, after several necessary corrections, he concludes that the force of the moon is to the force of the sun, in raising the waters of the ocean, as 4,4815 to 1; so that the force of the moon is able, of itself, to produce an elevation of 8 feet and $7\frac{5}{22}$ inches, and the sun and moon together may produce an elevation of about $10\frac{1}{2}$ feet, in their mean distances from the earth, and an elevation of about 12 feet when the moon is nearest the earth. The height to which the water is found to rise, upon the coasts of the open and deep ocean, is agreeable enough to this computation.

It is from this last calculation that he is able to make an estimate of the density and quantity of matter in the moon. Her influence on the tides is the only effect of the moon's attracting power which we have access to measure, and it enables us to estimate her density compared with that of the sun, which we find it exceeds in the proportion of 4891 to 1000; and since the density of the earth is to that of the sun as 4000 to 1000, it follows that the moon must be more dense than the earth in the proportion of 4891 to 4000, or of 11 to 9 nearly. The proportion of the diameter of the earth to that of the moon is known, from astronomical observations, to be that of 365 to 100; and from these two proportions it easily follows, that the quantity of matter in the moon is to the matter in the earth as 1 to 39,788; and the centre of gravity of the earth and moon must be, therefore, almost 40 times nearer to the earth than to the moon: and the situation of their centre of gravity being known, the motions in their system may be determined with great preciseness.

Our author enquires into the figure of the moon : and because the earth contains near 40 times more matter than the moon, the elevation produced by the action of the earth in the parts of the moon that are nearest to it, and in the parts opposite to these, would be near 40 times greater than that which the moon produces in our seas, if this elevation was not to be diminished in proportion as the semidiameter of the moon is less than the semidiameter of the earth, that is, in the proportion of 100 to 365. By compounding these proportions he finds, that the diameter of the moon that passes through the centre of the earth, must exceed those that are perpendicular to it, by about 186 feet. He thinks the solid parts of the moon must have been formed into such a spheroidal figure, having its longest diameter directed towards the earth ; and this may be the reason why the moon always turns the same side towards the earth. If there were great seas in the moon, and if she revolved on her axis so as to turn different sides towards the earth, there would have been very great tides produced in them, such as would exceed our tides ten times ; but, by her keeping one side always towards the earth, there are no tides produced in her seas, but what proceed from the differences of their distances from the earth, and from the moon's librations ; for the action of the sun can have very little effect upon them.

C H A P. VIII.

Of the comets.

1. **H**itherto we have treated of the planets: but, besides these, we find in the expanse of heaven many other bodies belonging to the system of the sun, that seem to have much more irregular motions. These are the *comets*, which, descending from the far distant parts of the system with great rapidity, surprize us with the singular appearance of a train or tail, which accompanies them; become visible to us in the lower parts of their orbits, and, after a short stay, are carried off again to vast distances, and disappear. Though some of the ancients had more just notions of them, yet the opinion having prevailed, that they were only meteors generated in the air, like to those we see in it every night, and in a few moments vanishing, no care was taken to observe or record their phænomena accurately, till of late. Hence this part of astronomy is very imperfect. The number of the comets is far from being known: many have been noted by historians formerly, and not a few of late observed by astronomers; and some have been discovered accidentally by telescopes, passing by us, that never became visible to the naked eye: so that we may conclude their number to be very great. Their periods, magnitudes, and the dimensions of their orbits, are also uncertain. This is a part of science, the perfection of which may be reserved for some distant age, when these numerous bodies, and their vast orbits, by long and accurate observation, may be added to the known parts of the solar system. Astronomy will appear as a new science, after all the discoveries we

now boast of: but then it will be remembered, even in those flourishing days of astronomy, that it was *Sir Isaac Newton* who discovered and demonstrated the principles by which alone such great improvements could be made; and that he begun and carried this work so far, that he left to posterity little more to do, but to observe the heavens, and compute after his models.

Having this part of astronomy to deduce almost from its elements, he begins with shewing, against the scholastic philosophers, that the comets are above the moon; because they participate of the apparent diurnal motion, rising and setting daily, as all things that are not appendant to the earth do, and that without any sensible diurnal parallax. But, as they are all affected by the annual motion of the earth, appearing, like the planets, sometimes direct, sometimes retrograde, he concludes that, when they become visible to us, they must be in the regions of the planets. As they are all affected by the motion of the earth, and it is impossible to bring their motions to any regularity without allowing that motion; and it, alone, suffices for explaining the irregularities of every comet, as well as of every planet; we obtain from this a new confirmation of the motion of the earth, and find all the parts of this philosophy perfectly consistent.

Our author having shewed that the comets descend into the planetary regions when they are visible to us, against the opinion of *Des Cartes*, he proceeds to trace them in their courses. It follows, from the general law of gravity already established, that they must move either in *parabolic*, or very excentric *elliptical*, orbits, that have one focus in the centre of the sun. He then enquires, with his usual skill

skill and a great deal of labour, how a motion in a parabola may agree with the observations that have been made upon the comets; and, for this end, shews how, from three observations, the parabolic trajectory which a comet describes may be determined: and, from several examples which he has given, there appears so perfect a harmony between his theory and the observations, as adds a new evidence to it, and shews its use in carrying on the knowledge of our system.

He insists particularly on the celebrated comet that appeared near the end of the year 1680, and in the beginning of 1681. He determines its trajectory, or curve, from three observations made by Mr. *Flamsteed*; and then compares all the observations, that were made by himself or others, with the motion of a body in that curve, and finds the differences betwixt the observed places of this comet and those computed for it in the curve, for the same time, to be very small. It was the same comet that was seen in *November* 1680, and in *December*, *January*, *February*, and *March* following, though they had been generally esteemed two different comets. In *November* it was descending towards the sun; it passed very near the sun on the 12th day of *December*; where, having been heated to a prodigious degree, tho' the light of the head or *nucleus* was duller, yet, while it ascended in the other half of its orbit, its tail was vastly greater than before, extending sometimes 70° in length, and continuing visible even after the head or nucleus was carried out of sight.

Dr. *Halley*, to whom every part of astronomy, but this in a particular manner, is highly indebted, has joined his labours to our author's on this subject; nor is it necessary for us to separate them. Finding

three observations of comets recorded in history, agreeing with this in remarkable circumstances, and returning at the distance of 575 years from each other, he suspected that these might be one and the same comet, revolving in that period about the sun. He therefore supposed the *parabola* to be changed into such an excentric *ellipse* as the comet might describe in 575 years, and as should nearly coincide with the parabola in its lowest part; and, having computed the places of the comet in this elliptic orbit, he found them to agree so well with those in which the comet was observed to pass, that the variations did not exceed the differences which are found betwixt the computed and the observed places of the planets, whose motions had been the subject of astronomical calculation for some thousand years. This comet may, therefore, be expected again after finishing the same period, about the year 2255. If it then return, it will give a new lustre and evidence to Sir *Isaac Newton's* philosophy, in that distant age. And should the inconstancy of human affairs, and the perpetual revolutions to which they are subject, occasion any neglect of our philosophy in the intervening ages; this comet will revive it, and fill every mouth again with this great man's name. Nor need this be esteemed a vain prediction; for we cannot but suppose that the attention of the astronomers of those days, to this comet must be raised to a great pitch, because in one part of its orbit it approaches very near to the orbit of our earth; so that, in some revolutions, it may approach near enough to have very considerable, if not fatal effects upon it. Nor is it to be doubted but that, while so many comets pass among the orbits of the planets, and carry such immense tails along with them, we should have been called, by very extraordinary consequences, to attend to these bodies long ago, if the motions in the universe

verse had not been at first designed, and produced by a Being of sufficient skill to foresee their most distant consequences. Our earth was out of the way when this comet last passed near her orbit; but it requires a perfect knowledge of the motion of the comet, to be able to judge if it will always pass by us with so little effect. We may here observe, that these great periods, and distant depending observations, promise this good effect, that they must contribute to preserve the relish for learning from the revolutions it has been formerly subject to. By them, distant ages are connected together, and perpetual matter for reviving the curiosity of men is provided, from time to time.

But we are not to wait for the return of this distant comet to have our author's theory verified, and to see predictions of this kind begin to take place. By comparing together the orbits of the comets that appeared in 1607 and 1682, they are found so coincident, that we cannot but suppose them to be one and the same comet, revolving in 75 years about the sun. If this comet, according to this period, return in 1758, astronomy will then have something new to boast of. It seems to be of those that rise to the least height from the sun, its greatest distance being only 35 times greater than the distance of the earth from the sun; so that, at the farthest, it does not run out four times farther from us than *Saturn*. It will probably be the first that will be added to the number of the revolving planets, and establish this part of our author's theory.

Besides these comets we have mentioned, our author has considered the motions of several others, and finds his theory always consonant with observation. He particularly computes the places of a re-

markable comet that appeared in 1664 and 1665. It moved over 20° in one day, and described almost six signs in the heavens before it disappeared; its course deviated from a great circle, towards the north, and its motion, that had been before retrograde, became direct towards the end: and notwithstanding so unusual a course, its places, computed from our author's theory, agree with the observed places, as well as those of the planets agree with their theory.

The phænomena of all the comets, but especially of the comet of 1680, shew them to be solid, fixed, and durable bodies. This comet was, in its *perihelium*, 166 times nearer to the sun than our earth is: and, from this, our author computes that it must have conceived a heat 2000 times greater than that of iron almost going into fusion, and that, if it was equal to our earth, and cooled in the same manner as terrestrial bodies, it would take 50,000 years to cool: to bear so prodigious a heat, it must surely be a very solid and fixed body.

There is a phænomenon that attends each comet, and is peculiar to them, called its *tail*: some have imputed this appearance to the refraction of the sunbeams passing through the nucleus or head, which they supposed to be transparent: others to the refraction of the beams reflected from the head, as they pass through the intermediate spaces to us. Our author refutes both these opinions, and shews that the tail consists of a vapour arising continually from the body of the comet, towards those parts that are opposite to the sun, for a like reason that vapour or smoke rises in the atmosphere of the earth. Because of the motion of the body of the comet, the tail is bent a little towards those parts which the comet leaves

leaves in its motion. These tails are found greatest after it has passed its perihelium, or least distance from the sun, where its heat is greatest, and the atmosphere of the sun is most dense. The head appears after this, obscured by the thick vapour that rises plentifully from it. The tail of the comet of 1680 was of a prodigious size: it was extended from the head to a distance scarcely inferior to the vast distance of the sun from the earth. As the matter of the tail participates of the motion of the comet, it is thereby carried along with the comet in its motion, and some part of it returns again with it: and as the matter in the tail rises, it becomes more and more rarified; as appears from the tail's increasing in breadth upwards. By this rarefaction a great part of the tail must be dilated and diffused over the system; some of this, by its gravity, may fall towards the planets, mix with their atmospheres, and supply the fluids, which, in natural operations, are consumed; and may, perhaps, supply that subtile spirit in our air, which is necessary for the life of animals, and for other natural operations.

We are not to expect that the motions of the comets can be so exact, and the periods of their revolutions so equal, as those of the planets; considering their great number, and their great distance from the sun in their aphelia, where their actions upon each other must have some effect to disturb their motions. The resistance which they meet with in the atmosphere of the sun, when they descend into the lower parts of their orbits, will also affect them. By the retardation of their motion in these lower parts, their gravity will be enabled to bring them nearer the sun in every revolution, till at length they fall into him, and supply fuel to that immense body of fire. The comet of 1680 passed at a distance
from

from the surface of the sun, no greater than the 6th part of his diameter; it will approach still nearer in the next revolution, and fall into his body at length. The fixed stars may receive supplies, in the same manner, by comets falling into them; and some of them, whose light and heat are almost exhausted, may receive new fuel in this way. Of this kind those stars seem to be, which have been observed to break out at once with great splendor, and to vanish gradually afterwards. Such was the star in *Cassiopeia*, that was not visible on the 8th of *November* 1572, but shone the following night with a brightness almost equal to that of the planet *Venus*, and decreased continually afterwards, till in 16 months time it vanished. Another of the same kind appeared to *Kepler's* scholars in the right foot of *Serpentarius*, on the 30th of *September* 1604, brighter than *Jupiter*, though it was not visible the preceding night; which also decreased gradually, and vanished in fifteen or sixteen months. By such a new star appearing with an extraordinary brightness in the heavens, *Hipparchus* is said to have been induced to make his catalogue of the fixed stars. But those stars which appear and disappear, gradually increasing and decreasing by turns, seem to be of a different kind; and to have a luminous and an obscure side, which by their rotation on their axis, they turn towards us alternately.

The argument against the eternity of the universe, drawn from the decay of the sun, still subsists; and even acquires a new force from this theory of the comets: since the supply which they afford must have been long ago exhausted, if the world had existed from eternity. The matter in the comets themselves, that supplies the vapour which rises from them in every revolution to the perihelium, and
forms

forms their tails, must also have been exhausted long ere now. In general, all quantities that must be supposed to decrease or increase continually, are repugnant to the eternity of the world; since the first had been exhausted, and the last had grown into an infinite magnitude, at this time, if the world had been from eternity: and of both kinds there seem to be several sorts of quantities in the universe.

The descent of the comets into the planetary regions shews that the solid orbs, in which the planets were supposed, by the schoolmen, to move, are imaginary. And the regularity of their motions, while they are carried in very excentric orbs, in all directions, into all parts of the heavens, conspire with many other arguments to overthrow the *Cartesian* vortices.

Sir *Isaac Newton* further observes, that while the comets move in all parts of the heavens, with different directions, and in very excentric orbits, whose planes are inclined to one another in large angles; it cannot be attributed to blind fate that the planets move round the sun, and the satellites round their respective primaries, all with one direction, in orbits nearly circular, and almost in the same plane. The comets, by moving in very excentric orbits, descend with a vast velocity, and are carried quickly thro' the planetary regions, where they approach the nearest to each other, and to the planets, so as to have as little time as possible to disturb their own motions, or those of the planets. By their moving in very different planes, they are carried to a vast distance from each other in the highest parts of their orbits, or aphelia; where, because of the slowness of their motions, and the weakness of the sun's action at so great distances, their mutual actions, but for this precaution,

precaution, would produce the greatest disorders. Thus we always find, that what has, at first sight, the appearance of irregularity and confusion in nature, is discovered, on further enquiry, to be the best contrivance, and the most wise conduct.

Sir *Isaac Newton* proceeds to make some reflections on the nature of the supreme *cause*, and infers, from the structure of the visible world, that it is governed by *One Almighty and All-wise Being*, who rules the world, not as its *Soul* but as its *Lord*, exercising an absolute sovereignty over the universe, not as over his *own body* but as over his *work*; and acting in it according to his pleasure, without suffering any thing from it. What he has delivered concerning the Deity will be further explained in the next chapter.

C H A P. IX.

*Of the Supreme Author and Governor of the universe,
the True and Living God.*

I. *ARISTOTLE* concludes his treatise *de mundo*, with observing, that “to treat of the world without saying any thing of its Author would be impious;” as there is nothing we meet with more frequently and constantly in nature, than the traces of an All-governing Deity. And the philosopher who overlooks these, contenting himself with the appearances of the material universe only, and the mechanical laws of motion, neglects what is most excellent; and prefers what is imperfect to what is supremely perfect, finitude to infinity, what is narrow and weak to what is unlimited and almighty, and what is perishing to what endures for ever. Such

who attend not to so manifest indications of supreme wisdom and goodness, perpetually appearing before them wherever they turn their views or enquiries; too much resemble those antient philosophers who made night, matter, and chaos, the original of all things.

2. As we have neither ideas nor words sufficient to describe the first cause, so *Aristotle*, in the conclusion of the above-mentioned treatise, is obliged to content himself with comparing him with what is chief and most excellent, in every kind *. Thus we say he is the king or lord of all things, the parent of all his creatures, the soul of the world, or great spirit that animates the whole. Such expressions, though well meant at first, were sometimes abused afterwards; particularly, that of his being the *anima mundi*, which was apt to represent him not only as the active and self-moving principle, but likewise as passive and suffering from the actions and motions of bodies. The abstruse nature of the subject gave occasion to the latter *Platonists*, particularly to *Plotinus*, to introduce the most mystical and unintelligible notions concerning the Deity and the worship we owe to him; as when he tells us that intellect or understanding is not to be ascribed to the Deity, and that our most perfect worship of him consists, not in acts of veneration, reverence, gratitude or love; but in a certain mysterious self-annihilation, or total extinction of all our faculties. These doctrines, however absurd, have had follow-

* Καθόλου δὲ, ὅπερ ἐν νῇ κυβερνήτης, ἐν ἄρματι δὲ ἡνίοχος, ἐν χορῷ δὲ κορυφαῖος, ἐν πολεὶ δὲ νόμος, ἐν στρατοπέδῳ δὲ ἡγεμῶν τῷ θεῷ ἐν κόσμῳ· πλὴν καθ' ὅσον, τοῖς μὲν καμαλητὴρ τὸ ἄρχειν, πολυκίνητον τε καὶ πολυμέριμον· τῷ δὲ, ἄλυπον, ἀπονόν τε καὶ πάσης κεχωρισμένον σωματικῆς ἀσθενείας· ἐν ἀκινήτῳ γὰρ ἰδρυμένον πάντα κινεῖ, καὶ περιάγει ὅπου βέλεται, καὶ ὅπως, ἐν διαφόροις ἰδέαις τε καὶ φύσεσιν. Cap. 6.

ers, who, in this, as in other cases, by aiming too high, far beyond their reach, overstrain their faculties, and fall into folly or madness; contributing, as much as lies in them, to bring true piety and devotion into contempt.

3. Neither are they to be commended, who, under the pretence of magnifying the essential power of the supreme cause, make truth and falsehood entirely to depend on his will; as we observed of *Des Cartes*, Book I. Chap. 4. Such tenets have a direct tendency to introduce the absurd opinion, that intellectual faculties may be so made, as clearly and distinctly to perceive that to be true, which is really false. They judge much better, who, without scruple, measure the divine omnipotence itself, and the possibility of things, by their own clear ideas concerning them; affirming that God himself cannot make contradictions to be true at the same time; and represent the certain part of our knowledge, in some degree, as the knowledge and wisdom of the Deity imparted to us, in the views of nature which he has laid before us.

4. The sublimity of the subject is apt to exalt and transport the minds of men, beyond what their faculties can always bear: therefore, to support them, allegorical and enigmatical representations have been invented, which, in process of time, have produced the greatest abuses. When metaphorical figures and names came to be considered as realities, in place of the true God, false deities were substituted without number, and, under the pretence of devotion, a worship was paid to the most detestable characters, that tended to extinguish the notions of true worth and virtue amongst men.

5. As there are no enquiries of a more arduous nature than those that relate to the Deity, or of near so great importance to intellectual beings, that discern betwixt truth and falshood, betwixt right and wrong; so it is manifest, that there are none in which the utmost caution and soberness of thought is more requisite. Hence it is a very unpleasant prospect to observe with how great freedom, or rather licentiousness, philosophers have advanced their rash and crude notions concerning his nature and essence, his liberty and other attributes. What freedoms were taken by *Des Cartes* in describing the formation of the universe without his interposition, and in pretending to deduce from his attributes consequences that are now known to be false, we explained in the first book, almost in his own words. A manner of proceeding so unjustifiable, in so serious and important a subject, ought, one would think, to have disgusted the sober and wise part of mankind. *Spinoza*, while he carried the doctrine of absolute necessity to the most monstrous height, and surpassed all others in the weakness of his proofs as well as the impiety of his doctrines, yet affects to speak, on several occasions, in the highest terms of veneration for the Deity. Mr. *Leibnitz* and many of his disciples have likewise maintained the same doctrine of absolute necessity, extending it to the Deity himself, of whom our ideas are so inadequate, and whom it so much concerns us not to misrepresent. But Sir *Isaac Newton* was eminently distinguished for his caution and circumspection, in speaking or treating of this subject, in discourse as well as in his writings; tho' he has not escaped the reproaches of his adversaries, even in this respect. As the Deity is the supreme and first cause, from whom all other causes derive their whole force and energy, so he
thought

thought it most unaccountable to exclude *Him only* out of the universe. It appeared to him much more just and reasonable, to suppose that the whole chain of causes, or the several *series* of them, should centre in him as their source and fountain; and the whole system appear depending upon him the only independent cause.

6. The plain argument for the existence of the Deity, obvious to all and carrying irresistible conviction with it, is from the evident contrivance and fitness of things for one another, which we meet with throughout all parts of the universe. There is no need of nice or subtle reasonings in this matter: a manifest contrivance immediately suggests a contriver. It strikes us like a sensation; and artful reasonings against it may puzzle us, but it is without shaking our belief. No person, for example, that knows the principles of optics and the structure of the eye, can believe that it was formed without skill in that science; or that the ear was formed without the knowledge of sounds; or that the male and female in animals were not formed for each other, and for continuing the species. All our accounts of nature are full of instances of this kind. The admirable and beautiful structure of things for final causes, exalt our idea of the *Contriver*: the unity of design shews him to be *One*. The great motions in the system, performed with the same facility as the least, suggest his *Almighty Power*, which gave motion to the earth and the celestial bodies, with equal ease as to the minutest particles. The subtilty of the motions and actions in the internal parts of bodies, shews that his influence penetrates the inmost recesses of things, and that He is equally *active* and *present* every where. The simplicity of the laws that prevail in the world, the excellent disposition
of

of things, in order to obtain the best ends, and the beauty which adorns the works of nature, far superior to any thing in art, suggest his consummate *Wisdom*. The usefulness of the whole scheme, so well contrived for the intelligent beings that enjoy it, with the internal disposition and moral structure of those beings themselves, shew his unbounded *Goodness*. These are the arguments which are sufficiently open to the views and capacities of the unlearned, while at the same time they acquire new strength and lustre from the discoveries of the learned. The Deity's acting and interposing in the universe, shew that he *governs* it as well as formed it, and the depth of his counsels, even in conducting the material universe, of which a great part surpasses our knowledge, keep up an inward veneration and awe of this great Being, and dispose us to receive what may be otherwise revealed to us concerning him. It has been justly observed, that some of the laws of nature now known to us, must have escaped us if we had wanted the sense of seeing. It may be in his power to bestow upon us other senses of which we have at present no idea; without which it may be impossible for us to know all his works, or to have more adequate ideas of himself. In our present state, we know enough to be satisfied of our dependency upon him, and of the duty we owe to him the lord and disposer of all things. He is not the object of sense; his essence, and indeed that of all other substances, is beyond the reach of all our discoveries; but his attributes clearly appear in his admirable works. We know that the highest conceptions we are able to form of them are still beneath his real perfections; but his power and dominion over us, and our duty towards him, are manifest.

7. Sir *Isaac Newton* is particularly careful, always to represent him as a free agent; being justly appre-

hensive of the dangerous consequences of that doctrine which introduces a fatal or absolute necessity presiding over all things. He made the world, not from any necessity determining him, but when he thought fit: matter is not infinite or necessary, but he created as much of it as he thought proper: he placed the systems of the fixed stars at various distances from each other, at his pleasure: in the solar system, he formed the planets of such a number, and disposed them at various distances from the sun, as he pleased: he has made them all move from west to east, though it is evident from the motions of the comets, that he might have made them move from east to west. In these and other instances, we plainly perceive the vestiges of a wise agent, but acting freely and with perfect liberty.

As caution was a distinguishing part of Sir *Isaac Newton's* character, but no way derogatory from his penetration and the acuteness and sublimity of his genius; so we have particular reason on this occasion to applaud it, and to own that his philosophy has proved always subservient to the most valuable purposes, without ever tending to hurt them.

8. As in treating of this unfathomable subject we are at a loss for ideas and words, in any tolerable degree, adequate to it, and, in order to convey our notions with any strength, are obliged to have recourse to figurative expressions, as was observed already; so it is hardly possible for the most cautious to make use of such as may not be liable to exceptions, from angry and captious men. Sir *Isaac Newton*, to express his idea of the divine *Omnipresence*, had said, that the Deity perceived whatever passed in space fully and intimately, as it were in his *Sensorium*. A clamour was raised by his adversaries, as if he meant that

that space was to the Deity what the *Sensorium* is to our minds. But whoever considers this expression without prejudice, will allow that it conveys a very strong idea of the intimate presence of the Deity every where, and of his perceiving whatever happens in the completest manner, without the use of any intermediate agents or instruments, and that Sir *Isaac* made use of it with this view only ; for he very carefully guards against our imagining that external objects act upon the Deity, or that he suffers any passion or reaction from them. It is commonly supposed that the mind is intimately conscious of the impressions upon the sensorium, and that it is immediately present there, and there only ; and as we must derive our ideas of the attributes of God from what we know of our minds, or of those of others, in the best manner we can, by leaving out all imperfection and limitation ; so it was hardly possible to have represented to us the divine *Omnipresence* and *Omniscience* in a stronger light, than by this comparison. But the fondness of philosophers for their favourite systems, often irritates them against those, who, in the pursuit of truth, innocently overturn their doctrines, and provokes them to catch at any occasion of finding fault.

9. But the greatest clamour has been raised against Sir *Isaac Newton*, by those who have imagined that he represented *infinite space* as an attribute of the Deity, and that He is present in all parts of space by diffusion. The truth is, no such expressions appear in his writings : he always thought and spoke with more veneration of the divinity than to allow himself such liberties. On the contrary, he tells * us that
 “ the

* *Æternus est & infinitus, omnipotens & omnisciens id est, durat ab æterno in æternum, & adest ab infinito in infinitum: om-*

“ the Deity endures from eternity to eternity, and is present from infinity to infinity ; but that he is not eternity or infinity, space or duration.” He adds indeed, that as the Deity exists necessarily, and by the same necessity exists every where and always, he constitutes space and duration : but it does not appear that this expression can give any just ground of complaint ; for it is saying no more than that since he is essentially and necessarily present in all parts of space and duration, these of consequence, must also necessarily exist.

10. This idea is so far from giving any just ground of complaint, that it accounts for the necessary existence of space, in a way worthy of the Deity, and suggests the noble improvement we may make of this doctrine, which lies so plain and open before us. Sir *Isaac Newton* is so far from representing the Deity as present in space by diffusion (as some have advanced very unjustly) that he expressly tells us * there are successive parts in duration, and co-existent parts in space. But that neither are found in the soul or principle of thought which is in man ; and that far less can they be found in the divine substance. As man is one and the same in all the periods of his life, and thro' all the variety of sensations and passions to which he is subject ; much more must we allow the supreme Deity to be one and the same in all time, and

nia regit, & omnia cognoscit, quæ sunt aut fieri possunt. Non est æternitas & infinitas, sed æternus & infinitas ; non est duratio & spatium, sed durat & adest. Durat semper, & adest ubique, & existendo semper & ubique, durationem & spatium constituit. *Neut. Princip Scholium Generale*, pag. 528.

* Partes dantur successivæ in duratione, coexistentes in spatio, neutræ in persona hominis seu principio ejus cogitante ; & multo minus in substantia cogitante Dei. Omnis homo quatenus res sentiens, est unus & idem homo durante vitâ suâ in omnibus & singulis sensuum organis. Deus est unus & idem Deus semper & ubique, *ibid.*

in all space, free from change and external influence. He adds, that the Deity is present every where, *non per virtutem solam sed etiam per substantiam, sed modo prorsus incorporeo, modo nobis penitus ignoto*. It is plain therefore, that he was far from meaning that the Deity was present every where by the diffusion of his substance, as a body is present in space by having its parts diffused in it. Nor is it surprizing that we should be at a loss to give a satisfactory account of the manner of God's omnipresence. Our knowledge of things penetrates not into their substance: we perceive only their figure, colour, external surface, and the effects they have upon us, but no sense, or act of reflection, discovers to us their substance; and much less is the divine substance known to us. As a blind man knows not colours, and has no idea of the sensation of those who see, so we have no notion how the Deity knows and acts.

II. His existence and his attributes are, in a sensible and satisfactory manner, displayed to us in his works; but his essence is unfathomable. From our existence and that of other contingent beings around us, we conclude that there is a *first cause*, whose existence must be necessary, and independent of any other being; but it is only *a posteriori* that we thus infer the necessity of his existence, and not in the same manner that we deduce the necessity of an eternal truth in geometry, or the property of a figure from its essence: nor is it even with that direct self-evidence which we have for the necessary existence of space. We mention this only to do justice to Sir *Isaac Newton's* notion, when he suggests that the necessary existence of space is relative to the necessary existence of the Deity. Philosophers have had always disputes about infinite space and duration; and probably their contests on these subjects will never

have an end: all we want to represent is only, that what is so briefly and modestly advanced by this great man on those subjects, is, at least, as rational and worthy of the Deity, and as well founded in true philosophy, as any of their schemes; though it must be expected that the best account we can form of matters of so arduous a nature, will be liable to difficulties and objections. As for those who will not allow space to be any thing real, we observed above that the reality of motion, which is known by experience, argues the reality of absolute space; without admitting which, we should have nothing but confusion and contradictions in natural philosophy. Many other arguments, particularly those drawn from the axiom, *non entis nulla sunt attributa*, for the reality of space, whose parts are subject to mensuration and various relations, have been treated of largely by others.

12. We observed above, that as the Deity is the first and supreme cause of all things, so it is most unaccountable to exclude him out of nature, and represent him as an *intelligentia extramundana*. On the contrary, it is most natural to suppose him to be the chief mover throughout the whole universe, and that all other causes are dependent upon him; and conformable to this is the result of all our enquiries into nature; where we are always meeting with powers that surpass mere mechanism, or the effects of matter and motion. The laws of nature are constant and regular, and, for ought we know, all of them may be resolved into one general and extensive power; but this power itself derives its properties and efficacy, not from mechanism, but, in a great measure, from the immediate influences of the first mover. It appears, however, not to have been his intention, that the present state of things should continue for ever without alteration; not only from
what

what passes in the moral world, but from the phænomena of the material world likewise ; as it is evident that it could not have continued in its present state from eternity.

13. The power of gravity, by which the celestial bodies persevere in their revolutions, penetrates to the centres of the sun and planets without any diminution of virtue, and is extended to immense distances, decreasing in a regular course. Its action is proportional to the quantity of solid matter in bodies, and not to their surfaces, as is usual in mechanical causes : this power therefore, seems to surpass mere mechanism. But, whatever we say of this power, it could not possibly have produced, at the beginning, the regular situation of the orbs and the present disposition of things. Gravity could not have determined the planets to move from west to east in orbits nearly circular, almost in the same plane ; nor could this power have projected the comets with all variety of directions. If we suppose the matter of the system to be accumulated in the centre by its gravity, no mechanical principles, with the assistance of this power of gravity, could separate the vast mass into such parts as the sun and planets, and, after carrying them into their different distances, project them in their several directions, preserving still the equality of action and reaction, or the state of the centre of gravity of the system. Such an exquisite structure of things could only arise from the contrivance and powerful influences of an intelligent, free, and most potent agent. The same powers, therefore, which at present govern the material universe, and conduct its various motions, are very different from those which were necessary to have produced it from nothing, or to have disposed it in the admirable form in which it now proceeds.

14. As we cannot but conceive the universe, as depending on the first cause and chief mover, whom it would be absurd, not to say impious, to exclude from acting in it; so we have some hints of the manner in which he operates in nature, from the laws which we find established in it. Though he is the source of all efficacy, yet we find that place is left for second causes to act in subordination to him; and mechanism has its share in carrying on the great scheme of nature*. The establishing the equality of action and reaction, even in those powers which seem to surpass mechanism, and to be more immediately derived from him, seems to be an indication that those powers, while they derive their efficacy from him, are however, in a certain degree, circumscribed and regulated in their operations by mechanical principles; and that they are not to be considered as mere immediate volitions of his (as they are often represented) but rather as instruments made by him, to perform the purposes for which he intended them. If, for example, the most noble phænomena in nature be produced by a rare elastic *ætherial medium*, as Sir *Isaac Newton* conjectured, the whole efficacy of this medium must be resolved into his power and will, who is the supreme cause. This, however, does not hinder, but that the same medium may be subject to the like laws as other elastic fluids, in its actions and vibrations; and that, if its nature was better known to us, we might make curious and useful discoveries concerning its effects, from those laws. It is easy to see that this conjecture no way derogates from the government and influences of

* Ἀλλὰ τοῦτο ἦν τὸ θεϊότατον, τὸ μετὰ ραγώνης καὶ ἀπλῆς κινήσεως παντοδατὰς ἀποτελεῖν ἰδέας, ὥσπερ ἀμελεῖ δρῶσιν οἱ μηχανοποιοὶ διὰ μιᾶς ὀργάνου σχαστηρίας, πολλὰς καὶ ποικίλας ἐνεργείας ἀποτελεῦντες. *Aristot. ubi supra.*

the Deity ; while it leaves us at liberty to pursue our enquiries concerning the nature and operations of such a medium. Whereas they who hastily resolve those powers into immediate volitions of the supreme cause, without admitting any intermediate instruments, put an end to our enquiries at once ; and deprive us of what is probably the most sublime part of philosophy, by representing it as imaginary and fictitious : by which means, as we observed above *, they hurt those very interests which they appear so sanguine to promote ; for the higher we rise in the scale of nature, towards the supreme cause, the views we have from philosophy appear more beautiful and extensive. Nor is there any thing extraordinary in what is here represented concerning the manner in which the Supreme Cause acts in the universe, by employing subordinate instruments and agents, which are allowed to have their proper force and efficacy ; for this we know is the case in the common course of nature ; where we find gravity, attraction, repulsion, &c. constantly combined and compounded with the principles of mechanism : and we see no reason why it should not likewise take place in the more subtile and abstruse phænomena and motions of the system.

15. It has been demonstrated by ingenious men, that great revolutions have happened in former times on the surface of the earth, particularly from the phænomena of the *Strata* ; which sometimes are found to lie in a very regular manner, and sometimes to be broken and separated from each other to very considerable distances, where they are found again in the same order ; from the impressions of plants left upon the hardest bodies dug deep out of

* Book I. Chap. 5. Sect. 3.

the earth, and in places where such plants are not now found to grow; and from bones of animals both of the land and sea, discovered some hundreds of yards beneath the present surface of the earth, and at very great distances from the sea. Some philosophers explain these changes by the revolutions of comets, or other natural means: but as the Deity has formed the universe dependent upon himself, so as to require to be altered by him, though at very distant periods of time; it does not appear to be a very important question to enquire whether these great changes are produced by the intervention of instruments, or by the same immediate influences which first gave things their form.

16. We cannot but take notice of one thing, that appears to have been designed by the author of nature: he has made it impossible for us to have any communication from this earth with the other great bodies of the universe, in our present state; and it is highly probable, that he has likewise cut off all communication betwixt the other planets, and betwixt the different systems. We are able, by telescopes, to discover very plainly mountains, precipices and cavities in the moon: but who tread those precipices, or for what purposes those great cavities (many of which have a little elevation in the middle) serve, we know not; and are at a loss to conceive how this planet, without any atmosphere, vapours, or seas, (as is now the common opinion of astronomers) can serve for like purposes as our earth. We observe sudden and surprizing revolutions on the surface of the great planet *Jupiter*, which would be fatal to the inhabitants of the earth. We observe, in them all, enough to raise our curiosity, but not to satisfy it. From hence, as well as from the state of the moral world, and many other
confi-

considerations, we are induced to believe, that our present state would be very imperfect without a subsequent one; wherein our views of nature, and of its great author, may be more clear and satisfactory. It does not appear to be suitable to the wisdom that shines throughout all nature, to suppose that we should see so far, and have our curiosity so much raised concerning the works of God, only to be disappointed at the end. As man is undoubtedly the chief being upon this globe, and this globe may be no less considerable, in the most valuable respects, than any other in the solar system, and this system, for ought we know, not inferior to any in the universal system; so, if we should suppose man to perish, without ever arriving at a more complete knowledge of nature, than the very imperfect one he attains in his present state; by analogy, or parity of reason, we might conclude, that the like desires would be frustrated in the inhabitants of all the other planets and systems; and that the beautiful scheme of nature would never be unfolded, but in an exceedingly imperfect manner, to any of them. This, therefore, naturally leads us to consider our present state as only the dawn or beginning of our existence, and as a state of preparation or probation for farther advancement: which appears to have been the opinion of the most judicious philosophers of old. And whoever attentively considers the constitution of human nature, particularly the desires and passions of men, which appear greatly superior to their present objects, will easily be persuaded that man was designed for higher views than of this life. These the author of nature may have in reserve to be opened up to us, at proper periods of time, and after due preparation. Surely it is in his power to grant us a far greater improvement of the faculties we already possess, or even to endow us with new

faculties,

faculties, of which, at this time, we have no idea, for penetrating farther into the scheme of nature, and approaching nearer to himself, the first and supreme cause. We know not how far it was proper or necessary that we should not be let into knowledge at once, but should advance gradually, that, by comparing new objects, or new discoveries, with what was known to us before, our improvements might be more complete and regular; or how far it may be necessary or advantageous, that intelligent beings should pass through a kind of infancy of knowledge. For new knowledge does not consist so much in our having access to a new object, as in comparing it with others already known, observing its relations to them, or discerning what it has in common with them, and wherein their disparity consists. Thus our knowledge is vastly greater than the sum of what all its objects separately could afford; and when a new object comes within our reach, the addition to our knowledge is the greater, the more we already know; so that it increases not as the new objects increase, but in a much higher proportion. * * *

F I N I S.

